International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

Analysis of MHD Convective Flow Along A Moving Semi Vertical Plate With Internal Heat Generation

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Abstract **- The objective of this work is to investigate steady convection flow of a viscous incompressible electrically conducting fluid along a semi infinite vertical plate in the presence of internal heat generation and a convective surface boundary condition. The non-linear partial differential equations, governing the problem under consideration, have been transformed using similarity transformation into a system of ordinary differential equations, which is solved numerically applying fourth order Runge-Kutta integration scheme together with shooting iteration technique. The effects of various flow parameters affecting the flow field are discussed with the help of figures and tables. Numerical data for the local skin friction coefficient and the local Nusselt number have been tabulated for various values of parametric conditions. Graphical results for the velocity and temperature profiles based on the numerical solutions are presented and discussed.**

Key words- Internal heat generation, MHD, Semi vertical plate, Boundary layer flow, Local biot number.

INTRODUCTION

The phenomenon of convective flow with internal heat transfer under the influence of magnetic field has received considerable attention of several workers because of its varied applications in different field of science, technology and industry. Although heat transfer analysis is most important for the proper sizing of fuel elements in the nuclear reactors cores to prevent burnout. Kao (1976) considered locally nonsimilar solution for laminar free convection adjacent to vertical wall. Sharma (1991) solved free convection effects on the flow of an ordinary viscous fluid past and infinite vertical porous plate with constant suction and constant heat flux. Effects of transverse magnetic field, Prandtl number and Reynolds number on non-Darcy mixed convective flow of an incompressible viscous fluid past a porous vertical flat plate in a saturated porous medium studied by Takhar and Beg (1997). Crepeau and Clarksean (1997) discussed similarity solutions of natural convection with internal heat generation. Hossain, Khanafer and Vafai (2001) considered the effect of radiation on free convention flow of fluid with variable viscosity from a porous vertical plate. Alam, Rahman and Samad (2006) reported numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Sharma and Singh (2009) studied numerical solution of transient MHD free convection flow of an incompressible viscous fluid along an inclined plate with ohmic dissipation. Ibrahim and Makinde (2010) discussed chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Makinde (2011) studied similarity solutions for a natural convection from a moving vertical plate with internal heat generation and a convective boundary condition. Sarmal and Hazarika (2011) considered effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field. Makinde (2011) reported similarity solutions for a natural convection from a moving vertical plate with internal heat generation and a convective boundary condition. Aiyesimi, Abah and Okedayo (2012) disussed radiative effects on the unsteady double diffusive MHD boundary layer flow over a stretching vertical plate.

Aim of the paper is to investigate steady free convection flow of an incompressible viscous electrically conducting fluid along a semi vertical plate in the presence of interior heat generation and a convective surface boundary condition.

Mathematical analysis

Consider two dimensional steady laminar incompressible and natural convection boundary layer flow over the right surface of the vertical plate moving with uniform velocity U_0. A uniform magnetic field of strength B_0 is applied normal to the direction of fluid flow. It is assumed that the left surface of the plate in contact with a hot fluid while quiescence cold fluid at temperature T_∞ on the right surface of the plate. The hot fluid is at temperature T_f which provides a heat transfer coefficient h_f and cold fluid generates heat internally at the volumetric rate q. Under the above assumptions, the governing equations are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad , \qquad (1)$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\sigma\frac{B_0^2}{\rho}u \quad , \quad (2)$$

Energy equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}+q \quad , \quad (3)$$

where u and v are the x and y components of the velocity, respectively; T is fluid temperature, υ is the kinematics viscosity of the fluid, ρ is the fluid density, c_p is the specific heat at constant pressure, κ is the thermal conductivity of the fluid, and β is the thermal expansion coefficient.

The boundary conditions are

$$u(x,0)=U_0, v(x,0)=0, -k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right]$$

(4)

$$u(x,\infty)=0, T(x,\infty)=T_\infty \quad (5)$$

Introducing dimensionless variables, parameters and similarity variable

$$\eta=\frac{y}{x}\sqrt{Re}\,, u=U_0 f'\,, v=\frac{\upsilon}{2x}\sqrt{Re}(\eta f'-f)\,, \theta=\frac{T-T_\infty}{T_f-T_\infty},$$

$$Bi_x=\frac{h_f}{k}\left(\frac{\upsilon x}{U_0}\right)^{1/2}, Gr_x=g\beta x\frac{(T_f-T_\infty)}{U_0^2}, \lambda_x=\frac{qx^2e^\eta}{\kappa Re(T_f-T_\infty)},$$

$$\Pr=\frac{\mu c_p}{\kappa}, M=B_0\sqrt{\frac{\sigma x}{\rho U_0}}, Re=\frac{U_0 x}{\upsilon} \quad (6)$$

into the equations (1) to (3), we get

$$f'''+\frac{1}{2}ff''+Gr_x\theta-M^2 f'=0 \quad , \quad (7)$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\lambda_x e^{-\eta}=0 \quad , \quad (8)$$

where the prime denotes differentiation with respect to η and is the Bi_x Biot number, Gr_x is the Grashof number, λ_x is the internal heat generation parameter, $\Pr$ is the Prandtl number, Re is the Reynolds number and M is the magnetic parameter.

The boundary conditions in dimensionless form are

$$f(0)=0, f'(0)=1, \theta'(0)=-Bi_x\left[1-\theta(0)\right], f'(\infty)=0 \quad (9)$$

Moreover, equations (7) and (8) with the boundary condition (9) will definitely produce a local similarity solution for the problem. In order to have a true similarity solution, the parameters Gr_x, Bi_x and λ_x must be constant and this condition will be satisfied if we assume

$$h_f=cx^{-1/2}, \beta=mx^{-1}, q=lx^{-1}e^{-\eta} \quad (10)$$

where c, m and l are constants but have the appropriate dimensions. Substituting (10) into the parameters Gr_x, Bi_x and λ_x, we obtain

$$Bi=\frac{c}{\kappa}\sqrt{\frac{\upsilon}{U_0}}, Gr=\frac{mg(T_f-T_\infty)}{U_0^2}, \lambda=\frac{l\upsilon}{\kappa U_0(T_f-T_\infty)} \quad (11)$$

The equations (7) and (8) are a set of coupled differential equations and solved using with the shooting iteration technique under the boundary condition (9). The numerical values of surface temperature $\theta(0)$, skin-friction coefficient $f''(0)$ and Nusslet number $\theta'(0)$ for various values of physical parameters are derived, discussed and presented through table.

Results and discussion

Effects of physical parameters on velocity profiles

Figure1 shows that the velocity boundary layer thickness decreases fast with an increase in the intensity of magnetic field as the magnetic field presents a damping effect on the velocity field by creating a drag force that opposes the fluid motion. Figure 2 and 3 show that the velocity boundary layer thickness decreases with increase in the values of the Prandtl number or intensity of Biot number (Bi_x).

Figures 4 and 5 depict that the velocity boundary layer thickness increases with increase in the values of the Grashof number (Gr_x) or internal heat generation parameter (λ_x).

Effects of physical parameters on temperature profiles

Figure 6 depicts that fluid temperature decreases with increase in the intensity of magnetic field. Figures 7 and 8 show that the fluid temperature decreases with increase in the values of the Prandtl number or intensity of local Biot number (Bi_x).It is seen from figure 9 that fluid temperature decreases with the increase of Grashof number due to an increase in the intensity of buoyancy force. An increase in internal heat generation parameter causes increase in the fluid temperature as shown in figure 10.

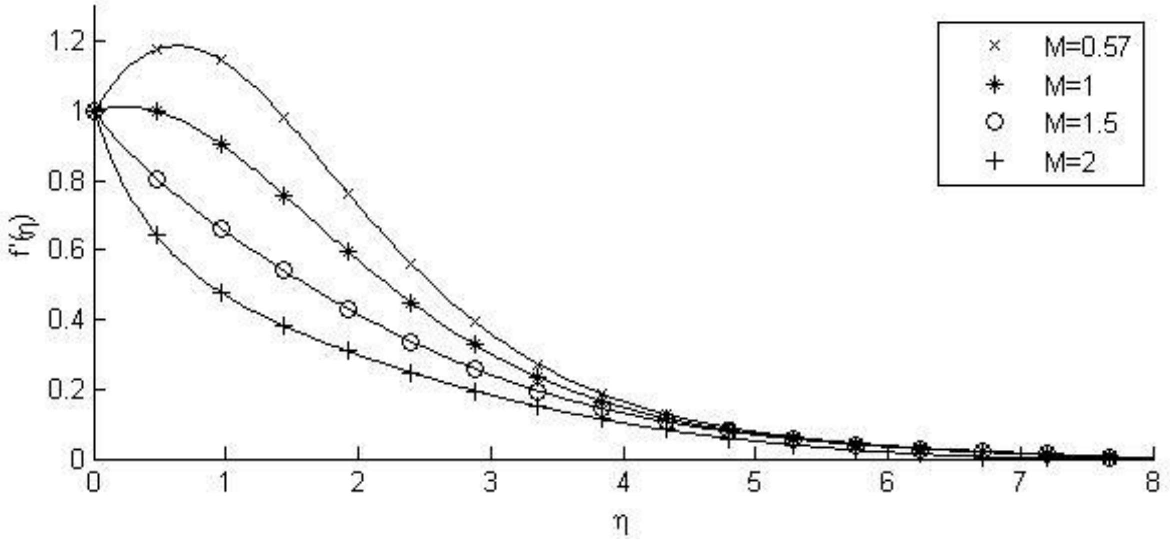

Figure 1. Velocity profiles versus η when $\Pr = 0.72, Bi_x = 0.1, Gr_x = 0.1, \lambda_x = 10$

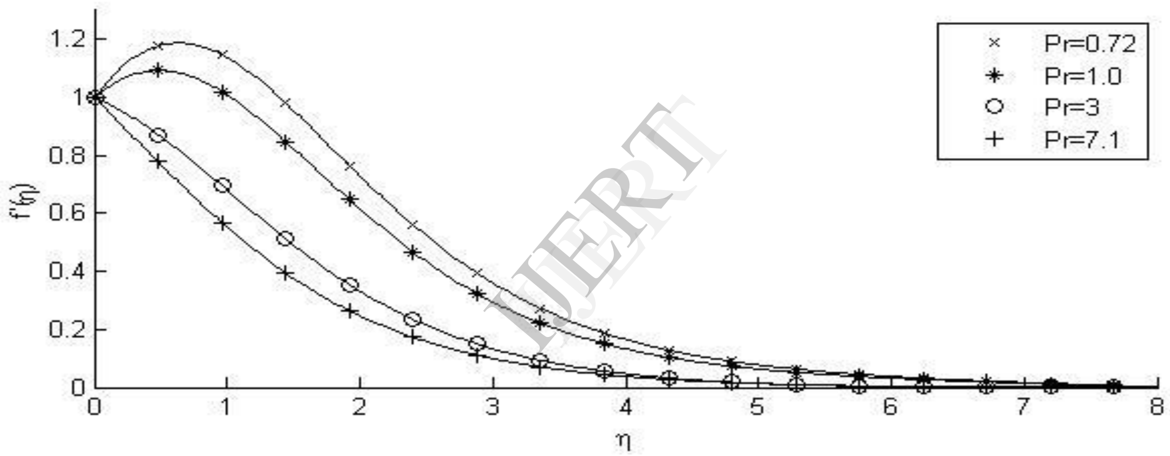

Figure 2. Velocity profiles versus η when $M = 0.57, Bi_x = 0.1, Gr_x = 0.1, \lambda_x = 10$

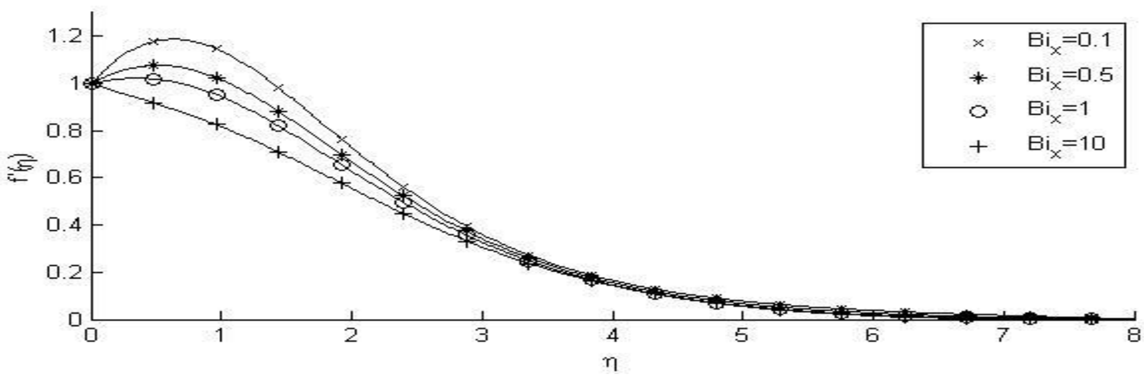

Figure 3. Velocity profiles versus η when $M = 0.57, \Pr = 0.72, Gr_x = 0.1, \lambda_x = 10$

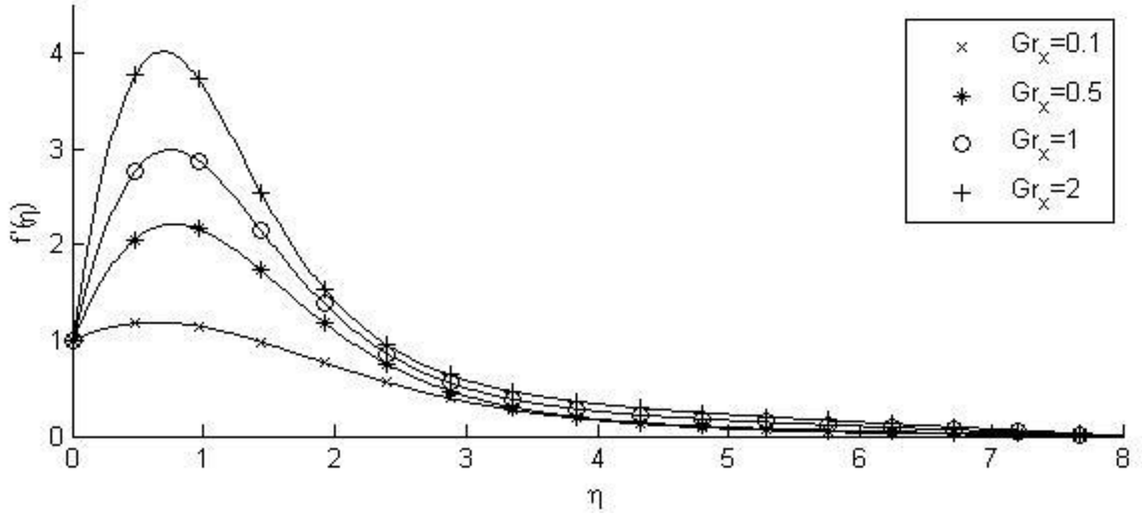

Figure 4. Velocity profiles versus η when $\Pr = 0.72, Bi_x = 0.1, M = 0.57, \lambda_x = 10$

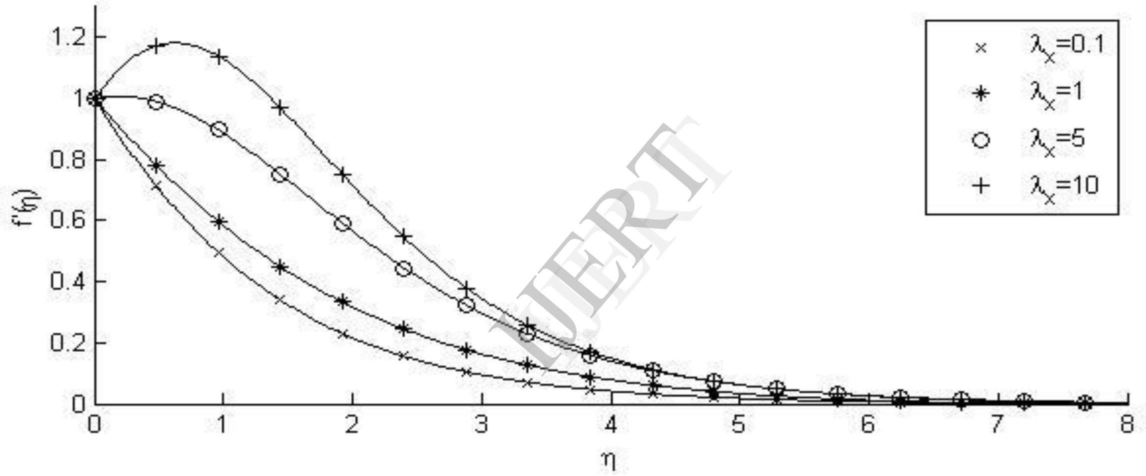

Figure5. Velocity profiles versus η when $\Pr = 0.72, Bi_x = 0.1, Gr_x = 0.1, M = 0.57$

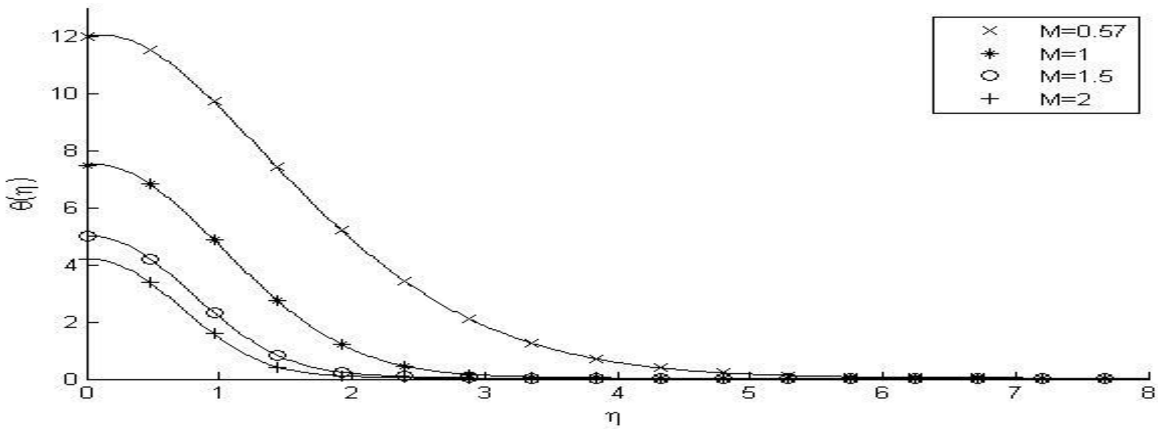

Figure 6.Temperature profiles versus η when $\Pr = 0.72, Bi_x = 0.1, Gr_x = 0.1, \lambda_x = 10$

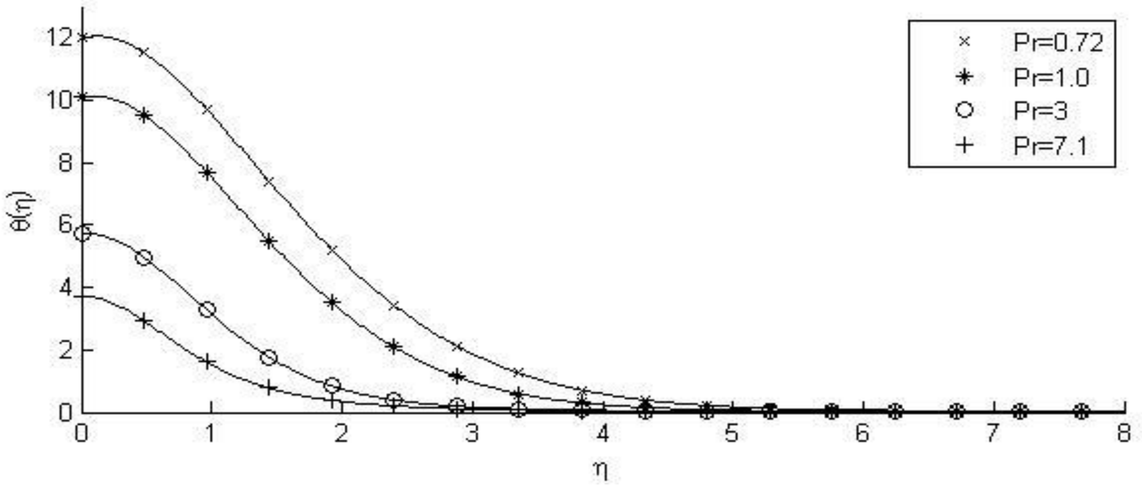

Figure7. Temperature profiles versus η when $M = 0.57, Bi_x = 0.1, Gr_x = 0.1, \lambda_x = 10$

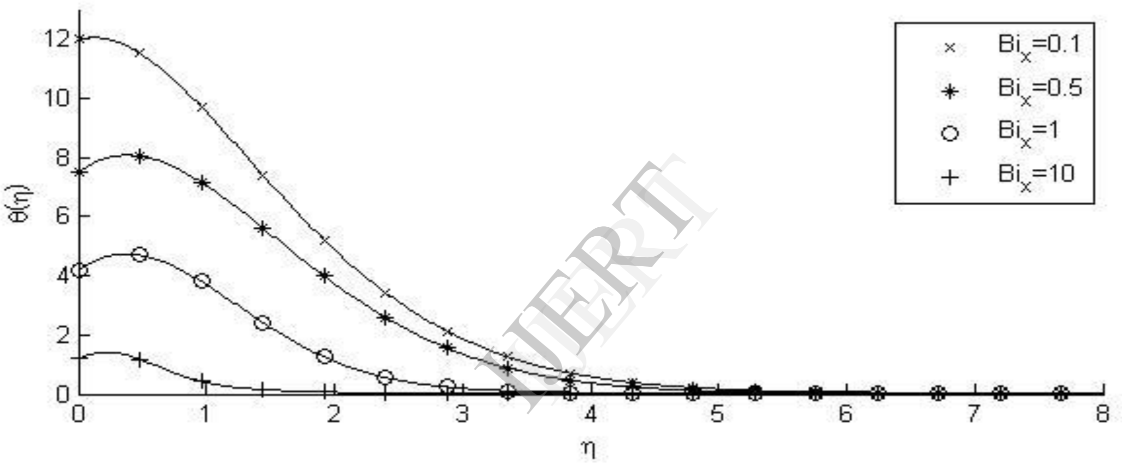

Figure 8. Temperature profiles versus η when $M = 0.57, \Pr = 0.72, Gr_x = 0.1, \lambda_x = 10$

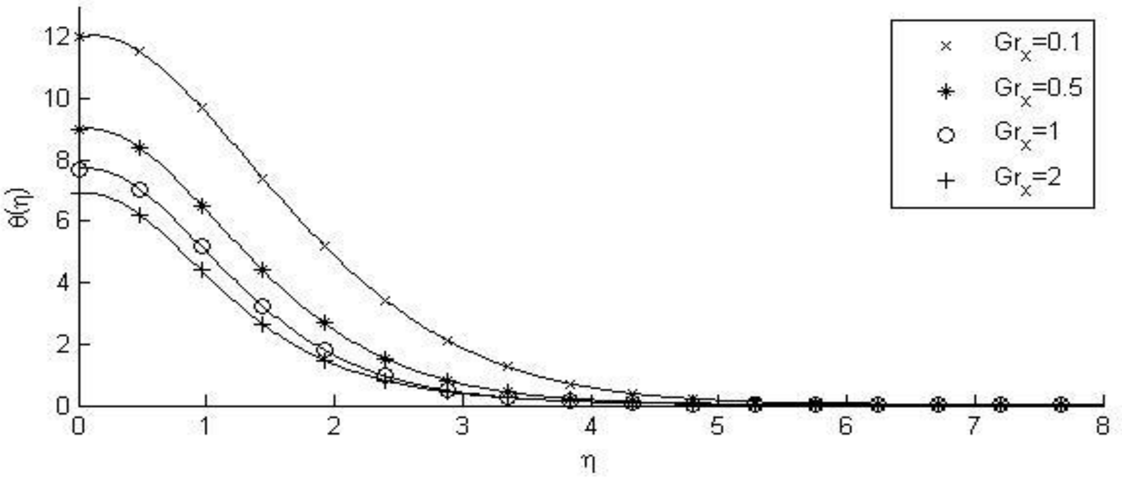

Figure 9. Temperature profiles versus η when $\Pr = 0.72, Bi_x = 0.1, M = 0.57, \lambda_x = 10$

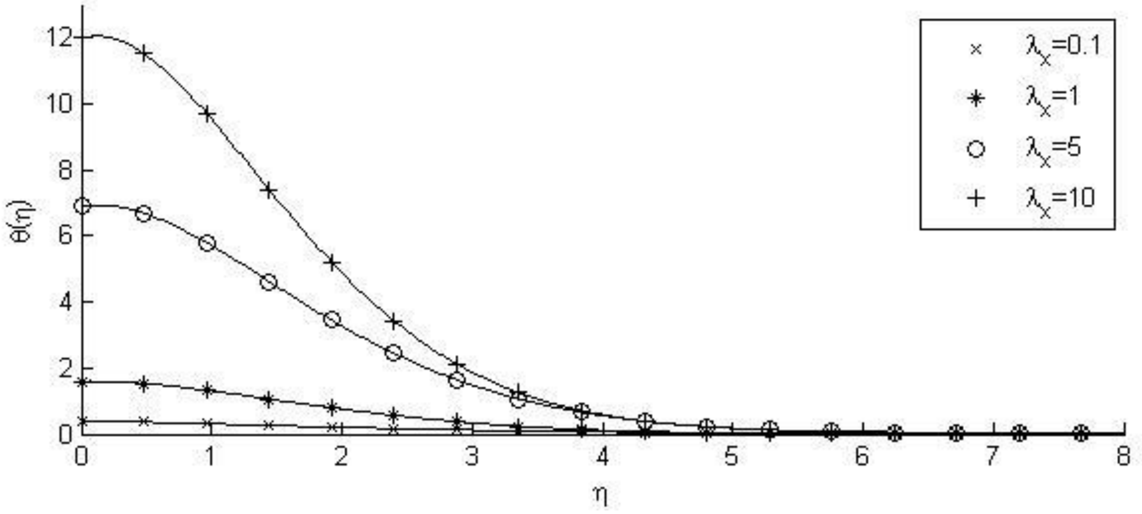

Figure10. Temperature profiles versus η when $\Pr = 0.72, Bi_x = 0.1, Gr_x = 0.1, M = 0.57$

Table1. Numerical values of $f''(0), \theta'(0)$ and $\theta(0)$ for different values of parameters.

M	Bi_x	Gr_x	Pr	λ_x	$f''(0)$	$\theta'(0)$	$\theta(0)$
0.57	0.1	0.1	0.72	10	0.64010	1.10012	12.0012
1.00	0.1	0.1	0.72	10	1.93502	0.65019	7.50190
2.00	0.1	0.1	0.72	10	7.82950	0.31980	4.1980
0.57	0.1	0.5	0.72	10	3.26410	0.80103	9.01030
0.57	0.1	1.0	0.72	10	5.53501	0.67090	7.70901
0.57	0.1	0.1	3.00	10	-0.19190	0.55938	6.59388
0.57	0.1	0.1	7.10	10	-0.45196	0.28514	3.85142
0.57	0.1	0.1	0.72	5	0.106220	0.59091	6.90914
0.57	0.1	0.1	0.72	1	-0.33420	0.06017	1.60172
0.57	1.0	0.1	0.72	10	2.13121	3.18112	4.18112

CONCLUSION

In this paper we have studied MHD natural convection internal heat generation on boundary layer flow over a vertical plate with a convective surface boundary condition with the surrounding in the presence of magnetic field. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results for the prescribed local skin friction coefficient and Nusslet number at the plate surface are presented and discussed. We can conclude that

1. Skin friction coefficient at the plate surface increases rapidly with an increase in intensity of magnetic field but Nusslet number decreases slightly with an increase in intensity of magnetic field.

2. Skin friction coefficient increases with the increase of magnetic number, Biot number, Grashoff number or heat generation parameter.

3. Skin friction coefficient decreases with the increase of the Prandtl number.

4. The rate of heat transfer decreases due to increase in magnetic number, Grashoff number, Prandtl number or Biot number.

5. The rate of heat transfer increases due to increase in heat generation parameter.

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