# Analysis of Composite Laminate Plates under UD Load using CLPT 

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#### Abstract

In this paper the strength of a composite material configuration is obtained from the properties of the constituent laminate by using Classical Laminate Plate Theory. For the analysis simply supported 4 Ply Orthotropic laminate plate of Boron/Epoxy with uniformly distributed load is considered. C-Programme was developed to calculate the deflection, stress, strain and failure load of the laminate of different configurations. The strengths are calculated for Uniformly Distributed Loads by varying the dimensions and configurations of laminate. From the analysis it is found that as the number of layers of a laminate increases in a given thickness, the strength of the laminate increases. It is also found that the location and angles of the principle material direction affect the strength of rectangular laminate, but it is not affecting the strength of a square orthotropic laminate plate. Keywords:Composite, Laminate, Classical Laminate Plate Theory.


## 1. Introduction:

A composite is a combination of two or more dissimilar materials on a macroscopic scale, with the aim to get properties better than the individual constituentmaterialslike Strength, stiffness, corrosion resistance, wear resistance, fatigue life thermal insulation, thermal conductivity, acoustical insulation, temperature dependent behavior etc.The Properties of a composite material depend on the properties of the constituents, geometry, and distribution of the phases. One of the most important parameters is the volume (or weight) fraction of reinforcement, or fiber volume ratio. We can find the applications of composite in automobiles as vehicle bodies, engine components, racing
boat bodies, propellershaft and hulls for small boats,civilian and military aircraft components, rocket components, heat shields for satellites and hypersonic aeroplanes, sports goods, musical instruments, safety protections etc.

Composite materials can be broadly classified into phase composites and layered composites. Layered composites are called laminates. Depending on the number of layers they are classified as Uni-layer composites and multilayer composites. A composite laminate is two or more laminae bonded together to act as an integral structural element. Laminae principal material directions are oriented to produce a structural element capable of resisting load in several directions.

Each layer of a unidirectional composite is known as layer, ply or lamina. The behaviourof aLaminae is the basic building block for laminate construction. [1]


Fig. 1 Schematic diagram of unidirectional composite.

It consists of parallel fibers embedded in a matrix. The direction parallel to fibers is known as longitudinal direction (x-axis). The direction perpendicular to the fibers is called the transverse direction (along y-axis).

A lamina has the strongest properties in the longitudinal direction.

| Nomenclature |  |
| :---: | :---: |
| A | :Area of plate, $\mathrm{m}^{2}$ |
| $\mathrm{A}_{\mathrm{ij}}$ | Extensionalstiffnesses |
| A | :Length of plate in longitudinal direction, $m$ |
| $\mathrm{B}_{\mathrm{ij}}$ | :Couplingstiffnesses |
| b | :Length of plate in transverse direction, $m$ |
| $\mathrm{D}_{\mathrm{ij}}$ | :Bendingstiffnesses, KN-m |
| $\mathrm{E}_{11}$ | $\mathrm{E}_{1}$ :Longitudinal young's modulus of a laminae, Gpa |
| $\mathrm{E}_{22}$ | $\mathrm{E}_{2}$ :Transverse young's modulus of a laminae, Gpa |
| $\mathrm{E}_{\mathrm{f}}$ | :Young's modulus of fiber, Gpa |
| $\mathrm{E}_{\mathrm{m}}$ | :Young's Modulus of matrix,Gpa |
| fx | :Stress in longitudinal direction, |
| fy | :Stress in transverse direction, |
| fxy | :Shear stress, |
| P | :Uniformly distributed load, $\mathrm{N} / \mathrm{m}^{2}$ |
| $\mathrm{Q}_{\mathrm{ij}}$ | :Reducedstiffnesses, GPa |
| S | :In plane shear strength, Gpa |
| t | :thickness of each ply of the laminate, mm |
| $\mathrm{X}_{4}$ | :Longitudinal tensile strength, Gpa |
| $\mathrm{X}_{\mathrm{c}}$ | :Longitudinal compressive strength, Gpa |
| $\mathrm{Y}_{\mathrm{c}}$ | :Transverse compressive strengths, Gpa |
| $\mathrm{Y}_{\mathrm{t}}$ | :Transverse tense strengths, Gpa |
| $v_{\text {f1 }}$ | :Fiber volume ratio in longitudinal direction |
| $v_{\text {f } 2}$ | :Fiber volume ratio in transverse direction |
| $v_{\text {m } 1}$ | :Matrix volume ratio in longitudinal direction |
| $v_{\text {m } 2}$ | :Matrix volume ratio in transverse direction |
| $v_{12}$ | :Majorpoisson Ratio |
| $\mathrm{v}_{21}$ | :Minorpoisson Ratio |
| $\omega$ | :Deflection, mm |
| $\rho$ | :Density, kg/m ${ }^{3}$ |

Volume fraction of the matrix
$\left(\mathrm{V}_{\mathrm{m})}=\frac{v_{m}}{v_{m}+v_{f}}=\frac{v_{m}}{v_{c}}\right.$
Similarly, volme fraction of the fibres

$$
\begin{equation*}
\left(\mathrm{V}_{\mathrm{f}}\right) \quad=\frac{v_{f}}{v_{c}} \tag{2}
\end{equation*}
$$

## 2. Theory:

### 2.1 Structural composites:

Structural Composites are used for structural applications. These are classified into Laminar andSandwichcomposites.Laminar Composites are two-dimensional sheets layered one on another.[1]


Fig.2Laminar composites.

### 2.2 Orthotropy or Anisotropy :

The materials which of their properties at a point vary with direction.The composite materials are Orthotropic in nature i.e., they exhibit different properties in three different directions. These three mutually perpendicular axes, called principal axes of material symmetry.

### 2.3 Laminate designations:

Unidirectional 6-ply : [0/0/0/0/0/0] $=\left[0_{6}\right]$
Crossplysymmetric : [0/90/90/0] $=[0 / 90]_{\mathrm{s}}$

### 2.4 Deflection of simply supported laminated plates under uniformly distributed lateral load:

Consider the general Class of laminated rectangular Plates that are simply supported along edges $\mathrm{x}=0, \mathrm{x}=\mathrm{a}, \mathrm{y}=0$ and $\mathrm{y}=\mathrm{b}$ and subjected to a distributed lateral load $\mathrm{p}(\mathrm{x}, \mathrm{y})$.

The lateral load can be expanded in a double Fourier series
$\mathrm{P}(\mathrm{x}, \mathrm{y})=\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \mathrm{P}_{\mathrm{mn}} \sin \frac{m \pi x}{a} \cdot \sin \frac{n \pi y}{b}---(3)$


Fig. 3 showing simply supported rectangular plate subjected to U.D. Load

### 2.4 Specially orthotropic laminates :

A specially orthotropic laminate has either a single layer of a specially orthotropic material/multiple specially orthotropic layers that are symmetrically arranged about the laminate middle surface.

Laminate stiff nesses consist solely of $\mathrm{A}_{11}, \mathrm{~A}_{12}, \mathrm{~A}_{22}, \mathrm{~A}_{66}, \mathrm{D}_{11}, \mathrm{D}_{12}, \mathrm{D}_{22}, \mathrm{D}_{66}$

There is neither shear/twist coupling nor bending extension, coupling exists. Thus, for plate problems, the lateral deflections are described by only one D.E. of e.g.
$P(x, y)=D_{11} W W_{x x x x}+2\left(D_{12}+2 D_{66}\right) W,{ }_{x x y y}+D_{22} W_{, y y y y}$

Subject to Boundary Conditions of simply supported edge
$X=0, a: w=0 m_{x}=-D_{11} w,{ }_{x x}-D_{12} w,{ }_{y y}=0$
$\mathrm{Y}=0, \mathrm{~b}: \mathrm{w}=0 \mathrm{~m}_{\mathrm{y}}=-\mathrm{D}_{12} \mathrm{w}, \mathrm{xx}^{-}-\mathrm{D}_{22} \mathrm{w}, \mathrm{yy}=0$

### 2.5 Navier'sSolution :

Deflection equation is give by,
$\omega=C \cdot \sin \frac{m \pi x}{a} \cdot \sin \frac{n \pi y}{b}--(5)$
Lateral load
$\mathrm{P}(\mathrm{x}, \mathrm{y})=\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \mathrm{fm} \sin \frac{m \pi x}{a} \cdot \sin \frac{n \pi y}{b}$

$$
\begin{align*}
& \mathrm{P}(\mathrm{x}, \mathrm{y})= \\
& \left.\frac{16 \mathrm{p}_{\mathrm{o}}}{\pi^{6}} \cdot a^{4} \sum_{m=1,3, n=1,3 .}^{\infty} \sum_{m\left(D_{11} m^{4}+2\left(D_{12}+2 D_{66}\right) m^{2} n^{2}+D_{22} n^{4}\right)}^{(-1)^{\frac{m+n}{2}}-1}\right\} \tag{7}
\end{align*}
$$

The maximum deflection occurs at center of
plate i.e, at $x=a / 2, y=b / 2$
ThereforeCenterDeflection $(\omega)=$
$\left.\left.\frac{16 \mathrm{p}_{\mathrm{o}}}{\pi^{6}} \sum_{m=1,3, n=1,3.3}^{\infty} \sum_{\sum_{11}^{\infty}}^{D_{11}\left(\frac{m}{a}\right)^{4}+2\left(D_{12}+2 D_{66}\right)\left(\frac{m}{a}\right)^{2}\left(\frac{n}{b}\right)^{2}+D_{22}\left(\frac{n}{b}\right)^{4}}\right)\right\}$

### 2.6 Stress - Strain Relations : [2]

The stress-strain relations for an anistropic body can be written as in contracted nations as

$$
\left[\begin{array}{c}
\sigma_{1}  \tag{9}\\
\sigma_{2} \\
\sigma_{3} \\
\tau_{4} \\
\tau_{5} \\
\tau_{6}
\end{array}\right]=\left[\begin{array}{llllll}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
& c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
& & c_{33} & c_{34} & c_{35} & c_{36} \\
& & & c_{44} & c_{45} & c_{46} \\
& s y m & & & c_{55} & c_{56} \\
& & & & & c_{66}
\end{array}\right]\left[\begin{array}{l}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{3} \\
\gamma_{4} \\
\gamma_{5} \\
\gamma_{6}
\end{array}\right]
$$

$$
\begin{gather*}
\text { (Or) } \\
{[\varepsilon]=\left[\begin{array}{cccccc}
s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\
& s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\
& & s_{33} & s_{34} & s_{35} & s_{36} \\
& & & s_{44} & s_{45} & s_{46} \\
& \text { sym } & & & s_{55} & s_{56} \\
& & & & & s_{66}
\end{array}\right]\{\tau\},} \tag{10}
\end{gather*}
$$

### 2.7Special orthotropic material:

In the case of an orthotropic material (which has mutually perpendicular planes of material symmetry) the stress-strain relations in general have the same for as equation-9. However, the number of independent elastic constants is reduced to nine, as various stiffness and compliance terms are interrelated. When the reference system of coordinates is selected along principal planes of material symmetry, i.e., in the case of a specially orthotropic material, then

$$
\begin{gather*}
{\left[\begin{array}{l}
\sigma_{1} \\
\sigma_{2} \\
\sigma_{3} \\
\tau_{4} \\
\tau_{5} \\
\tau_{6}
\end{array}\right]=\left[\begin{array}{cccccc}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\
c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{array}\right]\left[\begin{array}{l}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{3} \\
\gamma_{4} \\
\gamma_{5} \\
\gamma_{6}
\end{array}\right]}  \tag{11}\\
{[\varepsilon]=\left[\begin{array}{cccccc}
s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\
& s_{22} & s_{23} & 0 & 0 & 0 \\
& & s_{33} & 0 & 0 & 0 \\
& s y m & & s_{44} & 0 & 0 \\
& & & s_{55} & 0 \\
& & --(12)
\end{array}\right]\{\tau\}}
\end{gather*}
$$

for $\theta=0$, the shear coupling coefficients are zero and the orthotropic lamina becomes specially orthotropic lamina.
$\left\{\begin{array}{lll}\varepsilon_{x x} & =\varepsilon_{11} \\ \varepsilon_{y y} & =\varepsilon_{22} \\ \varepsilon_{x y} & =\varepsilon_{12}\end{array}\right\}=\left[\begin{array}{lll}\frac{1}{E_{11}} & \frac{v_{12}}{E_{22}} & 0 \\ & \frac{1}{E_{22}} & 0 \\ s y m & & \frac{1}{G_{12}}\end{array}\right]\left\{\begin{array}{l}\tau_{x x} \\ \tau_{y y} \\ \tau_{x y}\end{array}\right\}$

The stress strain relations for a general orthotropic lamina are

$$
\left\{\begin{array}{l}
\tau_{x x} \\
\tau_{y y} \\
\tau_{x y}
\end{array}\right\}=\left[\begin{array}{lll}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
Q_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{array}\right]\left\{\begin{array}{l}
\varepsilon_{x x} \\
\varepsilon_{y y} \\
\varepsilon_{x y}
\end{array}\right\}--(14)
$$

$\bar{Q}_{11}=\mathrm{U}_{1}+\mathrm{U}_{2} \cos 2 \theta+\mathrm{U}_{3} \cos 4 \theta$
$Q_{12}=\mathrm{U}_{4}-\mathrm{U}_{3} \cos 4 \theta$
$\bar{Q}_{16}=\frac{1}{2} U_{2} \sin 2 \theta+U_{3} \sin 4 \theta$
$\bar{Q}_{22}=U_{1}-U_{2} \cos 2 \theta+U_{3} \cos 4 \theta$
$\bar{Q}_{26}=\frac{1}{2} U_{2} \sin 2 \theta-U_{3} \sin 4 \theta$
$\bar{Q}_{66}=U_{5}-U_{3} \cos 4 \theta$

$$
\text { Where }_{U_{1}}=\frac{1}{8}\left(3 Q_{11}+3 Q_{22}+2 Q_{12}+4 Q_{66}\right), \begin{aligned}
U_{2} & =\frac{1}{2}\left(Q_{11}-Q_{22}\right) \\
U_{3} & =\frac{1}{8}\left(Q_{11}+Q_{22}-2 Q_{12}-4 Q_{66}\right) \\
U_{4} & =\frac{1}{8}\left(Q_{11}+Q_{22}+6 Q_{12}-4 Q_{66}\right) \\
U_{5} & =\frac{1}{2}\left(U_{1}-U_{4}\right)^{---(16)}
\end{aligned}
$$

### 2.8 Laminate Geometry:

The geometry of an N layer laminate is shown in Fig 4. The z -coordinate is measured positive downward from the mid plane of the laminate. The total thickness of the laminate is $h$. the first lamina, of thickness $t_{1}$ is located at the top of the stacking. The jth lamina of thickness $\mathrm{t}_{\mathrm{j}}$ is at a distance $\mathrm{h}_{\mathrm{j}-1}$. The laminate itself is considered thin as in the case of thin place theory and a perfect inter laminar bond exists between all the laminae. Each lamina is also assumed to be macroscopically homogeneous and behaves in a linear elastic manner.


Fig. 4 Distances of lamina from middle plane
Depending on the individual lamina properties, the stresses are determined from equation, [3]

$$
\begin{align*}
& \left\{\begin{array}{l}
\tau_{x x} \\
\tau_{y y} \\
\tau_{x y}
\end{array}\right\}=\left[\begin{array}{lll}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{array}\right]\left\{\begin{array}{l}
\varepsilon_{x x} \\
\varepsilon_{y y} \\
\varepsilon_{x y}
\end{array}\right\}--(17) \\
& \left\{\begin{array}{l}
\tau_{x x} \\
\tau_{y y} \\
\tau_{x y}
\end{array}\right\}=\left[\begin{array}{lll}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{array}\right]\left\{\begin{array}{lll}
\varepsilon_{x x}^{0} & +Z \varepsilon_{x} \\
\varepsilon_{y y}^{0} & +Z \varepsilon_{y} \\
\gamma_{x y}^{0} & +Z \gamma_{x y}
\end{array}\right\} \tag{18}
\end{align*}
$$

Stress-displacement relations for the $\mathrm{K}^{\text {th }}$ layer can be written as:

$$
\begin{align*}
{\left[\begin{array}{c}
\sigma_{x} \\
\sigma_{y} \\
\tau_{x y}
\end{array}\right]_{k} } & =-z\left[Q_{i j}\right]_{k}\left[\begin{array}{c}
\frac{\partial^{2} \omega}{\partial x^{2}} \\
\frac{\partial^{2} \omega}{\partial y^{2}} \\
2 \frac{\partial^{2} \omega}{\partial x \partial y}
\end{array}\right]--(19) \\
& =\left[Q_{i j}\right]_{k}\left[\begin{array}{c}
\varepsilon_{x} \\
\varepsilon_{y} \\
\tau_{x y}
\end{array}\right] \tag{20}
\end{align*}
$$

stiffness matrices are given by,
Extensional stiffness matrix:
$[\mathrm{A}]=\left[\begin{array}{lll}A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66}\end{array}\right]$
$\mathrm{A}_{\mathrm{ij}}=\sum_{K=1}^{N}\left(\overline{Q_{i j}}\right)_{k}\left(Z_{k}-Z_{k-1}\right)--$
Coupling stiffness matrix:
$[\mathrm{B}]=\left[\begin{array}{lll}B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66}\end{array}\right]$
$\mathrm{B}_{\mathrm{ij}}=\frac{1}{2} \sum_{K=1}^{N}\left(\overline{Q_{i j}}\right)_{k}\left(Z_{k}^{2}-Z_{k-1}^{2}\right)--(22)$
Bending stiffness matrix:
$[\mathrm{D}]=\left[\begin{array}{lll}D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66}\end{array}\right]$
$\mathrm{D}_{\mathrm{ij}}=\frac{1}{3} \sum_{K=1}^{N}\left(\overline{Q_{i j}}\right)_{k}\left(Z_{k}^{3}-Z_{k-1}^{3}\right)-$

### 2.9 Orthotropic material under plane stress:

In most structural applications composite materials are used in the form of thin laminates loaded in the plane of the laminated. Thus composite laminae or laminates can be considered to be under a condition of plane stress with all stress components in the out of plane direction ( 3 direction) being Zero i.e.,

$$
\sigma_{3}=0, \tau_{4}=0, \tau_{5}=0
$$

The orthotropic stress-strain relations reduced to

$$
\begin{gathered}
{\left[\begin{array}{c}
\sigma_{1} \\
\sigma_{2} \\
\tau_{6}
\end{array}\right]=\left[\begin{array}{ccc}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{array}\right]\left[\begin{array}{l}
\varepsilon_{1} \\
\varepsilon_{2} \\
\gamma_{6}
\end{array}\right]--(24) \sigma_{12}} \\
=[\mathrm{Q}]_{12} \quad[\varepsilon]_{12} \\
\text { Where }_{\mathrm{ij}}=\mathrm{C}_{\mathrm{ij}}-\frac{c_{i 3} c_{j 3}}{c_{33}} \quad(\mathrm{i}, \mathrm{j}=1,2,6)
\end{gathered}
$$

The inverse relation can be written as,

$$
[\varepsilon]_{1,2}=[\mathrm{s}]_{1,2}[\sigma]_{1,2}
$$

Reduced stiffnesses in terms of elastic constants,
$\mathrm{Q}_{11}=\frac{E_{1}}{1-v_{12} v_{21}}$
$\mathrm{Q}_{22}=\frac{E_{2}}{1-v_{12} v_{21}}$
$\mathrm{Q}_{12}=\frac{v_{21} E_{1}}{1-v_{12} v_{21}}=\frac{v_{12} E_{2}}{1-v_{12} v_{21}}$
$\mathrm{Q}_{66}=\mathrm{G}_{12}$

### 2.10 Tsai - Hill criterion or Deviatoric Strain Energy Theory :[4]

$$
\begin{equation*}
\frac{f_{x}^{2}}{X^{2}}+\frac{f_{y}^{2}}{Y^{2}}-\frac{f_{x} f_{y}}{X^{2}}+\frac{f_{x y}^{2}}{S^{2}}=1 \tag{26}
\end{equation*}
$$

The Tsai-Hill failure theory is expressed in terms of a single criterion instead of the three sub criteria required in the maximum stress and maximum strain theories. The Tsai- Hill theory allows for considerable interaction among the stress components. It is based on Hill's theory for ductile anisotropy and adopted to more brittle heterogeneous composites.

## 3. Results \& Discussion:

In this work, the strength of Composite Laminated plates of varying number of layers, configurations and dimensions are calculated using Classical Laminate Plate Theory. Numerical results are generated for the Boron Epoxy laminated composite plate and the properties are listed in Table 1.

Table 1 Material properties of Boron/Epoxy composite:

| Longitudinal Modulus | E11 | 206.84 Gpa |
| :--- | :---: | :--- |
| Transverse Modulus | E22 | 20.68 Gpa |
| Major Poisson's ratio $\quad$ v12 | 0.30 |  |
| Minor Poisson's ratio $\quad \mathrm{V} 21$ | 0.30 |  |
| Planar shear modulus $\quad \mathrm{G} 12$ | 6.89 Gpa |  |
| Ultimate longitudinal <br> strength X | tensile | 1.378 Gpa |
| Ultimate transverse tensile <br> strength Y | 0827 Gpa |  |
| Ultimate shear strength $\quad \mathrm{S}$ | 0.124 Gpa |  |

## Details of the plates :

Thickness of laminate : 10 mm
Plate Dimensions : Rectangular plates of 2 m along principal plane direction, 1 m across principal plane direction.
Square plate $1.414 \mathrm{~m} \times 1.414 \mathrm{~m}$
Laminate Material : Boron / Epoxy
Ply angle : $0^{\circ}$ and $90^{\circ}$ only (i.e., crossply)
Numbers of layers : 2,4,6,8 with equal thickness of laminae.
Edge conditions : Simply supported on all sides.
Loading : Uniformly Distributed Lateral Load.
An efficient C-program [7] is developed to evaluate the response of the laminate, initial failure load and ultimate failure load under different load and boundary conditions. The developed program is run for the problem given in the book mechanics of laminated composites by J.N.Reddy in the chapter 5 Analysis of specially orthotropic laminates using CLPT. And the results are compared here under.
Given data: [3]
Simply supported Square plate under
distributed mechanical loading laterally applied
With Material properties

$$
\begin{aligned}
& \mathrm{E} 11=25 \mathrm{E} 22 \\
& \mathrm{v} 12=0.25
\end{aligned}
$$

and G12 $=0.5 \mathrm{E} 22$;
Table(2) comparison of the results with published one

|  | Published <br> result | Out put of <br> the <br> program |
| :--- | :--- | :--- |
| Non dimensional <br> deformation | .666 | .699 |
| Non dimensional <br> Longitudinal stress | .8075 | .8549 |
| Non dimensional <br> Transverse stress | .0306 | .0324 |

The results are found satisfactory.

The program is run for the material properties and lamina dimensions under consideration and the following results are obtained.

## Reduced stiffness matrices for the given

 material(Boron/Epoxy): Q[ij] Matrix for 0Degrees| 208.72 | 6.26 | 0.00 |
| :---: | :---: | :---: |
| $6.26 \quad 20$. | 20.872 | -0.00 in GPa |
| 0.00 | -0.00 | 6.89 |

Q[ij] Matrix for 90Degrees

| ------------------------------------ |  |  |
| ---: | ---: | ---: |
| 20.87 | 6.26 | 0.00 |
| 6.26 | 208.72 | -0.00 |
| 0.00 | -0.00 | 6.89 |
| --------------------------------- |  |  | in Gpa

4-Layer Symmetric Laminate[90|0|0|90]:
Coupling Matrix (D) - Matrix in KN-m:

| 3.70 | 0.52 | 0.00 |
| :---: | :---: | :---: |
| 0.52 | 15.44 | 0.00 |
| 0.00 | 0.00 | 0.57 |

The maximum allowable load : 294.0000 KPa and deflection $\quad: 0.297 \mathrm{~m}$ 4-Layer antisymmetriclaminate $(0|90| 0 \mid 90)$ :


The maximum allowable load : 263. KPa and deflection $\quad: 0.377 \mathrm{~m}$


Fig. 5 ) Failure Load of Laminates Vs Deflection curve for 4-Ply laminate of various configurations

## 6-PLY LAMINATE



Fig.(6) Failure Load of 6-plyLaminates Vs Deflection curve for 6-Ply laminate

ORTHROTOPIC SQUARE PLATE:
Dimensions: $\mathrm{a}=1 \mathrm{~m} ; \mathrm{b}=1 \mathrm{~m} ; \quad \mathrm{t}=.01 \mathrm{~m}$

|  | Failure load <br> Kpa | Deflection , <br> mm |
| :--- | :--- | :--- |
| 2-Ply laminate <br> $(0 \mid 90)$ <br> (90\|0) | 76 | .224 |
| 4-Ply laminate <br> $(0\|90\| 0 \mid 90)$ <br> $(0\|90\| 90 \mid 0)$ <br> $(90\|0\| 0 \mid 90)$ | 152 | .441 |
| 6-Ply Laminate <br> $(0\|90\| 0\|90\| 0 \mid 90)$ | 228 | .665 |
| $0\|90\| 0\|0\| 90 \mid 0)$ <br> $(90\|0\| 90\|90\| 0 \mid 90)$ |  |  |

Table 3 Failure loads and Deflections of square laminated plates


Fig 7 Number of layers Vs Ultimate failure load of square laminate plates of varying layers


Fig. (8) Longitudinal stress Vs Strain in outer layer of a four ply laminate ( $90|0| 0 \mid 90$ )


Fig. (9) Lateral stress Vs strain in outer layers of a four ply laminate ( $90|0| 0 \mid 90$ )


Fig. (10) Longitudinal stress Vs Strain in inner layers of a four plylaminate (90|0|0|90)


Fig. (11) Lateral stress Vs Strain in inner layers of a four plylaminate ( $90|0| 0 \mid 90$ )

## 4. ANALYSIS:

Based on the results the graphs are drawn :
Fig. (6)indicates the failure of 4-ply laminates with different configurations. The first point on each of the lines indicates the First ply failure which occurred in outer layers and the second point indicates the last ply failure which is in inner layers. Out off the three configurations $(90|0| 0 \mid 90)$ is the best configuration that an with stand a higher ultimate load of 294 KPa with small deflection of .297 m .

Table (3) presents ultimate loads with varying number of layers of aOrthotropic Square laminate with different configurations. It is observed that the ultimate loads forall the configurations is giving same failure loads and deflections for a given number of layers.

Fig. (7) presents the ultimate loads for varying number of layers in a Square laminate plate. It is observed that as the number of layers increases the strength of the plate also increases.

Fig. (8) to (11) presents the stress and strain relations in both longitudinal and transverse in a 4ply laminate ( $90|0| 0 \mid 90$ ). It is observed that there is a linear relation ship in stress and strain and the stresses are equal in magnitude at equal distance from mid plane on both sides but in opposite nature i.e., tensile in lower laminae and compressive at upper laminae.

## 5. CONCLUSIONS

From the study of the work carried out for simply supported orthotropic laminated plates under uniformly distributed load, the following conclusions are made.

As the number of layers increases in a laminated composite plate for a given thickness, it's strength also increases.

The strength of the laminated composite plate depends on the number of layers and also the arrangement of the layers.

The strength of an orthotropic square laminated plate is independent of the arrangement of the layers.

## 6. References:

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