

Analysis of Cold Plasma Injection on Whistler Mode Instability with AC Field in Magnetosphere of Saturn

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Abstract- Whistlers generated by lightning have been observed in the magnetosphere of Saturn by the Cassini Radio and Plasma Wave Science Investigation (RPWS). Two whistlers were observed as the spacecraft flew over the rings on July 1, 2004, and the third was observed on October 28, 2004, during the inbound pass of orbit at a radial distance of 6.18 Rs (Saturn radii). The effect of cold plasma injection on whistler mode instability has been studied for ring distribution function with perpendicular AC electric field. The dispersion relation of the parallel propagating electromagnetic whistler mode wave has been applied to the magnetosphere of Saturn. An expression for the growth rate of a system with added cold plasma injection has been calculated. The effects of electron density, number density, temperature anisotropy, A.C frequency and relativistic factor on the growth rate of whistler mode emission were investigated. The observation is in very good agreement with the results. Analysis shows that an increase in temperature anisotropy, A.C. frequency, and energy density increases the rate of increase in whistler waves with significant changes in wavenumbers. It has been shown that whistler mode waves have increased due to the loss of vertical kinetic energy of the ring electrons. In Saturn's magnetosphere, calculations were made at a radial distance of 6.18 Rs. This result is important for the analysis of VLF emissions observed over a broad spectrum of frequencies in the Saturn magnetosphere.

Keywords- Whistler mode instability; Ring distribution function; Dispersion relation; Cold injection

I. INTRODUCTION

Whistler is an audio-frequency electromagnetic wave generated by lightning, which propagates along the magnetic field lines at a frequency lower than the electron cyclotron frequency and the electron plasma frequency. Whistling was first discovered in 1918 by [1] using a basic vacuum tube audio amplifier. Later, [2] found that the arrival time is inversely proportional to the square root of the frequency. [3] was the first person to correctly explain the origin of whistler blowing. He showed that the whistler travels along the magnetic field lines from one hemisphere to the other, and this plasma propagation mode is now called the whistler mode. Whistlers have also been found on Jupiter [4-7].

There are two common types of Whistler-mode emissions in the planet's magnetosphere: hiss and chorus. The frequency of the Whistler mode emission is less than the lower of the plasma frequency f_p or the cyclotron frequency f_c . At Saturn, the hiss usually has an uncharacteristic spectrum, and its frequency is usually lower than the frequency of the whistler chorus ($f < 3$ kHz). It is known that the latter has a source region near the magnetic equator, usually a narrow-band structure, and shows many fine structures of drifting frequency tones or at high resolution, which is larger than the typical spectral density. For a long time, people have been well aware of the developed linear theory of chorus generation [8-10]. The fine structure of the chorus is the result of electronic nonlinearity and resonance wave capture, as described by [11-16].

Diffusion pitch angle scattering and electron acceleration occur on Earth due to Whistler emission [17-19], and may also occur at Saturn [20, 21]. Recently, it has been demonstrated on Saturn that the same general process of pitch angle scattering by Z mode is important [22]. On the earth, whistlers near the electromagnetic equator will have cyclotron resonance and wave growth [23, 24]. Earlier theories also believed that whistlers would be generated in the plasma layer [25-27].

The long duration of the Cassini mission has given us a deeper understanding of Saturn's plasma conditions and wave characteristics. The ratio of plasma frequency to gyro frequency, f_{pe}/f_{ce} , is very important for electron acceleration generated by wave-particle interaction, and a value of $f_{pe}/f_{ce} < 4$ will result in good electron acceleration [28]. The region of interest here is 6.18 Rs. It is reasonable to assume that the magnetic field is a dipole magnetic field near Saturn, which is very small [29].

Voyager 1's encounter with Saturn in November 1980 provided the first opportunity to study plasma wave interactions in Saturn's magnetosphere. Inspired by various pioneers, in this work, we try to study the Whistler mode

instability in the inner magnetosphere of Saturn at a radial distance of 6.18 Rs. The study is based on the effect of cold beam injection at a specific radial distance. Further sections describe the mathematical formulation, followed by graphical descriptions and conclusions.

II. MATHEMATICAL FORMULATION

Spatially homogeneous anisotropic, collisionless plasma subjected to external magnetic field $\mathbf{B} = B_0 \hat{\mathbf{e}}_z$ and an electric field $\mathbf{E}_{ox} = (E_0 \sin vt) \hat{\mathbf{e}}_x$ has been considered to get dispersion relation. Negligible inhomogeneities are assumed in the zone of interaction. In order to obtain a general dispersion relation for the case relativistic whistler waves in the presence of A.C electric field, linearized Vlasov-Maxwell equations are attained after neglecting higher order terms and separating the equilibrium and nonequilibrium parts by small perturbations in magnetic field, electric field and distribution function. Following the technique of [30], the dielectric tensor can be written as:

$$\epsilon_{ij}(\mathbf{k}, \omega) = 1 + \sum_s \left\{ \frac{4e_s^2 \pi}{(\beta m_e)^2 \omega^2} \right\} \int \frac{d^3 P_j (\lambda_3) \| S_s^* \|}{\left(\omega - k_{\parallel} \frac{P_{\parallel}}{\beta m_e} - (n + g) \frac{\omega_{cs}}{\beta} + pv \right)} \quad (1)$$

For the propagation and instability of whistler mode waves with $k_{\perp} = 0$ [31], the branch of general dispersion relation (1) reduces to:

$$\epsilon_{11} \pm i\epsilon_{12} = N^2 \quad (2)$$

Where $N^2 = (\kappa^2 c^2) / \omega^2$ is refractive index. Therefore, dispersion relation for $n=1$ may be written as:

$$N^2 = 1 + \frac{4e_s^2 \pi}{(\beta m_e)^2 \omega^2} \int \frac{d^3 P}{2} P_{\perp} [N_1 + N_2] \times \left[\frac{1}{\omega - \frac{\kappa_{\parallel} P_{\parallel}}{\beta m_e} \pm (n + g) \frac{\omega_c}{\beta} + pv} \right] \quad (3)$$

Where

$$N_1 = \frac{(\beta m_e)^2}{P_{\perp}} \frac{\partial f_o}{\partial P_{\perp}} \left(\omega - \frac{\kappa_{\parallel} P_{\parallel}}{\beta m_e} \right) \left(\frac{P_{\perp}}{\beta m_e} - \frac{v \Gamma_x}{\beta \left(\frac{\omega_c^2}{\beta^2} - v^2 \right)} \right) \quad (4)$$

$$N_2 = \beta m_e \kappa_{\parallel} \frac{\partial f_o}{\partial P_{\parallel}} \left(\frac{P_{\perp}}{\beta m_e} - \frac{v \Gamma_x}{\beta \left(\frac{\omega_c^2}{\beta^2} - v^2 \right)} \right) \quad (5)$$

The distribution function for trapped electron is taken as Maxwellian ring momentum distribution function [31]

$$f(P_{\perp}, P_{\parallel}) = \frac{n_e/n}{\pi^{3/2} P_{o\parallel} P_{o\perp}^2 B} \exp \left[-\frac{(P_{\perp} - P_o)^2}{P_{o\perp}^2} - \frac{(P_{\parallel}^2)}{P_{o\parallel}^2} \right] \quad (6)$$

$$B = \exp(-P_o^2/P_{o\perp}^2) + \sqrt{\pi} \left(\frac{P_o}{P_{o\perp}} \right) \text{erfc}(-P_o/P_{o\parallel}) \quad (7)$$

Where,

$$P_{o\parallel} = \left(\frac{2k_b T_{\parallel}}{m_e} \right)^{1/2} \quad \text{and} \quad P_{o\perp} = \left(\frac{2k_b T_{\perp}}{m_e} \right)^{1/2}$$

Are the associated parallel and perpendicular electron thermal velocities.

In equation (6), n_e/n is the ratio of trapped energetic electrons to total electron density ($n_e/n = n_o$). Equation (7) gives the expression for complimentary error function. The P_{\parallel} and P_{\perp} are parallel and perpendicular thermal velocities in terms of momentum with respect to magnetic field. P_o is the drift speed in terms of momentum.

Substituting $d^3 P = 2\pi \int_{-\infty}^{\infty} P_{\perp} dP_{\perp} \int_{-\infty}^{\infty} dP_{\parallel}$

and using expression (7) in equation (6) and after solving the integrations, we get the dispersion relation as:

$$\frac{k^2 c^2}{\omega^2} = 1 + \frac{4\pi n_s^2}{(\beta m_e) \omega^2} \frac{(n_e/n)}{B} \left[X_1 \frac{\beta m_e \omega}{k_{\parallel} P_{o\parallel}} Z(\xi) + X_2 (1 + \xi Z(\xi)) \right] \quad (8)$$

Where

$$X_1 = 1 + \frac{P_o^2}{P_{o\perp}^2} - \frac{P_o}{P_{o\perp}} \sqrt{\pi} - \frac{(\beta m_e) v \Gamma_x}{\beta \left(\frac{\omega_c^2}{\beta^2} - v^2 \right)} \left(\frac{\sqrt{\pi}}{2} \frac{1}{P_{o\perp}} - \frac{P_o}{P_{o\perp}^2} \right)$$

$$X_2 = \left[X_1 + \frac{P_{o\perp}^2}{P_{o\parallel}^2} \left(1 - \sqrt{\pi} \frac{P_o^3}{P_{o\perp}^3} \text{erf} \left(\frac{P_{\perp}'}{P_{o\perp}} \right) + 3 \frac{P_o^2}{P_{o\perp}^2} - \frac{3}{2} \sqrt{\pi} \frac{P_o}{P_{o\perp}} \right) \right] - \left[\frac{(\beta m_e) v \Gamma_x}{\beta \left(\frac{\omega_c^2}{\beta^2} - v^2 \right)} \left(\frac{\sqrt{\pi}}{2} \frac{1}{P_{o\perp}} + \sqrt{\pi} \frac{P_o^2}{P_{o\perp}^3} \text{erf} \left(\frac{P_{\perp}'}{P_{o\perp}} \right) - 2P_o \right) \right]$$

Where,

$Z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t - \xi} dt$, is the plasma dispersion function with

$$\xi = \frac{\beta m_e}{k_{\parallel} P_{o\parallel}} \left(\omega - g \frac{\omega_c}{\beta} \pm \frac{\omega_c}{\beta} + pv \right)$$

Applying condition $\frac{k^2 c^2}{\omega^2} \gg 1$ for whistler waves

$$\omega_{ps}^2 = \frac{4e^2 \pi n_e/n}{B_o m_e}$$

$$\omega = \omega_r + i\gamma$$

The equation (8) reduces to

$$D(k, \omega) = \frac{-k^2 c^2}{\omega_{ps}^2} + \frac{1}{\beta} \left[\frac{X_1 \omega}{k_{\parallel} P_{o\parallel}} (\beta m_e) \left\{ -\frac{1}{\xi} - \frac{1}{2\xi^3} \right\} - \left\{ X_2 \frac{1}{2\xi^2} \right\} \right] + \left[\frac{(\beta m_e) \omega}{k_{\parallel} P_{o\parallel}} X_1 + X_2 \xi \right] \{ i\sqrt{\pi} \exp(-\xi^2) \} - \frac{\omega}{\omega \pm \omega_c} \cdot \frac{\omega_{pc}^2}{\omega_{pw}^2} \quad (9)$$

where $\omega_{pw} = (4\pi n_w e^2 / m)^{1/2}$ and $\omega_{pc} = (4\pi n_c e^2 / m)^{1/2}$ are the plasma frequencies of the warm background and cold injected plasma respectively.

Introducing the dimensionless parameters as $\tilde{k} = \frac{k_{\parallel} P_{o\parallel}}{\omega_{cs}}$

The growth rate in terms of the dimensionless parameters $\tilde{k}, \beta_1, X_2, X_1$ and X_4 is obtained as

$$\frac{\gamma}{\omega_c} = \frac{\frac{\sqrt{\pi}}{\beta \tilde{k}} \left(\frac{X_2}{X_1} - \frac{\beta X_3}{1 - \beta X_3 + \beta X_4} \right) (1 - \beta X_3 + \beta X_4)^3 \exp \left[- \left(\frac{1 - \beta X_3 + \beta X_4}{\tilde{k}} \right)^2 \right]}{1 + \beta X_4 + \frac{\tilde{k}^2 (1 + \beta X_4)}{2(1 - \beta X_3 + \beta X_4)^2} - \frac{\tilde{k}^2}{(1 - \beta X_3 + \beta X_4)} \left(\frac{X_2}{X_1} - \frac{\beta X_3}{1 - \beta X_3 + \beta X_4} \right) + \frac{(\delta - 1)k_3^2}{(1 + \beta X_4)(1 - \beta X_3)^2}} \quad (10)$$

The real part of eq. (9) is

$$X_3 = -\frac{\omega_r}{\omega_c} = \frac{\tilde{k}}{2\delta\beta_1} \left[k_2(1 + \beta X_4) + \frac{X_2}{X_1} \frac{\beta_1}{2\beta(1 + \beta X_4)} \right] \quad (11)$$

Where $k_2 = \frac{1}{2X_1}$ and $\beta_1 = \frac{4\pi\mu_0 \varepsilon_0 k_b T_{\parallel} (n_e/n)}{m_e^2 B B_o^2}$

$$X_4 = -\frac{pv}{\omega_c} \quad (12)$$

$$\delta = 1 + \frac{n_c}{n_w} k_2(1 + X_4) \text{ and } k_3 = 1 - \beta X_3 + \beta X_4$$

III. RESULT AND DISCUSSION

Analysis is attributed to observations and plasma conditions and limited to inner middle boundary ($< 7 R_s$). Here are discussing few relevant monitoring data. [33] first reported the global view of Saturn plasma electron environment by analysing observation from voyager 1 and Voyager 2. [34] performed observational study extensively about impulsive radio wave and plasma wave emission in

Saturn magnetosphere using electric and magnetic field antennas of RPWS ranging electric field from lower (1 Hz) to higher (16 MHz) and magnetic fields (1 Hz to 12 kHz). Saturn exhibited different temperature profile at radial distances.

Saturn's inner magnetosphere (5-12 R_s) is populated with two different components i.e. hot (>100 ev) and cold component (<100 ev) [35-37], radial diffusion of these population in inner magnetosphere is very much expected from the interchanged instability influenced by centrifugal forces. Study carryout the variation in dimensionless wave growth of parallel propagating whistler mode waves for the particles in ring distribution. Comprehensive analysis of the result has been performed by taking previous published data set based on Cassini and Voyager 1 spacecraft. Measured magnitude of ambient magnetic field $B_o=73$ nT. The thermal energy ($K_B T_{\parallel e}$) of electrons is 300 eV. Number density of electrons is considered to be $n_o=5 \times 10^7 \text{ m}^{-3}$ and magnitude of AC electric field $E_o=0.01$ mV/m has been considered. The range of electron energy used was 100-500 eV.

As described earlier, appropriate plasma and field parameters corresponding to $L=6.18$, and expression of the growth rate given in section II has been used to calculate growth rate with the wave number. The calculated growth rate as a function of wavenumber shows a main peak at normalized frequency <1 and is always accompanied by trailing growth at lower wave numbers (Fig. 1-5). Furthermore, it can be seen that the presence of relativistic electron tail results in a significant change in the normalized growth rate at the lower peak. In Fig. 1., it is shown that an increase in the AC frequency will increase the peak growth rate of whistler mode waves with peak shifting towards higher wavenumber side. In fig. 2. and fig. 3., growth rate has been plotted against wave number to see the effect of cold to hot electron number density ratio and energy density. From the graphs, it can be concluded that energy density supports growth rate significantly whereas number density does not seem to enhance the peak to much extent. Next fig. 4. shows that the increase in temperature anisotropy, significantly increases the peak value of growth rate. The graph indicates that, although temperature anisotropy enhances growth of whistlers but it does not show significant changes of wavenumber spectra. Similarly, fig. 5. shows that the increase in the relativistic factor results in a significant decrease in the peak value of whistler mode waves. These results are consistent with the literature. In this study, the ring distribution function used to illustrate the existence of relativistic effects has been used by other workers to study the effects of various instabilities.

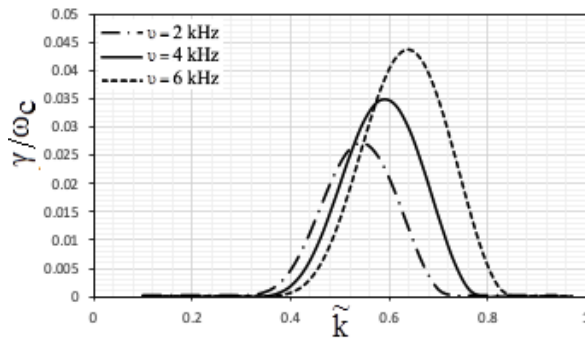


Fig. 1. Growth rate variation with wave vector \tilde{k} at numerous values of A.C. frequency keeping other plasma parameters constant such as $E_o = 0.01$ mV/m, $B_o = 73$ nT, $n_c/n_w = 10$, $T_{\perp}/T_{\parallel} = 1.5$, $\beta = 0.6$ etc.

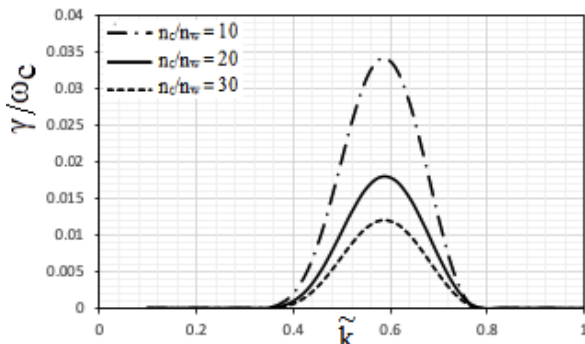


Fig. 2. Growth rate variation with wave vector \tilde{k} at numerous values of cold to hot electron density ratio keeping other plasma parameters constant such as $E_o = 0.01$ mV/m, $B_o = 73$ nT, $\nu = 4$ kHz, $T_{\perp}/T_{\parallel} = 1.5$, $\beta = 0.6$ etc.

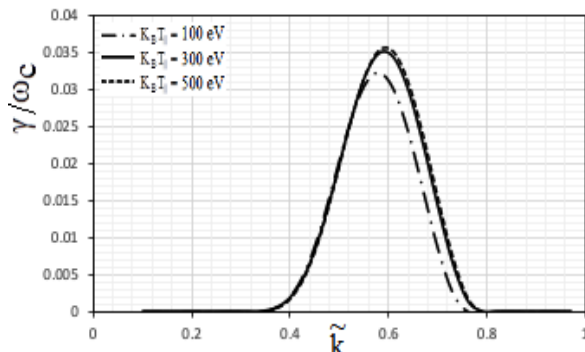


Fig. 3. Growth rate variation with wave vector \tilde{k} at numerous values of energy density keeping other plasma parameters constant such as $E_o = 0.01$ mV/m, $B_o = 73$ nT, $n_c/n_w = 10$, $T_{\perp}/T_{\parallel} = 1.5$, $\beta = 0.6$, $\nu = 4$ kHz etc.

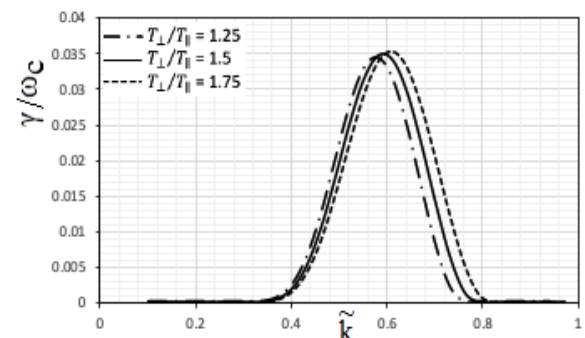


Fig. 4. Growth rate variation with wave vector \tilde{k} at numerous values of temperature anisotropy keeping other plasma parameters constant such as $E_o = 0.01$ mV/m, $B_o = 73$ nT, $n_c/n_w = 10$, $\nu = 4$ kHz, $\beta = 0.6$ etc.

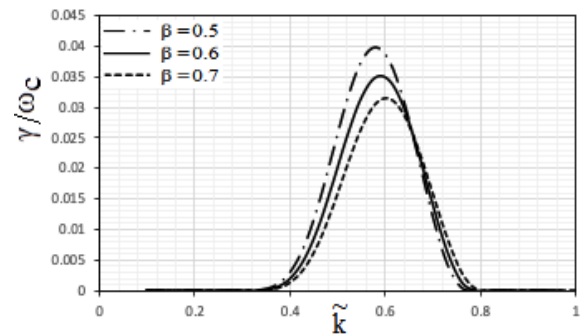


Fig. 5. Growth rate variation with wave vector \tilde{k} at numerous values of relativistic factor keeping other plasma parameters constant such as $E_o = 0.01$ mV/m, $B_o = 73$ nT, $n_c/n_w = 10$, $\nu = 4$ kHz, $T_{\perp}/T_{\parallel} = 1.5$ etc.

IV. CONCLUSION

In this work, whistler mode waves that propagates parallel in the presence of an electric field with a ring distribution function in the magnetosphere of Saturn is studied. The method is formed by a kinetic method to ensure efficiency, consistency etc. at a radial distance of 6.18 Rs. The expressions of dispersion relation, growth rate and real frequency are derived to analyze the effects of plasma temperature anisotropy, relativistic factor, number density, AC frequency and energy density. Parametric analysis inferred that growth rate of whistler waves increases with increasing value of temperature anisotropy and AC frequency, but the growth rate does not increase with number density ratio and relativistic factor. We find that a temperature anisotropy ($T_{\perp}/T_{\parallel} \sim 1.5$) can account for linear temporal growth rate of whistler mode waves, which provides a majority of the observed frequency-integrated power. Higher frequency chorus emissions differ from lower frequency whistler mode emissions and are sometimes associated with simultaneously observed low frequency electromagnetic ion cyclotron waves.

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