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# Analysis of Call Completion using the Time Diagram

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***Abstract*— Due to the ever expanding nature of the global system for mobile communication (GSM) network, there is a need for an improvement in the rate at which calls are completed. This suggests a need for a proper analysis of call completion. In this paper, call completion is analysed with the aid of a time diagram showing the block and drop probabilities with respect to the cell dwell time (CDT) and call holding time (CHT). A simple expression is derived which is a close approximation of the call completion probability in the GSM network. This derivation is made under exponentially distributed cell dwell time and call holding time with arbitrary distribution. This study shows that call completion probability can be measured in terms of some failed operations in the system.**

***Keywords*— GSM, call completion probability, time diagram, call holding time, cell dwell time;**



## I. Introduction

In the performance analysis of wireless networks, the duration of time that a mobile user (MU) dwells under the radio coverage of a base station is called the cell dwell time (CDT). The Call Holding Time (CHT) is the duration of time a mobile user occupies a channel (from start to the end of the call) [1]. The distributions of the CDT and CHT can be derived using several approaches such as the three parameter generalized gamma (GG3), exponential, lognormal, pareto and weibull distributions [1][2].

In analysing a complete call in a GSM network, the mobile user (MU) is usually tracked from the time that the call is started until the time that the call is completed, as the MU moves from cell to cell via a handoff process [3]. The CHT and CDT give a vivid idea of the effect of blocked and dropped calls in the system as is described by the time diagram.

In this paper we start by defining call completion and the possible causes of incomplete calls followed by a description of the outcome of GSM call cases using a flow diagram in section one.

In the next section, the time diagram is used for the analysis of call completion where a simple expression based on an exponential approximation of the CHT and CDT is derived.

It is followed by discussion of results and conclusion in section three.

## II. Related Works

Adegoke and Babalola, [4] did an appraisal of the performance of GSM operators using Nigeria as a case study and examined the problems facing the industry. Having evaluated the parameters that attributed to poor quality of service by operators, they came up with methods that are suggested towards improving network performance.

Ayeni, [5], made a quantitative study of the phenomenon of call set up failure. Test calls were made to different identified routes of the network vis-à-vis intra, inter, PSTN (Public Switched Telephone Network) and international. It was discovered that relatively low call failure rates were recorded on intra and international routes as against the inter and PSTN routes. It was also observed that definite call failure patterns could not be observed when viewed over the hours of the day, as is expected in normal traffic studies.

The authors in [6] derived simple closed-form expressions that closely approximate the call completion probability in a wireless cellular network under generalized gamma distributed cell dwell time and call holding time with arbitrary distribution.

[7] proposed a method to obtain upper and lower bounds of call completion probabilities in a mobile communication network having a large number of base stations. To overcome the difficulty of analysing overlapping zones in the network, two smaller size models were introduced and the upper and lower bounds were represented. It was seen that the values of the bounds could be gotten with less computation.

### *A. Call Completion*

A call is said to be complete when it is released by normal call clearing (i.e release message RL-M and release complete message RLC-M has been successfully exchanged in the signalling flow), be it during a ringing phase or conversation phase by either the caller or the called party [8].

Incomplete calls are a regular feature in GSM networks. This is due to a variety of reasons ranging from network to user-oriented causes. Emphasis will be laid on network-oriented causes such as; Assignment failure set up failure and hand-off failure.

*B. Outcome of All Call Cases*

The flow diagram below gives detailed information about the outcome of all call cases in GSM networks.

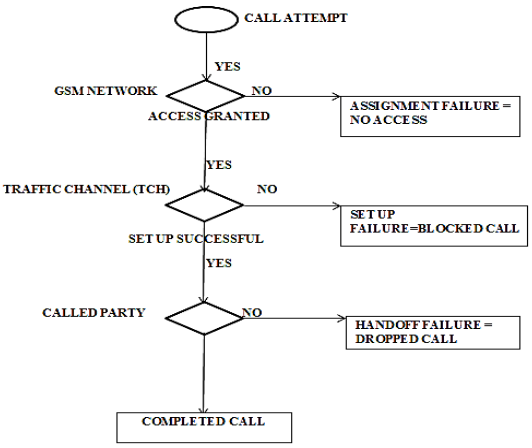


Figure 1.0; Flow diagram for the outcome of a call. [9]

The outcome of call cases in GSM networks as depicted by the flow diagram (figure 1.0) can be analysed in three stages;

**Stage 1;** when the mobile user initiates a call, the possibilities are that it is either granted access to the network via a signalling channel (RACH) or access is denied.

**Stage 2;** on accessing the network, it makes a request for a traffic channel (TCH) during the call set up process. The request is either granted or blocked.

**Stage 3;** after a successful call set up, the called party is alerted. While the conversation is taking place, the outcomes are that the call is either dropped or completed.

These are the possible outcome of call cases in GSM networks.

1) **Assignment Failure**: Assignment failure is a situation where an attempt to allocate a signalling channel to a caller fails.
2) **Setup Failure:** This is a situation whereby network access is reached, but the call does not reach the state of a usable two-way end-to-end connection. Here the call is blocked.
3) **Setup Success:** A call attempt is successful when it leads to a usable connection.
4) **Dropped call:** A call is said to be dropped when it is prematurely or forcefully terminated, mainly due to handoff failure [9][10].

Call completion rate (CCR), can be defined as the number of calls that are successfully served to completion per unit time by a network [11].

CCR according to [9] is derived from the relationship between

(i) New call arrival rate,

(ii) **(1- $P_b$)**; probability that a new call is admitted,

(iii) **(1-$P_{ft}$)**; probability that an on-going call is not forced to terminate.

In this paper, we will attempt to analyse call completion in terms of the number of failed operations in the network. Examples of such operations are the probability that an attempt to allocate a signalling channel fails, the probability that an attempt to allocate a traffic channel for a call fails, or the probability that a handover between two neighbouring base station transceivers fails. All of these have an adverse effect on the rate of call completion.

*C. The Timing Diagram*

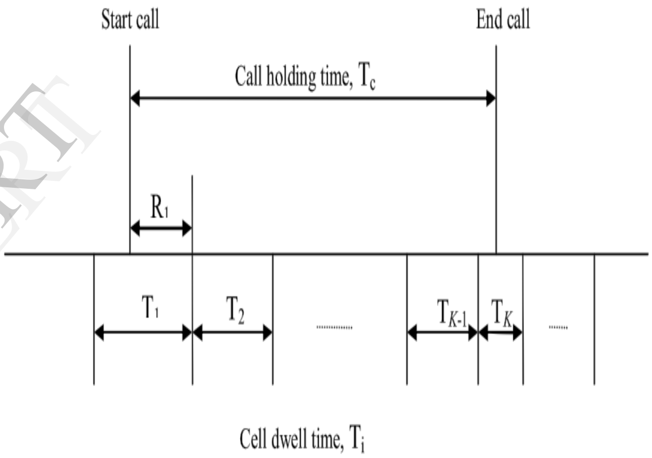


Figure 2.0; Timing Diagram, [3]

Figure 2.0 shows the timing diagram for tracking one complete call by the Mobile User (MU) in a GSM network.
The call starts at an arbitrary point in the first cell $T_1$ with excess life $R_1$, and traverses through different cells and ends at an arbitrary point at the K-th cell. The call holding time is denoted as $T_c$.

**Cell Dwell Time (CDT);** this is the duration of time a mobile user dwells under the radio coverage of a base station [3].

Let $T_l$ (l=1,2,3……) be independent and identically distributed (i.i.d.) non-negative random variables representing the Cell Dwell Time (CDT) in the l-th cell (i.e., the time that the MU resides in the l-th cell).

An arbitrary CDT is denoted by the random variable T that has probability distribution function (p.d.f) of $f_T$ (t) and finite r-th moment.

$$\mu_r = E[T^r] < \infty \qquad (1)$$

Assuming the MU starts a call at an arbitrary point in the first cell; let the excess life of the first CDT be denoted by the random variable $R_1$. The p.d.f of $R_1$ from equation 1.0 is given as [12],

$R_1 = f_T(\tau)$ (2)

Using the p.d.f of $R_1$ from eqn.2.1, the r-th moment of $R_1$ can be obtained via integration by parts as

$E[R_1^r] = 1/\mu_1 \int_0^\infty t^r \int_t^\infty f_T(\tau)\, d\tau = \mu_r + 1/(r+1)\mu_1$ (3)

Let the probability that a handoff attempt fails be denoted by *p*, then hand-off success probability is (1-*p*).

Assuming the call is completed in the random k-th cell, the sum of CDT's is a random variable $S_k$ given as

$S_k = R_1 + \sum_{l=2}^{k} T_l$ (4)

where K is a geometric random variable with parameter p .

The distribution of $S_k$ from [7] is given as

$P_r(S_k \leq x) = \sum_{n=1}^{\infty} p(1-p)^{n-1} \times P_r(R_1 + \sum_{l=2}^{n} T_l \leq x)$ (5)

Using exponential approximation [13][14], if limits of both sides is taken applying erdos-renyi's theorem, when p is sufficiently small (rare events), the tail of the geometric compound sum converges in distribution to the exponential distribution, i.e.,

$\mathrm{Lim}_{p\to 0} P_r(S_k \leq x \mid T_c = x) = \mathrm{Lim}_{p\to 0} \sum_{n=1}^{\infty} p(1-p)^{n-1} \times P_r(R_1 + T_2 + \ldots T_n \leq x \mid T_c = x) \approx 1 - \exp[x/E[S_k]]$ (6)

Where $E[S_k]$ is the mean or entropy of $S_k$

Call Holding Time (CHT) $T_c$; this is the duration of time a mobile user occupies a channel (from start to the end of the call).

Averaging (5) over the distribution of $T_c$,

We have $P_r(S_k \leq T_c) = \int_0^\infty [\mathrm{Lim}_{p\to 0} \sum_{n=1}^{\infty} p(1-p)^{n-1} \times P_r(R_1 + T_1 + T_2 \ldots\ldots T_n \leq x \mid T_c = x)] f_{Tc}(x)dx$

$= \int_0^\infty [1-\exp(-x/E[S_k])] f_{Tc}(x)dx = 1 - Ł_{Tc}[1/E[S_k]]$ (7)

Where $Ł_{Tc} = \int_0^\infty e^{-st} f_{Tc}(t)dt$ denotes the laplace transform function( LTF) of $T_c$. [3] shows that when the call originates at the beginning of the first cell, $R_1 = T_1$, then

$E[S_K] = E[K] \times E[T] = E[T]/p$ (8)

But when the call does not originate at the beginning of the first cell,

$E[S_k] = E[R_1] + (E[K] - 1) \times E[T] =$

$p\, E[R_1] + (1-p)\, E[T] / p$ (9)

**Call Completion Probability;** this is the probability that a new call which is not initially blocked is successfully connected to the network and is not dropped until the call is ended by the mobile user in any cell. It can be expressed as a laplace transform function of the call holding time $T_c$ as shown in [3].

Assuming all calls end at the k-th cell, it means CHT is less than the sum of first k CDT's. Let the new call blocking probability be denoted by $p_0$, then call completion probability,

$P_c = (1 - p_0) \times P_r(T_c < S_k)$ (10)

From (8)

$P_r(S_k \leq T_c) = 1 - Ł_{Tc}[1/E[S_k]]$

From (10), $E[S_k] = p\, E[R_1] + (1-p)\, E[T] / p$

Therefore, $P_r(S_k \leq T_c) = 1 - Ł_{Tc}[p/ p\, E[R_1] + (1-p)\, E[T]]$

Put into (10),

$P_c = (1 - p_0)(1 - Ł_{Tc}[p/ p\, E[R_1] + (1-p)\, E[T])$

$= (1 - p_0)1 - Ł_{Tc}[p/ p\, E[R_1] + (1-p)\, E[T]]$ (11)

Recall that $E[T^r] = \mu_r$ (1) and $E[R_1^r] = \mu_r + 1/(r+1)\mu_1$ (2)

When r=1, $E[T^1] = \mu_1$ and $E[R_1^1] = \mu_1 + 1/(1+1)\mu_1 = \mu_2/2\mu_1$

Put into (11),

$P_c = (1 - p_0)1 - Ł_{Tc}[p/ p(\mu_2/2\mu_1) + (1-p)\, \mu_1]$ (12)

| Generalized Gamma $(c, b, v)$ | $c$ | $b$ | $v$ |
|---|---|---|---|
| Exponential $(\mu)$ | 1 | $\mu$ | 1 |
| Erlang $(m, b)$ | Integer $m$ | $b$ | 1 |
| Gamma $(\alpha, \beta)$ | $\alpha$ | $\beta$ | 1 |
| Weibull $(\beta, \lambda)$ | 1 | $\beta$ | $\lambda$ |
| Lognormal $(m, \sigma)$ $v^2 c \to 1/\sigma^2$ $(bc)^{1/v} \to m$ | $c \to \infty$ | $m$ | $v \to 0$ |
| Pareto $(b, p)$ $vc \to -p$ | $c \to 0$ | $b$ | $v \to -\infty$ |

The following table shows the relationship between the different distributions

Table 2.0; Generalized gamma distribution and its special and limiting cases

Assuming the CHT follows a 3-parameter generalised gamma distribution [3] shows that its approximation has an arbitrary shape parameter c, with the following property;

$Ł_{Tc} = (1 + bs)^{-c} = G_{1,1}^{1,1}[bs \mid {}_0^{1-c}]$ (13)

Applying $(1 + bs)^{-c}$ property to (13), We have;

$P_c = (1 - p_0)1 - (1 + bp/p(\mu_2/2\mu_1) + (1-p)\,\mu_1)^{-c}$ (14)

However, for GSM calls the special case of exponentially distributed CHT's and CDT's is considered, where $R_1 = T_1$, and c=1

We have $P_c = (1 - p_0)1 - (1 + bp/p\mu_1 + (1-p)\,\mu_1)^{-1}$

$= (1 - p_0)(1 - bp/\mu_1)^{-1}$

Therefore, $P_c = 1 - p_0/(1 - bp/\mu_1)$ (15)

Where $P_c$ = Call completion Probability

$p_0$ = New call blocking probability

$\mu_1$ = Service Rate

b*p* =Drop probability

## III. Presentation Discussion of Results

In this section, numerical results are presented in order to illustrate the relationship between loss probabilities and call completion probability. The results provided in this section assume exponentially distributed CDT's and CHT's.

The derivation shows that call completion probability is a function of the block probability, the drop probability and the service rate. This means that the call completion probability ($P_c$), is dependent on the probability that a new call is not blocked ($1 - p_0$), and the probability that an on-going call is not dropped (1-b*p*), which both bear a relationship with the service rate ($\mu_1$).

The failed operations represented in the derivation are the block probability,$p_0$ and the drop probability, b*p*. The relationship between the failed operations and call completion probability is shown using MATLAB.

*po* and b*p* values fall within the same range ($10^{-3}$ and $10^{-2}$) and $\mu_1$is assumed to be 1.The response for different values of *po* and b*p* is illustrated below in MATLAB.

From the numerical results presented using MATLAB, (Figures 3.0, 4.0, and 5.0) it can be observed that call completion probability increases when loss probabilities (block and drop) decreases, at a constant service rate.

Table 3.0 shows the values for *po*, b*p*, Pc and $\mu_1$.

| ***po*, b*p*** | $\mu_1$ | **Pc** |
|---|---|---|
| **$10^{-3}$** | **1** | **0.6952** |
| **$10^{-2}$** | **1** | **0.9893** |

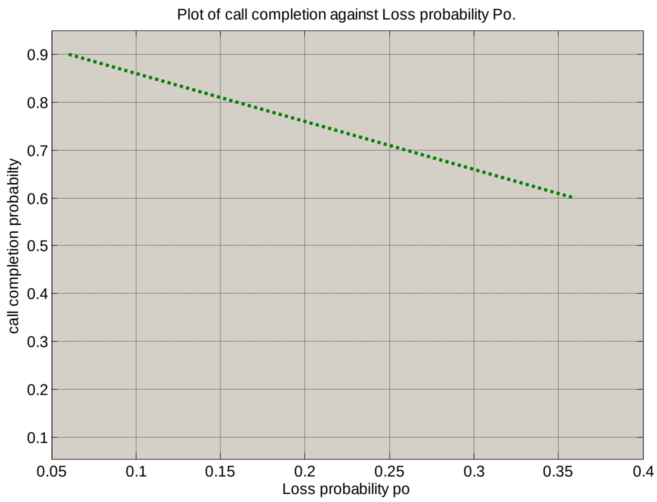


Figure 3.0: Plot of call completion against Loss probability Po.

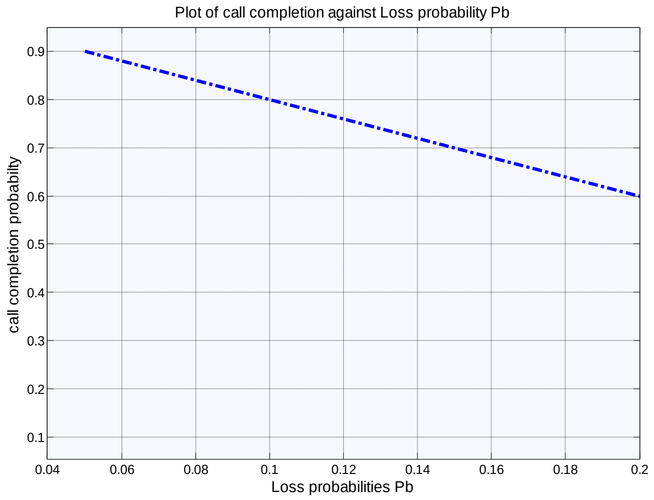


Figure 4.0: Plot of call completion against Loss probability Pb.

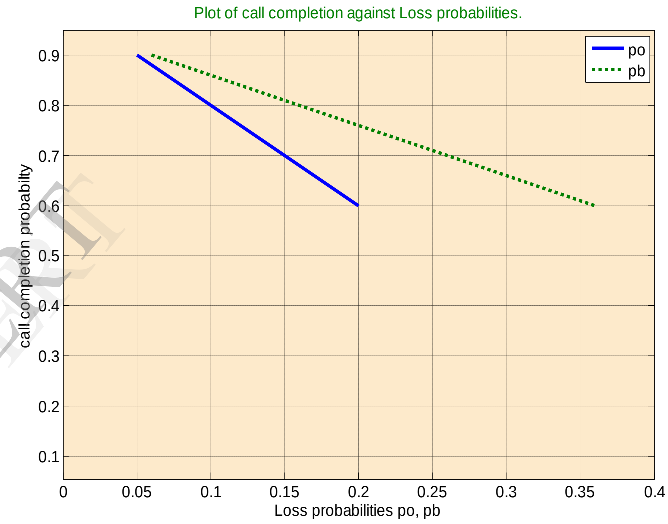


Figure 5.0: Plot of call completion against Loss probabilities Po, Pb.



## IV. Conclusion

The call completion probability for a GSM network under exponentially distributed call holding time and cell dwell times was derived using the random sum approach. The conditional distribution of the geometric random sum was approximated using exponential approximation which resulted in the call completion probability being expressed in terms of the LTF of the CHT distribution.

From the numerical results presented using MATLAB, it was shown that call completion probability increases when loss probabilities (block and drop) decreases, at a constant service rate.

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