

Analysis of a Computer System with Software Redundancy Subject to Hardware Preventive Maintenance

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Abstract :- The present work is devoted to the analysis of a computer system with software redundancy subject to hardware preventive maintenance. For this purpose, a system model has been developed by considering software redundancy. There is an independent hardware and software failure in the system model. The preventive maintenance of the hardware component has been conducted after a specific operation time 't' (called maximum operation time). A single server is provided immediately to the system for carrying out hardware repair, software up-gradation and hardware preventive maintenance as and when needed. The repair activities conducted by the server are perfect. The failure time of the hardware and software components follow negative exponential distribution whereas the distributions for hardware repair, software up-gradation and preventive maintenance times are taken as arbitrary with different probability density functions. Some important reliability characteristics of the system models have been examined stochastically in steady state for particular values of various parameters and costs by using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behaviour of mean time to system failure (MTSF), availability and profit function under different sets of assumptions on the parameters. The profit of the present model has also been compared with the model as discussed in the research paper of [5].

Keywords - Computer System; Software Redundancy; Up-gradation; Preventive Maintenance, Profit Analysis and Stochastic Modelling

I. INTRODUCTION

In past few decades, the importance of computer widely increases in most of areas such as companies, medicals, banks, etc. In the previous research based on computer system, some stochastic models for a computer system have been developed and analyzed in steady state by considering the aspects of component wise redundancy in cold standby, priority in repair disciplines and maximum repair time to hardware. And, a computer system would be

more profitable if redundancy is provided to the hardware along with maximum repair time rather than redundancy to the software. [1] examined a two-unit standby redundant system with preventive maintenance. [2] Analyzed a two dissimilar cold standby system with preventive maintenance and replacement of standby. On the other hand, the deterioration rate of a system can be reduced by conducting its preventive maintenance after a maximum operation time. [3] obtained reliability measures of a system under preventive maintenance. Hence, it becomes necessary to examine the effect of preventive maintenance on reliability measures of a computer system with the concepts of preventive maintenance and software redundancy. However, [4] tried to develop stochastic models of operating systems with preventive maintenance and priority subject to maximum operation and repair times. [6] established reliability measures of a computer system with hardware redundancy subject to preventive maintenance. The present work is devoted to the analysis of a computer system with software redundancy subject to hardware preventive maintenance. For this purpose, a system model has been developed by considering software redundancy in cold standby with the concept of hardware preventive maintenance. Hardware and software failures are independent in the system model. The preventive maintenance of the hardware component has been conducted after a specific operation time 't' (called maximum operation time). There is a single server who visits the system immediately for carrying out hardware repair, software up-gradation and hardware preventive maintenance as and when needed. The repair activities conducted by the server are perfect. The failure time of the hardware and software components follow negative exponential distribution whereas the distributions for hardware repair, software up-gradation and preventive maintenance times are taken as arbitrary with different probability density functions. Some important reliability characteristics of the system models have been examined

stochastically in steady state for particular values of various parameters and costs by using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behaviour of mean time to system failure (MTSF), availability and profit function under different sets of assumptions on the parameters. The profit of the present model has also been compared with the model as discussed in the research paper of [5].

II. SOME IMPORTANT NOTATIONS

O : Computer system is operative
Scs : Software is in cold standby
a/b : Probability that the system has hardware / software failure
 λ_1/λ_2 : Hardware/Software failure rate
 β_0 : The rate by which hardware component undergoes for preventive maintenance
HFUr/HFWr : The hardware is failed and under/waiting for repair
SFUG/SFWUG : The software is failed and under/waiting for up-gradation
HFUPm/HFWPm : The hardware is under/waiting for preventive maintenance
HFUR/HFWR : The hardware is failed and continuously under/ waiting for repair from previous state
SFUG/SFWUG : The software is failed and continuously under /waiting for up- gradation from previous state
HFUPM/HFWPM : The hardware is under/ waiting for preventive maintenance continuously from previous state
g(t)/G(t) : pdf/cdf of hardware repair time
f(t)/F(t) : pdf/cdf of software up-gradation time
m(t)/M(t) : pdf/cdf of hardware preventive maintenance time
q_{ij}(t)/ Q_{ij}(t) : pdf/cdf of first passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in (0, t]
q_{ij,k}(t)/Q_{ij,k}(t) : pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k once in (0, t]
M_i(t) : Probability that the system up initially in state S_i ∈ E is up at time t without visiting to any regenerative state
W_i(t) : Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
 μ_i : The mean sojourn time in state S_i which is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t) dt = \sum_j m_{ij},$$

where T denotes the time to system failure.

m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that

$$\mu_i = \sum_j m_{ij} \text{ and } m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q'_{ij}(0)$$

& /© : Symbol for Laplace - Stieltjes convolution
/ Laplace convolution

*/** : Symbol for Laplace Transformation (LT)
/Laplace Stieltjes Transformation (LST)

φ_i(t) : cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state

A_i(t) : Probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t=0.

B_i^H(t) : Probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state S_i at t = 0.

B_i^S(t) : Probability that the server is busy in up-gradation of the software at an instant 't' given that the system entered the regenerative state S_i at t = 0.

B_i^{Pm}(t) : Probability that the server is busy in preventive maintenance of the hardware given that the system entered state S_i at t=0.

NHR_i(t) : Expected number of hardware repairs by the server in (0, t] given that the system entered the regenerative state S_i at t = 0.

NSU_i(t) : Expected number of software up-gradations in (0, t] given that the system entered the regenerative state S_i at t = 0.

NHPm_i(t):Expected number of hardware preventive maintenances by the server in (0, t] given that the system entered the regenerative state S_i at t = 0.

K₀ : Revenue per unit up – time of the system

K₁ : Cost per unit time for which server is busy due hardware repair

K₂ : Cost per unit time for which server is busy due software Up-gradation

K₃ : Cost per unit repair of the failed hardware

K₄ : Cost per unit up-gradation of the failed software

K₅ : Cost per unit time for which server is busy due hardware preventive maintenance

K₆ : Cost per unit preventive maintenance of the failed hardware

P : Profit of the present model

P1 : Profit of the system model [5]

III. ANALYSIS OF SYSTEM MODEL

The state transition diagram is shown in the following figure:

State Transition Diagram

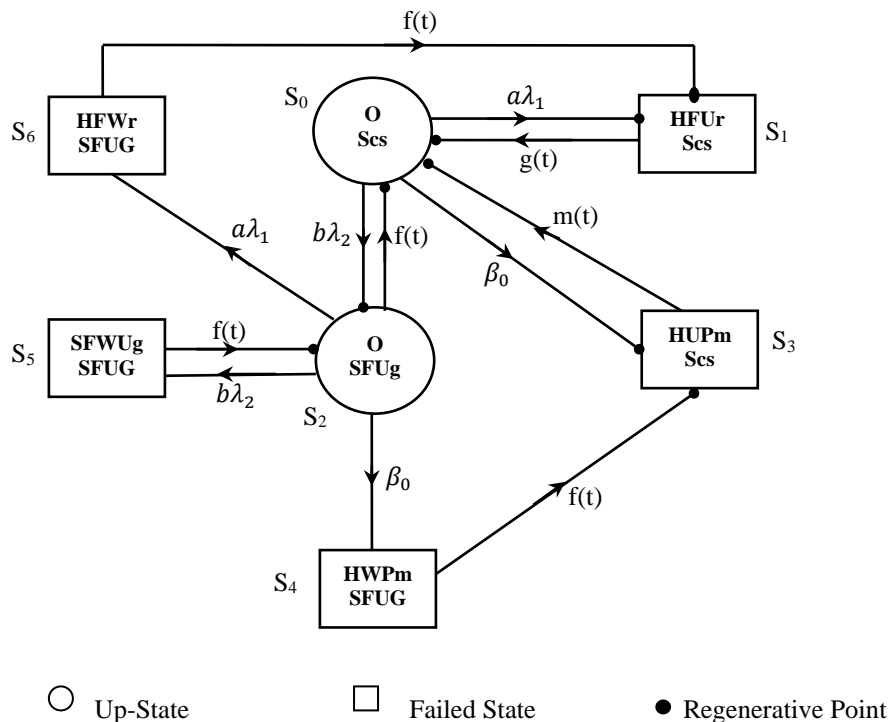


Fig. 1

A. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements.

$$p_{ij} = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t) dt$$

$$p_{01} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \beta_0}, \quad p_{02} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \beta_0}, \quad p_{03} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \beta_0}$$

$$p_{10} = g^*(0), \quad p_{20} = f^*(a\lambda_1 + b\lambda_2 + \beta_0), \quad p_{24} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \beta_0} \{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\},$$

$$p_{25} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \beta_0} \{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\}, \quad p_{26} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \beta_0} \{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\},$$

$$p_{30} = m^*(0), \quad p_{43} = p_{52} = f^*(0)$$

For $f(t) = \theta e^{-\theta t}$, $g(t) = \alpha e^{-\alpha t}$ and $m(t) = \gamma e^{-\gamma t}$

we have

$$p_{23.4} = \frac{\beta_0}{a\lambda_1 + b\lambda_2 + \beta_0} \{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\} f(t)$$

$$p_{22.5} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \beta_0} \{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\} f(t)$$

$$p_{21.6} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \beta_0} \{1 - f^*(a\lambda_1 + b\lambda_2 + \beta_0)\} f(t)$$

$$\text{But, } f^*(0) = g^*(0) = m^*(0) = 1 \text{ and } a + b = 1 \quad (1)$$

It can be easily verified that

$$p_{01} + p_{02} + p_{03} = p_{10} = p_{20} + p_{24} + p_{25} + p_{26} = p_{20} + p_{23.4} + p_{22.5} + p_{21.6} = p_{30} = p_{43} = p_{52} = 1 \quad (2)$$

The mean sojourn times (μ_i) in the state S_i are

$$\mu_0 = \frac{1}{a\lambda_1 + b\lambda_2 + \beta_0}, \quad \mu_1 = \frac{1}{\alpha}, \quad \mu_2 = \frac{1}{a\lambda_1 + b\lambda_2 + \beta_0 + \theta}, \quad \mu_3 = \frac{1}{\gamma}$$

Also

$$\mu_0 = m_{01} + m_{02} + m_{03}, \quad \mu_1 = m_{10},$$

$$\mu_2 = m_{20} + m_{24} + m_{25} + m_{26}$$

$$\text{And } \mu'_2 = m_{20} + m_{21.6} + m_{22.5} + m_{23.4} \quad (3)$$

B. Reliability and Mean Time to System Failure (MTSF)

The recursive relations for $\phi_i(t)$ are as follows:

$$\phi_0(t) = Q_{01}(t) + Q_{02}(t) \quad \& \quad \phi_2(t) = Q_{03}(t)$$

$$\phi_2(t) = Q_{20}(t) \quad \& \quad \phi_0(t) = Q_{24}(t) + Q_{25}(t) + Q_{26}(t) \quad (4)$$

Taking LST of above relations (4) and solving for $\phi_0^{**}(s)$, we have

$$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of the above equation.

The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \rightarrow 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N_1}{D_1} \quad (5)$$

$$\text{Where } N_1 = \mu_0 + p_{02} \text{ and } D_1 = 1 - p_{02}p_{20} \quad (6)$$

C. Steady State Availability

The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) \odot q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t)$$

$$A_1(t) = q_{10}(t) \odot A_0(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{21.6}(t) \odot A_1(t) + q_{22.5}(t) \odot A_2(t) + q_{23.4}(t) \odot A_3(t)$$

$$A_3(t) = q_{30}(t) \odot A_0(t)$$

where

$$M_0(t) = e^{-(a\lambda_1 + b\lambda_2 + \beta_0)t}$$

$$M_2(t) = e^{(a\lambda_1 + b\lambda_2 + \beta_0)t} \overline{F(t)} \quad (7)$$

Taking LT of equation (7) and solving for $A_0^*(s)$, the

steady state availability is given by $A_0(\infty) = \lim_{s \rightarrow 0} sA_0^*(s)$

$$= \frac{N_2}{D_2} \quad (8)$$

Where

$$N_2 = \mu_0(1 - p_{22.5}) + p_{02}\mu_2$$

$$D_2 = \mu_0(1 - p_{22.5}) + \mu_1(p_{01}(1 - p_{22.5}) +$$

$$p_{02}p_{21.6}) + p_{02}\mu_2 + \mu_3(p_{03}(1 - p_{22.5}) + p_{23.4}) \quad (9)$$

D. Busy Period of the Server

(a). Due to Hardware Repair

The recursive relations for $B_i^H(t)$ are as follows:

$$B_0^H(t) = q_{01}(t) \odot B_1^H(t) + q_{02}(t) \odot B_2^H(t) + q_{03}(t) \odot B_3^H(t)$$

$$B_1^H(t) = W_1^H(t) + q_{10}(t) \odot B_0^H(t)$$

$$B_2^H(t) = q_{20}(t) \odot B_0^H(t) + q_{21.6}(t) \odot B_1^H(t) + q_{22.5}(t) \odot B_2^H(t) + q_{23.4}(t) \odot B_3^H(t)$$

$$B_3^H(t) = q_{30}(t) \odot B_0^H(t) \quad (10)$$

where $W_1^H(t) = \overline{G(t)}$

(b). Due to Software Up-gradation

The recursive relations for $B_i^S(t)$ are as follows:

$$B_0^S(t) = q_{01}(t) \odot B_1^S(t) + q_{02}(t) \odot B_2^S(t) + q_{03}(t) \odot B_3^S(t)$$

$$B_1^S(t) = q_{10}(t) \odot B_0^S(t)$$

$$B_2^S(t) = W_2^S(t) + q_{20}(t) \odot B_0^S(t) + q_{21.6}(t) \odot B_1^S(t) + q_{22.5}(t) \odot B_2^S(t) + q_{23.4}(t) \odot B_3^S(t)$$

$$B_3^S(t) = q_{30}(t) \odot B_0^S(t) \quad (11)$$

where $W_2^S(t) = e^{(a\lambda_1 + b\lambda_2 + \beta_0)t} \overline{F(t)} +$

$$(a\lambda_1 e^{(a\lambda_1 + b\lambda_2 + \beta_0)t} \odot 1) \overline{F(t)} +$$

$$(b\lambda_2 e^{(a\lambda_1 + b\lambda_2 + \beta_0)t} \odot 1) \overline{F(t)} + (\beta_0 e^{(a\lambda_1 + b\lambda_2 + \beta_0)t} \odot 1) \overline{F(t)}$$

(c). Due to Preventative Maintenance

The recursive relations for $B_i^{Pm}(t)$ are as follows:

$$B_0^{Pm}(t) = q_{01}(t) \odot B_1^{Pm}(t) + q_{02}(t) \odot B_2^{Pm}(t) + q_{03}(t) \odot B_3^{Pm}(t)$$

$$B_1^{Pm}(t) = q_{10}(t) \odot B_0^{Pm}(t)$$

$$B_2^{Pm}(t) = q_{20}(t) \odot B_0^{Pm}(t) + q_{21.6}(t) \odot B_1^{Pm}(t) + q_{22.5}(t) \odot B_2^{Pm}(t) + q_{23.4}(t) \odot B_3^{Pm}(t)$$

$$B_3^{Pm}(t) = W_3^{Pm}(t) + q_{30}(t) \odot B_0^{Pm}(t) \quad (12)$$

where $W_3^{Pm}(t) = \overline{M(t)}$

Taking LT of relations (10), (11) & (12), solving

for $B_0^H(t)$, $B_0^S(t)$ and $B_0^{Pm}(t)$. The time for which server is busy due to repairs, up-gradations and preventative maintenances respectively are given by

$$B_0^H(t) = \lim_{s \rightarrow 0} s B_0^{H*}(t) = \frac{N_3^H}{D_2} \quad (13)$$

$$B_0^S(t) = \lim_{s \rightarrow 0} s B_0^{S*}(t) = \frac{N_3^S}{D_2} \quad (14)$$

$$B_0^{Pm}(t) = \lim_{s \rightarrow 0} s B_0^{Pm*}(t) = \frac{N_3^{Pm}}{D_2} \quad (15)$$

where $N_3^H = p_{01}(p_{10}(1 - p_{22.5}) + p_{02}p_{21.6})W_1^{H*}(0)$

$$N_3^S = p_{02}W_2^{S*}$$

$$N_3^{Pm} = (p_{02}p_{23.4}p_{30} + p_{03}p_{30}(1 - p_{22.5}^{(7)}))W_3^{Pm*}(0)$$

and D_2 is already mentioned. (16)

E. Expected Number of Hardware Repairs

The recursive relations for $NHR_i(t)$ are given as:

$$NHR_0(t) = Q_{01}(t) \& NHR_1(t) + Q_{02}(t) \& NHR_2(t) + Q_{03}(t) \& NHR_3(t)$$

$$NHR_1(t) = Q_{10}(t) \& (1 + NHR_0(t))$$

$$NHR_2(t) = Q_{20}(t) \& NHR_0(t) + Q_{21.6}(t) \& NHR_1(t)$$

$$+ Q_{22.5}(t) \& NHR_2(t)$$

$$+ Q_{23.4}(t) \& NHR_3(t)$$

$$NHR_3(t) = Q_{30}(t) \& NHR_0(t) \quad (17)$$

Taking LST of relations (17) and solving for $NHR_0^{**}(s)$.

The expected number of hardware repairs are given by

$$NHR_0 = \lim_{s \rightarrow 0} s NHR_0^{**}(s) = \frac{N_4}{D_2} \quad (18)$$

where

$$N_4 = p_{01}(p_{10}(1 - p_{22.5}) + p_{02}p_{21.6}) \text{ and } D_2 \text{ is already mentioned.} \quad (19)$$

F. Expected Number of Software Up-gradations

The recursive relations for $NSU_i(t)$ are given as:

$$NSU_0(t) = Q_{01}(t) \& NSU_1(t) + Q_{02}(t) \& NSU_2(t) + Q_{03}(t) \& NSU_3(t)$$

$$NSU_1(t) = Q_{10}(t) \& NSU_0(t)$$

$$NSU_2(t) = Q_{20}(t) \& (1 + NSU_0(t)) + Q_{21.6}(t) \&$$

$$(1 + NSU_1(t)) + Q_{22.5}(t) \& (1 + NSU_2(t)) + Q_{23.4}(t) \& (1 + NSU_3(t))$$

$$NSU_3(t) = Q_{30}(t) \& NSU_0(t) \quad (20)$$

Taking LST of relations (20) and solving for $NSU_0^{**}(s)$. The

expected numbers of software up-gradations are given by

$$NSU_0(\infty) = \lim_{s \rightarrow 0} s NSU_0^{**}(s) = \frac{N_5}{D_2} \quad (21)$$

Where $N_5 = p_{02}$ and D_2 is already mentioned. (22)

G. Expected Number of Preventative Maintenance

The recursive relations for $NHPm_i(t)$ are given as:

$$NHPm_0(t) = Q_{01}(t) \& NHPm_1(t) + Q_{02}(t) \&$$

$$NHPm_2(t) + Q_{03}(t) \& NHPm_3(t)$$

$$NHPm_1(t) = Q_{10}(t) \& NHPm_0(t)$$

$$NHPm_2(t) = Q_{20}(t) \& NHPm_0(t)$$

$$+ Q_{21.6}(t) \& NHPm_1(t)$$

$$+ Q_{22.5}(t) \& NHPm_2(t)$$

$$+ Q_{23.4}(t) \& NHPm_3(t)$$

$$NHPm_3(t) = Q_{30}(t) \& (1 + NHPm_0(t)) \quad (23)$$

Taking LST of relations (23) and solving for $NHPm_0^{**}(s)$. The expected numbers of hardware preventative maintenances are given by

$$NHPm_0 = \lim_{s \rightarrow 0} sNHPm_0^{**}(s) = \frac{N_6}{D_2} \quad (24)$$

where $N_6 = p_{02}p_{23.4}p_{30} + p_{03}p_{30}(1 - p_{22.5})$ and D_2 is already mentioned. (25)

IV. PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as:

$$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_3NHR_0 - K_4NSU_0 - K_5B_0^{Pm} - K_6NHPm_0 \quad (26)$$

where $A_0, B_0^H, B_0^S, NHR_0, NSU_0, B_0^{Pm}, NHPm_0$ are already defined.

V. PARTICULAR CASES

Suppose $g(t) = \alpha e^{-\alpha t}, f(t) = \theta e^{-\theta t}$ and $m(t) = \gamma e^{-\gamma t}$

We can obtain the following results:

$$MTSF(T_0) = \frac{N_1}{D_1}, \text{ Availability}(A_0) = \frac{N_2}{D_2}$$

Busy Period due to hardware

$$\text{failure } (B_0^H) = \frac{N_3^H}{D_2}$$

Busy Period due to software

$$\text{failure } (B_0^S) = \frac{N_3^S}{D_2}$$

Busy Period due to hardware preventive

$$\text{maintenance } (B_0^{Pm}) = \frac{N_3^{Pm}}{D_2}$$

Expected number of repair at hardware

$$\text{failure } (NHR_0) = \frac{N_4}{D_2}$$

Expected number of up – gradation at

$$\text{software failure } (NSU_0) = \frac{N_5}{D_2}$$

Expected number of preventive maintenance

$$\text{of the hardware } (NHPm_0) = \frac{N_6}{D_2}$$

Where

$$N_1 = \frac{1 + b\lambda_2}{(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

$$D_1 = \frac{(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta) - \theta b\lambda_2}{(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

$$N_2 = \frac{(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}{(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

$$D_2 = \frac{(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)(\alpha\gamma\theta(a\lambda_1 + \beta_0 + \theta) + \gamma\theta a\lambda_1(a\lambda_1 + b\lambda_2 + \beta_0 + \theta) + \alpha\theta(\beta_0(a\lambda_1 + \beta_0 + \theta) + \beta_0(a\lambda_1 + b\lambda_2 + \beta_0))) + \alpha\gamma b\lambda_2(\theta^2 + (a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + 2\theta))}{\theta\gamma\alpha(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

$$N_3^H = \frac{a\lambda_1((a\lambda_1 + \beta_0 + \theta)(a\lambda_1 + b\lambda_2 + \beta_0) + a\lambda_1 b\lambda_2)}{\alpha(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)(a\lambda_1 + b\lambda_2 + \beta_0)}$$

$$N_3^S = \frac{b\lambda_2}{(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

$$N_3^{Pm} = \frac{\beta_0(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}{\gamma(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

$$N_4 = \frac{a\lambda_1((a\lambda_1 + \beta_0 + \theta)(a\lambda_1 + b\lambda_2 + \beta_0) + a\lambda_1 b\lambda_2)}{(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)(a\lambda_1 + b\lambda_2 + \beta_0)}$$

$$N_5 = \frac{b\lambda_2}{(a\lambda_1 + b\lambda_2 + \beta_0)}$$

$$N_6 = \frac{\beta_0(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}{(a\lambda_1 + b\lambda_2 + \beta_0)(a\lambda_1 + b\lambda_2 + \beta_0 + \theta)}$$

VI. CONCLUSION

The reliability measures of a computer system have been obtained by assuming $g(t) = \alpha e^{-\alpha t}, f(t) = \theta e^{-\theta t}$ and $m(t) = \gamma e^{-\gamma t}$. And, graphs are drawn for the particular results of these measures as shown in figures 2 to 4. It is analyzed that the mean time to system failure (MTSF), availability and profit go on decreasing with the increase of failure rates (λ_1 and λ_2) and the rate (β_0) by which hardware undergoes for preventive maintenance after a pre specific operation time 't' while their values keep on moving up with the increase of hardware repair rate (α), software up-gradation rate (θ) and preventive maintenance rate (γ) provided system has more chances of hardware failure. Further, it is interesting to note that a computer system has more values of these reliability measures when it has more chances of software failure than that of hardware failure. The graphical presentation of the results related to these reliability measures obtained for the system model is shown in the figures 2 to 4.

VII. GRAPHICAL PRESENTATION OF RELIABILITY MEASURES FOR THE SYSTEM MODEL

Fig. 2: MTSF Vs Hardware Failure Rate (λ_1)

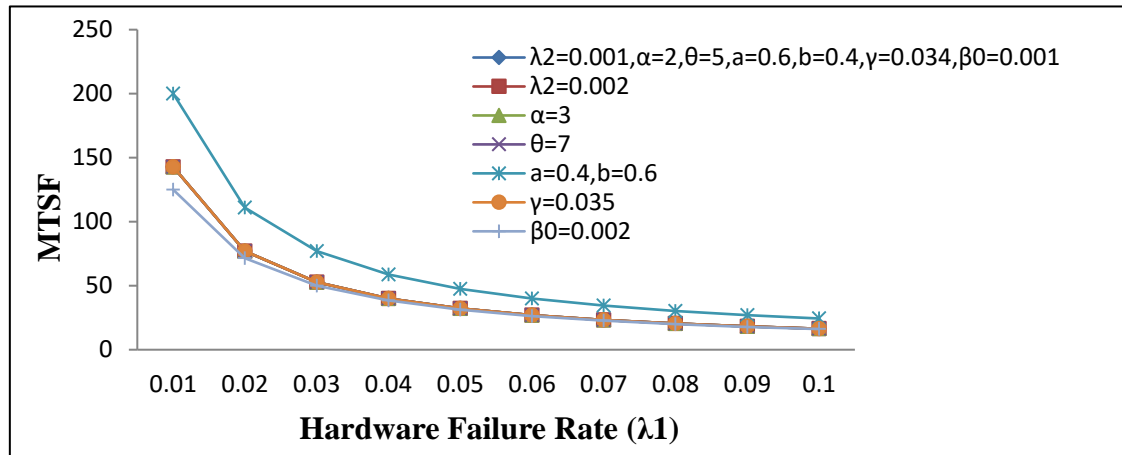


Fig. 3: Availability Vs Hardware Failure Rate (λ_1)

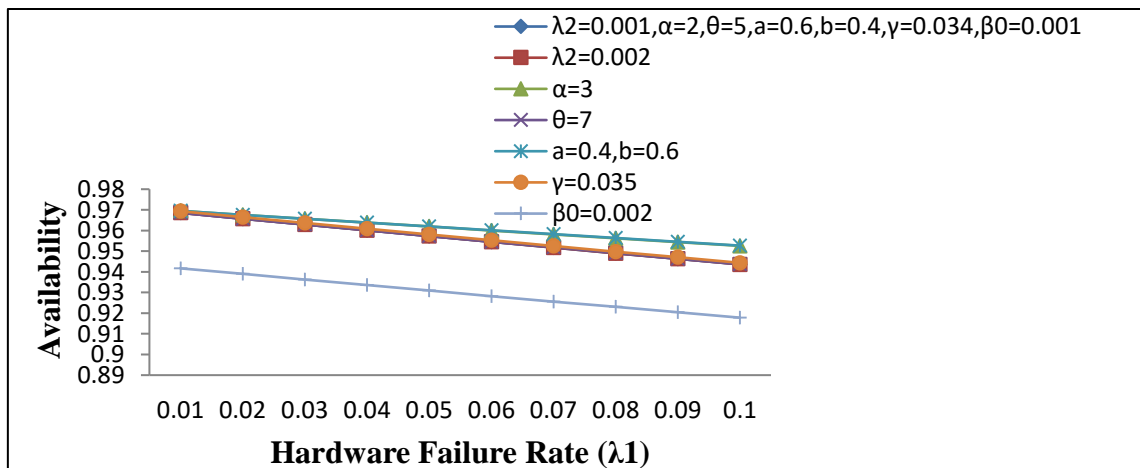
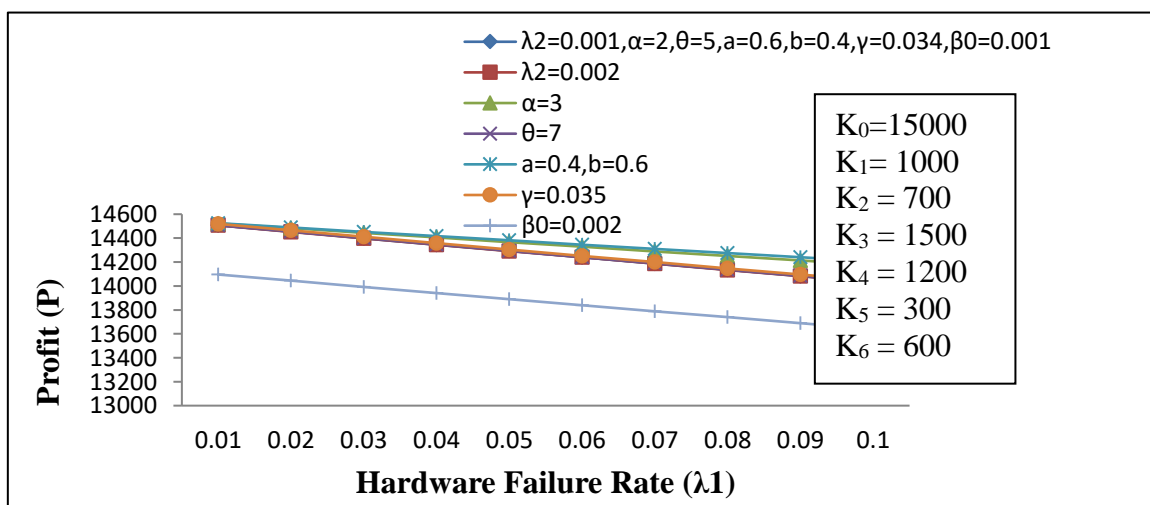


Fig. 4: Profit (P) Vs Hardware Failure Rate (λ_1)



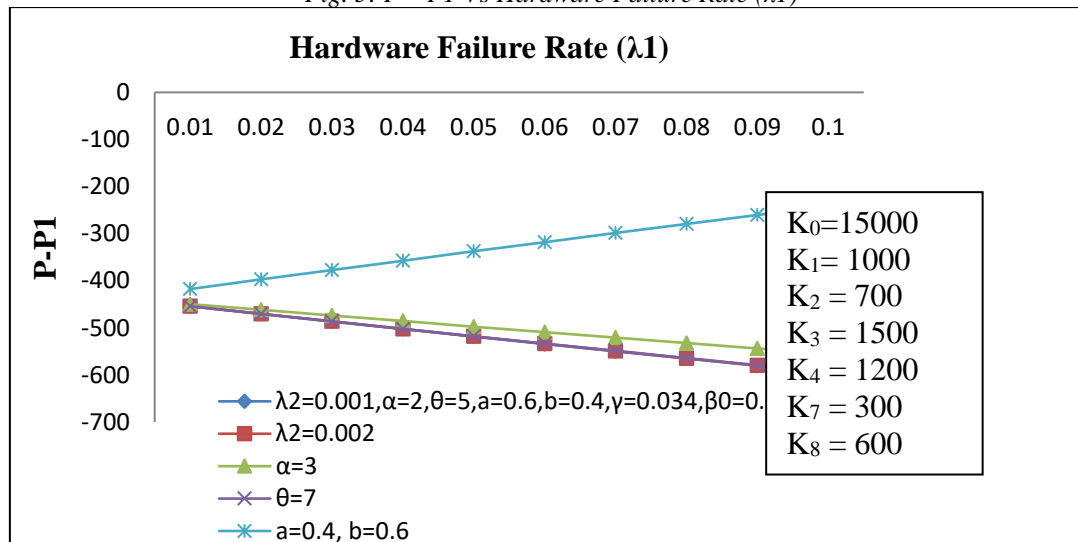
VIII. COMPARATIVE STUDY OF PROFIT OF SYSTEM MODELS

The profit of the present model has been compared with that of the model discussed in the research paper Munday and Malik (2015). It is revealed that the present model is less profitable. And, hence we can say that

the concept of hardware preventive maintenance in a computer system with software redundancy in cold standby is not much helpful in making the system more profitable. The behavior of the profit difference of the system model with respect to hardware failure rate (λ_1) has been shown graphically in figure 5.

IX. GRAPHICAL PRESENTATION OF PROFIT DIFFERENCES

Fig. 5: $P - P_1$ Vs Hardware Failure Rate (λ_1)



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