Analysis clonal algorithm based window Using Fractional Fourier Transform

K. Prasada Rao 1, A.S. Srinivasa Rao 2, P.V. Muralidhar 3

1. Aditya Institute of Technology and Management, Tekkali – 532001 (A. P., India)
2. Professor & HOD, Dept. of ECE. College of Engineering, AITAM, Tekkali – 532001 (A. P., India)
3. Associate Professor, Dept. of ECE. College of Engineering, AITAM, Tekkali – 532001 (A. P., India)

Abstract
An Approximated Exponential Fractional Fourier Transforms (FrFT) Mathematical derivation for clonal algorithm based window is proposed. By control the parameter of FrFT, it is possible to control the Spectral parameters of above windows like Half Bandwidth (HBW), Maximum Side Lobe Attenuation (MSLA) and Side Lobe Fall Of Ratio (SLFOR). This proposed derivation is also holds good for generalization of FrFT with Fourier Transform (FT).

Index Terms—Fractional Fourier transform, clonal algorithm based window.

NOMENCLATURE
FT: Fourier Transform
FrFT: Fractional Fourier Transform

1. INTRODUCTION
In order to reduce the effects of spectral leakages in Harmonic analysis, windows are used [1]. window functions successfully used in the areas like interpolation factors to design Anti-Imaging filters, speech processing systems, digital filter design and beam forming [2]-[3]. windows are also useful to solve reconstructive errors which are objective functions to design the prototype filters [4]. windows are essentially Applicable in spectral analysis of signals [5]-[6]. According to [3], as the parameter of FrFT, $\alpha = \frac{\pi}{2}$ which could not holds good for generalization of FrFT to FT [7]. In this proposed Derivation of FrFT, An attempt is made to study the variations of window parameters like HBW, MSLA and MSLFOR by different values of fluid parameter of FrFT to FT at $\alpha = \frac{\pi}{2}$. This paper is organized as follows. section-II gives an overview of FT, and mathematical model of windows by using FT. section-III gives an overview of FrFT, and mathematical model of windows by using FRFT. Later conclusive remarks are discussed in section-IV.

2. Fractional Fourier Transform
Fractional Fourier Transform widely used in quantum mechanics and quantum optics [13]. Fractional Fourier Analysis can obtain the mixed time and frequency components of signals [14]. It finds various applications like pattern recognisition with some spatial distortion, Image representation, compression and noise removal in signal processing [15]-[17]. FrFT used for Interpretation of sinusoidal signals and design of Digital FIR Filters [18]-[19]. The continuous time Fractional Fourier Transform of a signal

$\omega(t)$ is defined through an interval [3]

$$\omega_\alpha(u) = \int_{-\alpha}^{\alpha} \omega(t) K_\alpha(t, u) dt$$  (1)

Where the transform kernel $K_\alpha(t, u)$ of the FRFT is Given by

$$K_\alpha(t, u) = \sqrt{\frac{1 - j\cot(\alpha)}{2\pi}} e^{i\left(\frac{t^2 + u^2}{2}\right)\cot(\alpha) - iut\cosec(\alpha)}$$

if $\alpha$ is multiple of $\pi$ $\theta(t - u)$ if $\alpha$ is multiple of $2\pi$

$= \delta(t + u)$ if $\alpha + \pi$ is a multiple of $2\pi$  (2)

Where $\alpha$ indicates rotation of angle of the Transformed signal for FrFT.
2.1) window function based on Clonal Algorithm

The Expression for clonal Algorithm based window is [16]

\[ \omega(t) = 0.5154 - 0.4711 \cos(2\pi t) + 0.0135 \cos(4\pi t) \text{ for } |t| < 1 \]
\[ = 0 \text{ otherwise.} \]

\[ \omega_\alpha(u) = \int_0^1 \omega(t) \sqrt{1 - \cot(\alpha)} \exp \left( \frac{t^2 + u^2}{2} \cot(\alpha) - i u \cot(\alpha) \right) dt \]

Then equation-(21) becomes

\[ \omega_\alpha(u) = p \int_0^1 \omega(t) \exp \left( \frac{t^2}{2} \cot(\alpha) - i u \cot(\alpha) \right) dt \]

Substitute equation-(20) in equation-(23) then

\[ I_3 = \int_0^1 (0.5154 \cos(2\pi t) + 0.0135 \cos(4\pi t)) \exp \left( \frac{t^2}{2} \cot(\alpha) - i u \cot(\alpha) \right) dt \]

Now solving for I_3

\[ I_3 = 0.5154 \left( \exp(\frac{i2\pi t}{2}) - \exp(i2\pi t) \right) - \exp(\frac{i2\pi t}{2}) \]

Integration above equation and applying limits, you will get

\[ I_3 = 0.5154 \left( \exp(\frac{i2\pi t}{2}) - \exp(i2\pi t) \right) \]

Now solving for I_4

\[ I_4 = \int_0^1 (0.4711 \cos(2\pi t) + 0.0135 \cos(4\pi t)) \exp \left( \frac{t^2}{2} \cot(\alpha) - i u \cot(\alpha) \right) dt \]

According to [17]

\[ \cos(2\pi t) = \exp(i2\pi t) + \exp(-i2\pi t) \]

Substitute equation-(11) in equation-(10) then

\[ I_4 = 0.23555(\int_0^1 \exp(i2\pi t) \exp \left( \frac{t^2}{2} \cot(\alpha) - i u \cot(\alpha) \right) dt) \]

Now multiply both sides with \( e^{-\frac{1}{2}t^2\cot(\alpha)} \) which results to

\[ e^{-\frac{1}{2}t^2\cot(\alpha)} I_4 = 0.23555(\int_0^1 \exp(i2\pi t) \exp \left( \frac{t^2}{2} \cot(\alpha) - i u \cot(\alpha) \right) dt) \]

Now Integrate both sides with limits \( t_1 \) and \( t_2 \)
\[
\int_{t_1}^{t_2} \left( e^{-\frac{t^2}{2} \cot(\alpha)} \right) dt = \int_{t_1}^{t_2} \left( 0.23555 \left( \frac{\exp(i2\pi t + iut \csc(\alpha))}{i2\pi - iu \csc(\alpha)} - \frac{\exp(i2\pi t - iut \csc(\alpha))}{-i2\pi - iu \csc(\alpha)} \right) \right) dt
\]

The above equation can be written as

\[
I_4 = \int_{t_1}^{t_2} \left( 0.23555 \left( \frac{\exp(i2\pi t + iut \csc(\alpha))}{i2\pi - iu \csc(\alpha)} - \frac{\exp(i2\pi t - iut \csc(\alpha))}{-i2\pi - iu \csc(\alpha)} \right) \right) dt
\]

Now equation-(18) can be divided into two parts \( x_1 \) and \( x_2 \)

Where

\[
x_1 = \int_{t_1}^{t_2} \left( 0.23555 \left( \frac{\exp(i2\pi t + iut \csc(\alpha))}{i2\pi - iu \csc(\alpha)} - \frac{\exp(i2\pi t - iut \csc(\alpha))}{-i2\pi - iu \csc(\alpha)} \right) \right) dt
\]

\[
x_2 = \int_{t_1}^{t_2} \left( e^{-\frac{t^2}{2} \cot(\alpha)} \right) dt
\]

Now solving for \( x_1 \)

Integrating \( x_1 \) and applying limits

\[
x_1 = 0.23555 \left( \frac{\exp(i2\pi t_2 + iut \csc(\alpha))}{i2\pi - iu \csc(\alpha)} - \frac{\exp(i2\pi t_1 - iut \csc(\alpha))}{-i2\pi - iu \csc(\alpha)} \right) (t_2 - t_1) \]

Now solving for \( x_2 \)

\[
x_2 = \int_{t_1}^{t_2} \left( e^{-\frac{t^2}{2} \cot(\alpha)} \right) dt
\]

According to [17]

\[
e^{-\frac{t^2}{2} \cot(\alpha)} = 1 - i \frac{t^2}{2} \cot(\alpha) \]

Substitute equation-(21) in equation-(20) to get

\[
x_2 = \int_{t_1}^{t_2} \left( 1 - i \frac{t^2}{2} \cot(\alpha) \right) dt \]

Integrating and applying limits to get

\[
x_2 = \int_{t_1}^{t_2} dt - \frac{icot(\alpha)}{2} \int_{t_1}^{t_2} dt \]

\[
x_2 = (t_2 - t_1) - \frac{icot(\alpha)}{6} (t_2^3 - t_1^3)
\]

Finally \( I_5 \)

\[
I_5 = \frac{0.23555 \left( \frac{\exp(i2\pi t_2 - iut \csc(\alpha))}{i2\pi - iu \csc(\alpha)} - \frac{\exp(i2\pi t_1 - iut \csc(\alpha))}{-i2\pi - iu \csc(\alpha)} \right) (t_2 - t_1)}{t_2 - t_1}
\]

Now solving for \( I_5 \)

\[
I_5 = \int_{t_1}^{t_2} 0.01 \left( \exp(i4\pi t) \right) \cos(4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt
\]

According to [17]

\[
\cos(4\pi t) = \frac{\exp(i4\pi t) + \exp(-i4\pi t)}{2}
\]

Substitute equation-(27) in equation-(26) then

\[
I_5 = \int_{t_1}^{t_2} \left( \frac{0.4711}{2} \left( \exp(i4\pi t) + \exp(-i4\pi t) \right) \right) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt
\]

\[
I_5 = 0.23555 \left( \frac{\int_{t_1}^{t_2} \exp(i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt}{\int_{t_1}^{t_2} \exp(-i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt} \right)
\]

Now multiply both sides with \( e^{-\frac{t^2}{2} \cot(\alpha)} \) which results to

\[
e^{-\frac{t^2}{2} \cot(\alpha)} I_5 = \frac{0.23555 \left( \frac{\int_{t_1}^{t_2} \exp(i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt}{\int_{t_1}^{t_2} \exp(-i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt} \right)}{\int_{t_1}^{t_2} \exp(-i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt}
\]

Integration above equation and applying limits, you get

\[
e^{-\frac{t^2}{2} \cot(\alpha)} I_5 = 0.23555 \left( \int_{t_1}^{t_2} \exp(i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt \right)
\]

\[
I_5 = \frac{0.23555 \left( \frac{\int_{t_1}^{t_2} \exp(i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt}{\int_{t_1}^{t_2} \exp(-i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt} \right)}{\int_{t_1}^{t_2} \exp(-i4\pi t) \exp \left( i \frac{t^2}{2} \cot(\alpha) - iut \csc(\alpha) \right) dt}
\]
will get
\[
e^{-i\frac{t^2}{2}\cot(\alpha)}I_5
= 0.23555 \left( \frac{\exp(i4\pi t_2 - iut_2 \csc(\alpha))}{i4\pi - iucsc(\alpha)} \right. \\
\left. - \exp(-i4\pi t_1 - iut_1 \csc(\alpha)) \right) - (32)
\]

Now Integrate both sides with limits \(t_1\) and \(t_2\)

\[
\int_{t_1}^{t_2} \left( e^{-i\frac{t^2}{2}\cot(\alpha)}I_5 \right) dt \\
= \int_{t_1}^{t_2} \left( 0.23555 \left( \frac{\exp(i4\pi t_2 - iut_2 \csc(\alpha))}{i4\pi - iucsc(\alpha)} \right. \\
\left. - \exp(-i4\pi t_1 - iut_1 \csc(\alpha)) \right) \right) dt - - - (33)
\]

The above equation can be written as

\[
I_5 = 0.23555 \left( \frac{\exp(i4\pi t_2 - iut_2 \csc(\alpha))}{i4\pi - iucsc(\alpha)} \right. \\
\left. - \exp(-i4\pi t_1 - iut_1 \csc(\alpha)) \right) \right) \left( t_2 - t_1 \right) - - - (34)
\]

Now equation-(34) can be divided into two parts \(x_1\) and \(x_2\)

Where

\[
x_1 = \int_{t_1}^{t_2} \left( 0.23555 \left( \frac{\exp(i4\pi t_2 - iut_2 \csc(\alpha))}{i4\pi - iucsc(\alpha)} \right. \\
\left. - \exp(-i4\pi t_1 - iut_1 \csc(\alpha)) \right) \right) dt
\]

And

\[
x_2 = \int_{t_1}^{t_2} \left( e^{-i\frac{t^2}{2}\cot(\alpha)} \right) dt
\]

Now solving for \(x_1\)

Integrating \(x_1\) and applying limits

\[
x_1 = 0.23555 \left( \frac{\exp(i4\pi t_2 - iut_2 \csc(\alpha))}{i4\pi - iucsc(\alpha)} \right. \\
\left. - \exp(-i4\pi t_1 - iut_1 \csc(\alpha)) \right) \left( t_2 - t_1 \right) - - - (35)
\]

Now solving for \(x_2\)

\[
x_2 = \int_{t_1}^{t_2} \left( e^{-i\frac{t^2}{2}\cot(\alpha)} \right) dt - - - (36)
\]

According to [17]

\[
e^{-i\frac{t^2}{2}\cot(\alpha)} = 1 - i\frac{t^2}{2}\cot(\alpha) - - - (37)
\]

Substitute equation-(37) in equation-(20) to get

\[
x_2 = \left( t_2 - t_1 \right) - i\frac{\cot(\alpha)}{6} \left( t_2^3 - t_1^3 \right) - - - (39)
\]

Finally \(I_5\)

\[
I_5 = \left( \frac{\exp(i4\pi t_2 - iut_2 \csc(\alpha)) \exp(-i4\pi t_1 - iut_1 \csc(\alpha))}{i4\pi - iucsc(\alpha)} \right) \left( t_2 - t_1 \right) - - - (40)
\]

Finally

\[
\omega_4(\alpha) = I_3 - I_4 + I_5 - - - (41)
\]

Thus equation-(41) is the FRFT based clonal algorithm based window.

When substitute \(\alpha = \frac{\pi}{2}\) where \(\alpha = 1\) in equation-(41) results to generalized Fourier Transform based clonal algorithm based window.

The spectral parameters of above window is shown in Table and it’s spectral responses are shown from Fig1 to Fig6.

Table:

Spectral Parameters of FrFT Based clonal algorithm based window for variations in \(\alpha\)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>SLA in dB</th>
<th>HBW in dB</th>
<th>SLFOR in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.93</td>
<td>-57</td>
<td>0.0195</td>
<td>-73.94</td>
</tr>
<tr>
<td>0.94</td>
<td>-60.8</td>
<td>0.0195</td>
<td>-71.85</td>
</tr>
<tr>
<td>0.95</td>
<td>-65.5</td>
<td>0.0185</td>
<td>-71.87</td>
</tr>
<tr>
<td>0.96</td>
<td>-63.9</td>
<td>0.0185</td>
<td>-72.55</td>
</tr>
<tr>
<td>0.97</td>
<td>-62.2</td>
<td>0.0185</td>
<td>-72.09</td>
</tr>
<tr>
<td>0.98</td>
<td>-61.0</td>
<td>0.0185</td>
<td>-70.76</td>
</tr>
<tr>
<td>0.99</td>
<td>-60.3</td>
<td>0.0185</td>
<td>-69.49</td>
</tr>
<tr>
<td>1</td>
<td>-60</td>
<td>0.0185</td>
<td>-69.1</td>
</tr>
</tbody>
</table>
3 Conclusion:

From the study of Exponential derivation of FrFT for clonal algorithm based window, the controllability of window parameters like HBW, MSLA and SLFOR is possible..i.e. The MSLA increases from -60 dB to -65.5 dB for \( a = 1 \) to \( a = 0.95 \) decreases to -57 dB for \( a = 0.93 \).[Table] so that from the table it is observed that the spectral parameters of clonal based window is improved by controlling the ‘a’ parameter of FrFT. and also one of the property of FrFT is Generalization of FrFT to FT i.e. when \( a = \frac{\alpha \pi}{2} \) where \( a = 1 \); then The FrFT should equals to FT .This proposed Mathematical derivation of FrFT fulfills the Property of FT.

4:References


Prasada rao Koppala obtained bachelor degree under JNTU Hyderabad, pursuing M.Tech Under JNTU Kakinada His areas of interest are Automation of Industrial process, Tuning of Controllers, Signal Processing and Designing of Digital Filters
Adari Satya Srinivasa Rao received his M Tech. degree from Andhra University, Vishakapatnam in 2004 and he is a PhD student in Andhra University. He is having 15 years of teaching experience in various engineering colleges. Presently, he is working as Professor, Department of ECE, Aditya Institute of Technology and Management, Tekkali. His interests include signal processing, adaptive antenna arrays and communication systems.

First Other: P.V. Muralidhaar obtained M. Tech from JNTU, Hyderabad, pursuing PhD from Berhampur University. He is having an experience more than 10 years and also having more number of both national and international journals, conferences. His area of interest is signal processing, presently working with AITAM, Tekkali, Srikakulam, A.P.