Analysis and Synthesis of Wire Form Springs

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***Abstract*—Wire form springs are made by bending spring wires into various shapes and designed to perform many different functions. There are a multitude of applications for wire form springs, which include load carrying links, clips, clamps, electrical resistance units, decoration items, or furniture parts. Although wire form springs have been widely used for various applications, there are no general design formulas or equations for them. Wire form springs are mainly designed by try-and-error or intuitive approaches. In this paper, a systematic method is introduced for analyzing and synthesizing wire from springs. To analyze a wire form spring, a B-spline curve is first employed to interpolate the center curve of the wire form spring. The stress and deflection of the wire form spring are then analyzed based on its piecewise parametric interpolation. To synthesize a wire form spring, the center curve of the wire form spring is first modelled as a B-spline curve and described by the control polygon of the B-spline curve. The synthesis of a wire form spring then becomes the optimization of the control parameters of the B-spline curve. Examples on the analysis and synthesis of wire form springs are presented in the paper to verify the effectiveness and demonstrate the procedure of the introduced method.**

Keywords—wire form spring; analysis; synthesis; B-spline; interpolation; control parameter.

I. INTRODUCTION

Wire form springs are made by bending spring wires into various shapes and designed to perform many different functions [1]. There are a multitude of applications for wire form springs, which include load carrying links, clips, clamps, electrical resistance units, decoration items, or furniture parts. Springs are usually classified into four general categories based on their primary functions: push, pull, twist or energy storage [2]. Besides these primary functions, wire form springs are also applied for other functions such as decorating, clipping or clamping purposes.

There are a lot of different configurations for springs. Based on configuration features, springs are commonly divided into types that include helical, conical, spiral, washer, beam, leaf, volute, garter [3]. In a configuration type, a pattern is often followed and some feature dimensions are used to define the specific configuration (structural details or shape and size) of a spring in the type. For example, the configuration of a helical compression spring is defined by its coil diameter, coil pitch, number of coils and wire diameter. A spiral torsion spring has spring wire wound on itself (zero pitch) and open space between coils. The center curve of a spiral torsion spring is usually an Archimedean spiral curve, so the configuration of a spiral torsion spring is defined by its center Archimedean spiral curve, number of turnings and cross sectional sizes.

Wire form springs have various shapes for miscellaneous applications. It is difficult to use a single configuration pattern or feature (such as helix for helical springs) to characterize and define their shapes. Although wire form springs have been widely used for various applications and purposes, there are still no general design formulas or equations for them [4]. Wire form springs are mainly designed by try-and-error or intuitive approaches. Practical wire form springs are commonly composed of arcs and their tangent lines. Although arcs and their tangents are easy and convenient to apply, the shape varieties and flexibilities of wire form springs are compromised by using them only.

The research motivation of this work is to establish general formulas for analyzing and synthesizing wire form springs. The research objectives of this work are that: (1) the shape varieties and flexibilities of wire form springs are not compromised in the introduced analysis and synthesis formulas; (2) it is easy and convenient to use the introduced analysis and synthesis formulas. The research outcomes of this work are that: (1) any existing two-dimensional or three-dimensional wire form spring can be readily analyzed under the analysis formula presented in this paper; (2) a wire form spring can be satisfactorily synthesized to meet the needs by following the synthesis formula of the paper.

To analyze a wire form spring, the configuration of the analyzed wire form spring is already there. The stress and deflection of the wire form spring under the external loading condition usually need to be analyzed. In this paper, a B-spline curve is first employed to interpolate the center curve of the analyzed wire form spring. The stress and deflection of the wire form spring are then analyzed based on its piecewise parametric interpolation.

To synthesize a wire form spring, the center curve of the synthesized wire form spring is first modelled as a B-spline curve and described by the control polygon of the B-spline curve. The synthesis of a wire form spring then becomes the optimization of the control parameters of the B-spline curve.

There is a close relationship between analysis and synthesis of wire form springs. Analysis forms the foundation and synthesis is considered as the result. The purpose of analyzing a wire form spring is to evaluate the design and improve or further optimize the design of a wire form spring. Synthesizing a wire form spring relies on analyzing wire form springs since there are many candidates to be evaluated or analyzed during the synthesis process in order to select the optimal or satisfactory solution.

The remainder of the paper is organized as follows. The analysis and synthesis formulations on wire form springs are presented in section II. The analysis of wire form springs is provided in section III. Section IV is on the synthesis of wire form springs. Conclusions are derived in section V.

II. ANALYSIS AND SYNTHESIS FORMULATIONS OF WIRE FORM SPRINGS

There are various shapes for wire form springs to meet the needs of different applications. To analyze or synthesize a wire form spring, its center curve has to be described or modelled. Currently, Bezier, spline and B-spline curves have been extensively employed for shape descriptions. If a curve is described by a Bezier curve, the number of control points of the Bezier curve will increase in order to get more control over the shape of the curve, so is the polynomial degree of the Bezier curve since it is coupled with the number of control points (which is the total number of control points minus 1) [5]. High degree polynomial curves are inefficient to process and numerically unstable [6]. A spline curve is a set of polynomials of degree *p* that are smoothly connected at certain data points. At each data point where two polynomials connect, their derivatives up to (*p-1*)st are the same at the data point [7]. Regular spline curves do not have high degree problem but can only offer global shape control, i.e., change of any data point will cause the change of the entire spline curve. B-spline (B stands for Basis) curves eliminate this problem by using a special set of basis functions that has only local influence and depends only on a few neighboring control points [8]. Changes in one point of the control polygon of a B-spline curve affect the shape of at most *K* segments of the curve. *K* denotes the order (which is the degree of basis polynomials plus 1) of a B-spline curve. Because of their advantages, B-spline curves are adopted in the paper for shape descriptions of wire form springs.

A B-spline curve is defined by the following equation.

$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{P}_i \, N_{i,K}(t) \tag{1}$$

In (1), $\mathbf{P}_i$, $i = 0, 1, 2, \cdots, n$, are the (*n+1*) control points; parameter *K* is the order of the polynomial segments and controls the degree (*K-1*) of the basis polynomials and the continuity of the curve. $N_{i,K}(t)$ are the basis polynomials that are defined recursively by the following expressions [8].

$$N_{i,1}(t) = \begin{cases} 1 & if\ t_i \le t < t_{i+1} \\ 0 & otherwise \end{cases} \tag{2}$$

and if $k > 1$,

$$N_{i,k}(t) = \frac{(t - t_i)}{(t_{i+k-1} - t_i)} N_{i,k-1} + \frac{(t_{i+k} - t)}{(t_{i+k} - t_{i+1})} N_{i+1,k-1} \tag{3}$$

In (3), the integer values of *k* are: $k = 2, 3, \cdots, K-1$ where K is the order of the B-spline curve. The t_j ($j = 0, 1, 2, \cdots, n+K$) are knot values. The entire set of knot values comprises a knot vector that is denoted by T in this paper. When parameter t ranges through the interval of $[t_{K-1}, t_{n+1})$, the B-spline curve is produced. When any subscript in (3) is out of range of the summation limit or a denominator is zero, its associated fraction is not evaluated and becomes zero directly.

Fig. 1 shows an open quadratic B-spline curve. The standard knot vector is used for constructing the curve, which is:

$$\begin{aligned} T &= \{ t_j, j = 0, 1, 2, \cdots, n+K \} = \\ &\{ 0, 0, 0, 0.2, 0.4, 0.6, 0.8, 1, 1, 1 \} \end{aligned} \tag{4}$$

In (4), $n = 6$ and $K = 3$. The interior knots are evenly distributed in the normalized interval of [0, 1]. When *t* goes from 0 to 1, the entire curve is generated. The multiplicity of the first knot (0) is 3 (*K*), so the curve starts at $\mathbf{P}_0$. The curve ends at $\mathbf{P}_6$ because of the multiplicity (3) of the last knot (1). When t goes through the interval of $t_{j+1} \le t \le t_{j+2}$, $j = 1, 2, \cdots, n-1$, a curve segment (*j*) is constructed by three consecutive control points $\mathbf{P}_{j-1}$ $\mathbf{P}_j$ $\mathbf{P}_{j+1}$. Curve segment (*j*) is within the convex hull formed by its corresponding control points $\mathbf{P}_{j-1}$ $\mathbf{P}_j$ $\mathbf{P}_{j+1}$. Its starting and ending points are tangent to $\mathbf{P}_{j-1}$ $\mathbf{P}_j$ and $\mathbf{P}_j$ $\mathbf{P}_{j+1}$, respectively. There are totally five curve segments in Fig. 1. The ends of curve segments are all circled in Fig. 1.

The open cubic B-spline curve shown in Fig. 2 is from the same control polygon of Fig. 1. *K* is now 4, so its knot vector is

$$\begin{aligned} T &= \{ t_j, j = 0, 1, 2, \cdots, n+K \} = \\ &\{ 0, 0, 0, 0, 0.25, 0.50, 0.75, 1, 1, 1, 1 \} \end{aligned} \tag{5}$$

The curve starts at $\mathbf{P}_0$ and ends at $\mathbf{P}_6$ because of the multiplicity (4) of the first and last knots (0 and 1). When *t* changes in the interval of $t_{j+2} \le t \le t_{j+3}$, $j = 1, 2, \cdots, n-2$, a curve segment (*j*) is constructed by four consecutive control points $\mathbf{P}_{j-1}$ $\mathbf{P}_j$ $\mathbf{P}_{j+1}$ $\mathbf{P}_{j+2}$. Curve segment (*j*) is within the convex hull formed by its corresponding control points $\mathbf{P}_{j-1}$ $\mathbf{P}_j$ $\mathbf{P}_{j+1}$ $\mathbf{P}_{j+2}$. Different from the quadratic B-spline curve, the interior ends of the curve segments in the cubic situation are not located on the line segments formed by consecutive control points.

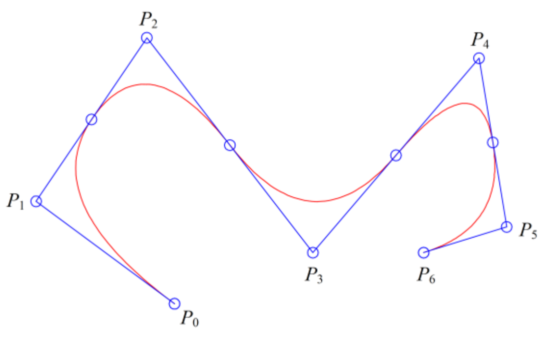

Fig. 1 An open quadratic B-spline curve.

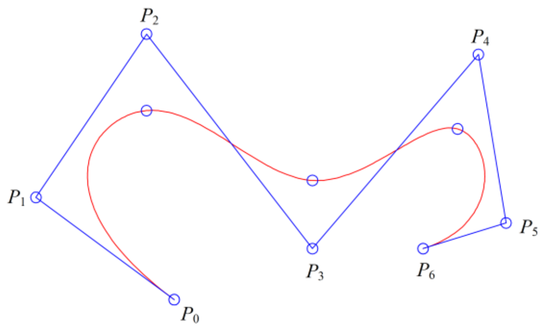

Fig. 2 An open cubic B-spline curve.

In light of the above two curves from the same control polygon, we can see that the quadratic B-spline curve is closer to its control polygon than the cubic B-spline curve. However, the cubic B-spline curve has C^2 continuity and the quadratic one has only continuity of C^1. Each segment in a quadratic B-spline curve is from the barycentric (or affine) combination of three control points. Its shape is always planar and can only be either concave or convex, and there is no inflection in the interior of the segment. The shape of a segment in a cubic B-spline curve can be planar or spacial, and is not restricted to be either concave or convex (inflection might appear within a curve segment as shown in Fig. 2) since the segment is from the barycentric combination of four control points. Although the shape flexibility of a quadratic B-spline curve segment is limited compared with that of a cubic B-spline curve segment, it has a unique feature, which is the tangency to the two legs of its control polygon. This feature makes the shapes of quadratic B-spline curves convenient to be controlled by their control polygons, which is beneficial for them to be applied for wire form springs.

Fig. 3 shows a closed quadratic B-spline curve, which is composed of four segments that come from control point triads of $\mathbf{P}_0$ $\mathbf{P}_1$ $\mathbf{P}_2$, $\mathbf{P}_1$ $\mathbf{P}_2$ $\mathbf{P}_3$, $\mathbf{P}_2$ $\mathbf{P}_3$ $\mathbf{P}_0$ and $\mathbf{P}_3$ $\mathbf{P}_0$ $\mathbf{P}_1$, respectively. The knot vector for this quadratic B-spline curve is as follows.

$$T = \{ t_j, j = 0, 1, 2, \cdots, n + K \} = \{ 0, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 1 \} \quad (6)$$

In (6), n is 5 (not 3) and K is 3. Four segments have to be used to close the B-spline curve. If the edge between $\mathbf{P}_3$ and $\mathbf{P}_0$ in Fig. 3 is removed, the control polygon formed by $\mathbf{P}_0$ $\mathbf{P}_1$ $\mathbf{P}_2$ $\mathbf{P}_3$ will become open and there will only be two segments for an open quadratic B-spline curve. The knot vector represented in (6) is uniform and different from the nonuniform knot vectors shown in (4) and (5). When t goes through the interval of $2/8 \le t \le 6/8$ (four subintervals for four segments), the entire closed quadratic B-spline curve is produced.

The closed cubic B-spline curve from the same control polygon of Fig. 3 is shown in Fig. 4. The four segments in Fig. 4 now come from control point quartets of $\mathbf{P}_0$ $\mathbf{P}_1$ $\mathbf{P}_2$ $\mathbf{P}_3$, $\mathbf{P}_1$ $\mathbf{P}_2$ $\mathbf{P}_3$ $\mathbf{P}_0$, $\mathbf{P}_2$ $\mathbf{P}_3$ $\mathbf{P}_0$ $\mathbf{P}_1$ and $\mathbf{P}_3$ $\mathbf{P}_0$ $\mathbf{P}_1$ $\mathbf{P}_2$, respectively. The knot vector for this cubic B-spline curve is as follows.

$$T = \{ t_j, j = 0, 1, 2, \cdots, n + K \} = \{ 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1 \} \quad (7)$$

n in (7) is 6 (not 3) and K is 4. When t goes through the interval of $0.3 \le t \le 0.7$ (four subintervals for four segments), the entire closed cubic B-spline curve is produced.

As shown in Figs. 3 and 4, the closed quadratic B-spline is tangent to its control polygon, but the closed cubic B-spline is not tangent to the same control polygon.

When a wire form spring is synthesized in this paper, its center curve is modelled by a B-spline curve (either open or closed based on its application and needs), its shape is modified through changing the locations of the control points in the control polygon.

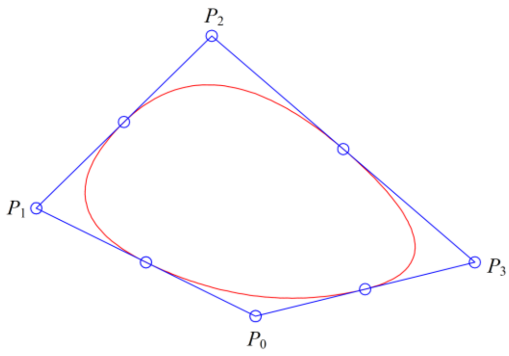

Fig. 3 A closed quadratic B-spline curve.

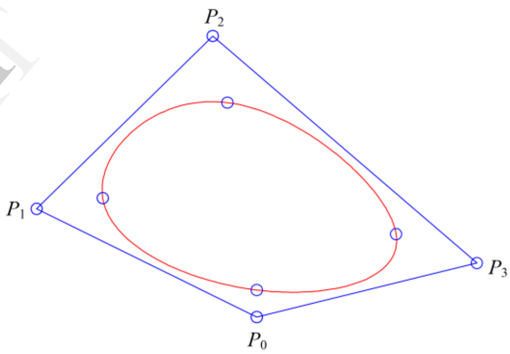

Fig. 4 A closed cubic B-spline curve.

When an existing wire from spring is analyzed, its center curve is already known. To analyze the stress and deflection, the center curve of the wire form spring is first interpolated by a B-spline curve. Suppose a set of points $\{ \mathbf{Q}_j, j = 0, 1, 2, \cdots, n \}$ on the known center curve of a wire form spring are chosen to be interpolated by a B-spline curve of order K. The parameter value for each $\mathbf{Q}_j$ is assumed to be $\{ u_j, j = 0, 1, 2, \cdots, n \}$. All u values form a special knot vector that is denoted by U in this paper. A set of $(n+1)$ x $(n+1)$ linear equations can then be set up with the $(n+1)$ unknowns of control points of the interpolation B-spline curve, $\{ \mathbf{P}_i, i = 0, 1, 2, \cdots, n \}$.

$$\mathbf{Q}_j = \sum_{i=0}^{n} \mathbf{P}_i \, N_{i,K}(u_j) \quad (8)$$

To solve equation set (8), two knot vectors U and T have to be decided. U is selected by the chord length method that has been widely used for curve interpolation. Let D be the total chord length that is calculated as follows.

$$D = \sum_{j=1}^{n} \left| \mathbf{Q}_j - \mathbf{Q}_{j-1} \right| \tag{9}$$

u_0 and u_n are set at 0 and 1, respectively. Other u_j values are calculated as follows.

$$u_j = u_{j-1} + \frac{\left| \mathbf{Q}_j - \mathbf{Q}_{j-1} \right|}{D}, \quad j = 1, 2, \cdots, n-1 \tag{10}$$

T is decided by the averaging method [6] as follows.

$$t_j = 0, \quad j = 0, 1, \cdots, K-1 \tag{11}$$

$$t_j = 1, \quad j = n+1, n+2, \cdots, n+K \tag{12}$$

$$t_{j+K-1} = \frac{1}{K-1} \sum_{i=j}^{j+K-2} u_i, \quad j = 1, 2, \cdots, n-K+1 \tag{13}$$

Fig. 5 shows the center curve of a wire form spring, which is composed of two circular arcs that have different radii and are connected by an inscribed tangent. The points to be interpolated on the curve are circles.

The dotted curve in Fig. 6 is the interpolation curve from a quadratic B-spline curve. The solid line in Fig. 6 is from Fig. 5 and is almost overlapping to the dotted curve.

When a cubic B-spline curve is employed to interpolate the solid curve in Fig. 5 through the circled points, the interpolation curve is shown in Fig. 7 as the dashed curve. Similar to the quadratic situation, it is difficult to separate the dashed curve from the solid on in Fig. 7.

After the center curve of an existing wire form spring is interpolated by a B-spline curve, the parametric interpolation curve can be conveniently used for the stress and deflection analysis of the wire form spring. To analyze the wire form spring, it needs to be discretized into beam elements for finite element analysis. Discretization of the interpolation curve is straightforward since its parameter t has been normalized and changes from 0 to 1. When the unit interval [0, 1] of parameter t is divided into multiple sub-intervals, the number of sub-intervals is the number of beam elements for analysis. ANSYS [9-10] is adopted in the paper for finite element analysis of wire form springs.

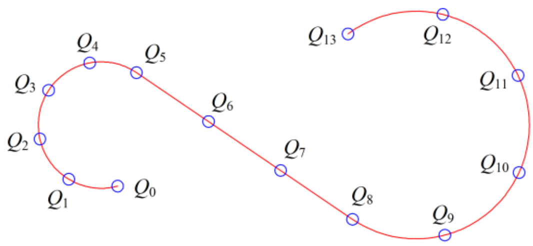

Fig. 5 A wire form curve to be interpolated.

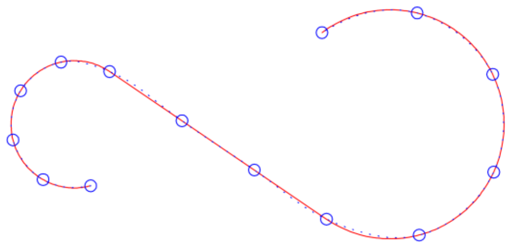

Fig. 6 Interpolation by a quadratic B-spline curve.

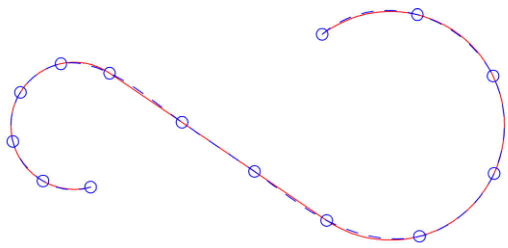

Fig. 7 Interpolation by a cubic B-spline curve.

III. ANALYSIS OF WIRE FORM SPRINGS

Fig. 8 shows a wire form spring to be analyzed. The spring is made of carbon steel wire with Young's modulus of 207 GPa, Poisson's ratio of 0.3, strength of 1700 MPa and wire diameter of 2 mm. As shown in Fig. 8, L is 90 mm, R is 13 mm and F is 20 N.

The center curve of the analyzed wire form spring is first interpolated by a quadratic B-spline curve, and then discretized into elements. The discretized wire form spring is shown in Fig. 9. Beam element (BEAM188) in ANSYS is employed for the deflection and stress analysis of the wire form spring. The input force of 20 N is divided into 4 even load steps and geometric nonlinearity command “NLGEOM” is turned on when the deflection and stress of the wire form spring are analyzed in ANSYS. The deflection and stress of the wire form spring are shown in Fig. 10. The horizontal and vertical deflections at the left force application point are 10.17 mm and 1.50 mm, respectively. The maximum stress in the deflected wire form spring is 963.38 MPa.

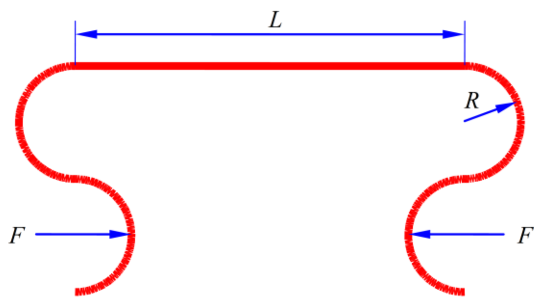

Fig. 8 A wire form spring to be analyzed.

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Fig. 9 The finite element discretization of the analyzed wire form spring.

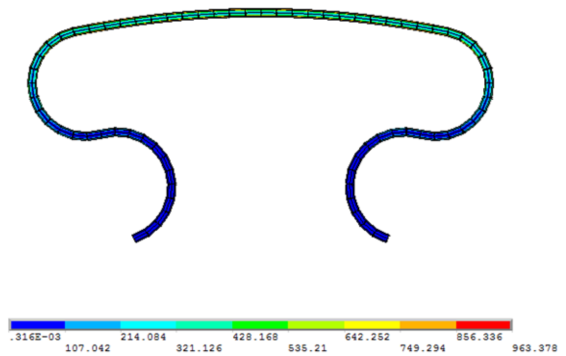

Fig. 10 The deflection and stress of the analyzed wire form spring.

IV. SYNTHESIS OF WIRE FORM SPRINGS

A wire form spring is synthesized here to act as a load carrying link. The design domain of the wire form spring is shown in Fig. 11 in which the length (L) and width (H) of the design domain are 80 mm and 30 mm, respectively. The upper right corner of the rectangular design domain is used as the fixed point of the wire form spring while the lower left corner of the design domain is the free end on which a downward load of 10 N is applied. The wire form spring is synthesized to connect the loading point to the fixed point in the specified design domain.

The material and diameter of the spring wire are the same as the analysis example. The center curve of the synthesized wire form spring is modelled as a quadratic B-spline curve whose control polygon is shown in Fig. 12. Six control points (P_0 to P_5) form the control polygon. P_0 and P_5 are at the loading and fixed points of the wire form spring, respectively and have their specified locations in the design domain. Other control points (P_1 to P_4) carry design variables. The wire form spring are perpendicular to the vertical line at both P_0 and P_5, so line segments P_0P_1 and P_4P_5 are horizontal. Thus, there are totally six independent design variables from the control polygon to define the wire form spring, which are represented as a design variable vector X as follows.

$$X = \lfloor P_{1x} \quad P_{2x} \quad P_{2y} \quad P_{3x} \quad P_{3y} \quad P_{4x} \rfloor \tag{14}$$

The wire form spring is synthesized to maximize its stiffness under the constraint that the maximum stress in the spring is below its allowable value. Stiffness is measured by the vertical deflection at the loading point (P_0). The optimization of the design variables is conducted by using the Global Optimization Toolbox of MATLAB [11-12]. ANSYS is used for the deflection and stress analysis during the optimization. The connection between MATLAB optimization and ANSYS finite element analysis is based on ANSYS Parametric Design Language [13].

Fig. 13 shows the synthesis result. Its finite element discretization is shown in Fig. 14. Fig. 15 shows the deflected wire form spring and its stress distribution. The vertical deflection of the spring is 9.06 mm at the loading point. The maximum stress is 946.94 MPa that happens at the fixed point of the wire form spring.

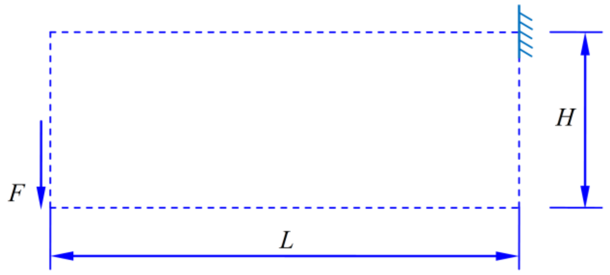

Fig. 11 The design domain of the synthesized wire form spring.

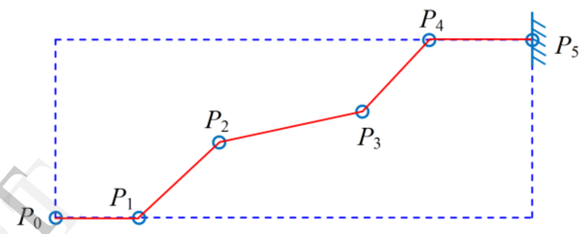

Fig. 12 The control polygon of the synthesized wire form spring.

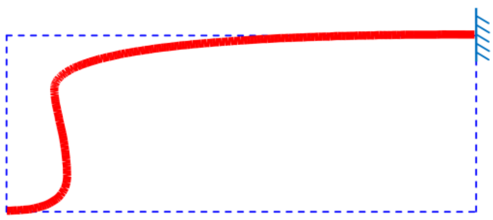

Fig. 13 The synthesis result of the wire form spring.

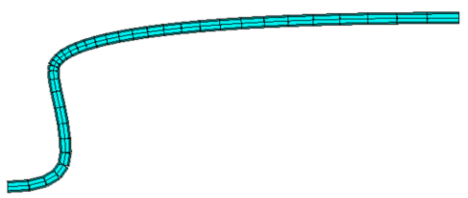

Fig. 14 The finite element discretization of the synthesized wire form spring.

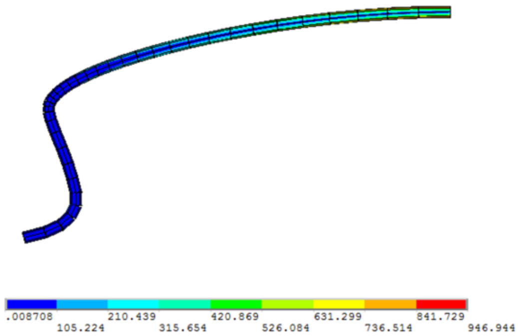

Fig. 15 The deflection and stress of the synthesized wire form spring.

V. CONCLUSIONS

A method for analyzing and synthesizing wire form springs is presented in the paper. To analyze an existing wire form spring, its center curve is first interpolated by a B-spline curve with normalized parameter interval of [0 1]. The parametric B-spline interpolation curve is then discretized by subdividing the unit interval into multiple sub-intervals. The number of sub-intervals is the number of elements for analyzing the wire form spring. ANSYS is adopted in the paper for finite element analysis. The beam element (Beam188) in ANSYS is employed for the deflection and stress analysis with its geometric nonlinearity command (NLGEOM) turned on.

To synthesize a wire form spring, the center curve is modeled as a B-spline curve and described by its control polygon. The shape of a synthesized wire form spring is decided by its control polygon. The shape change is realized through modifying the locations of the control points in the control polygon. The synthesis of a wire form spring thus becomes the optimization of the locations of the control points. The design variables are composed of the independent coordinates of the control points. The Global Optimization Toolbox of MATLAB is employed to optimize the design variables in this paper. The deflection and stress of design candidates are analyzed by using ANSYS during the optimization process. ANSYS Parametric Design Language is employed in the paper to communicate between MATLAB optimization and ANSYS finite element analysis.

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