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Analysis and Synthesis of Contact-Aided Cantilever Springs

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***Abstract*—A cantilever spring is a simple flat spring that is anchored at one end and loaded at the other end. There is a wide range of applications for cantilever springs that include automobiles, railway carriages, prosthetic and medical devices. A contact-aided cantilever spring decreases its cantilevered length through contacting a support when the spring is deflected. Because of the decreased length, the spring stiffness is increased since a larger force is required to deflect a shorter cantilever spring than a longer one. The more the cantilever spring is deflected, the higher its spring stiffness, which makes the contact-aided cantilever spring become a progressive nonlinear spring. Because of the contact support and nonlinear force-deflection feature, synthesizing contact-aided cantilever springs is challenging. In this paper, the shapes of analyzed or synthesized cantilever springs and contact supports are described by circular arcs and controlled by geometric parameters. Synthesizing contact-aided cantilever springs is systematized as optimizing geometric control parameters. Examples on analyzing and synthesizing contact-aided cantilever springs with different shapes are presented in the paper to verify the effectiveness and demonstrate the procedure of the introduced method.**

Keywords—Cantilever Spring; Contact; Analysis; Synthesis; Geometric Control Parameter; Optimization.

I. INTRODUCTION

Flat springs are commonly considered as beam-based springs that are made of sheet, plate or strip. Beams are usually designed to support load and have minimum deflection, but springs must deflect and meet desired load-deflection relationships [1]. The shapes of flat springs are different from helical, spiral, washer or power springs. To distinguish flat springs from other springs, term flat is used, but flat is not an accurate description of the shape of this type of springs. Their shapes are not restricted or limited as flat, and can be straight or curved [2]. A cantilever spring is a simple flat spring that is anchored at one end and loaded at the other end. There is a wide range of applications for cantilever springs that include automobiles, railway carriages, prosthetic and medical devices.

A cantilever spring is usually straight and has uniform rectangular cross section. such kind of cantilever springs are simple, easy and inexpensive to manufacture. The primary stress of a straight uniform cantilever spring is from its bending and the bending stress is proportional to the distance from the loading end. If a straight cantilever beam spring is subjected to small deflection, the bending stress along the spring axis can be calculated based on Bernoulli-Euler beam theory as follows [3].

$$\sigma(x) = \frac{6Px}{bt^2} \tag{1}$$

P in equation (1) is the force applied at the loading end of the cantilever spring, and perpendicular to the spring axis. t is the thickness of the rectangular cross section of the cantilever spring, and along the direction of force P. b is the width of the rectangular cross section of the spring, and perpendicular to the direction of force P. x is the distance from the loading end along the spring axis. The bending stress value calculated from equation (1) is for the two width edges of the rectangular cross section, among which one is tensile and another is compressive, and both share the same absolute value.

For straight cantilever beam springs with uniform cross section, the bending stress reaches its maximum at the clamping end and is zero at the loading end. Material is not efficiently utilized in the region close to the loading end. To make the bending stress constant throughout the spring, the uniform cross section can be made variable [4]. When thickness t is uniform, the bending stress will be constant if the ratio $b(x)/x$ is constant. The cantilever spring will then have a triangular width and uniform thickness. If b_0 is the width at the fixed end and L is the total length of the cantilever spring, the constant ratio needs to be $b(x)/x = b_0/L$, and the constant bending stress is $\sigma = \frac{6PL}{b_0 t^2}$. If width b does not change and thickness $t(x)$ is variable along the spring axis, the ratio $t^2(x)/x$ has to be constant in order to have a constant bending stress. The cantilever spring then has a parabolic thickness and uniform width. If t_0 is the thickness at the fixed end of the cantilever spring, the constant ratio is then $t^2(x)/x = t_0^2/L$, and the constant bending stress is $\sigma = \frac{6PL}{bt_0^2}$. A cantilever spring with a parabolic thickness is apparently much more difficult to manufacture than a cantilever spring with a triangular width.

Although bending stress is the primary stress for a loaded cantilever spring, shear stress also plays its role especially in the region close to the loading end where the bending moment is very small compared to the region around the clamping end. The shear stress distribution on the rectangular cross section of a straight cantilever spring can be decided by the following formula [5].

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$$\tau(y) = \frac{6P}{bt^3}\left(\frac{t^2}{4} - y^2\right) \tag{2}$$

y in equation (2) is the distance away from the neutral axis of the rectangular cross section. From equation (2), the maximum shear stress occurs on the neutral axis and can be calculated as follows [5].

$$\tau_{\max} = \frac{3P}{2A} \tag{3}$$

A in equation (3) is the area of the rectangular cross section and equals bt.

As previously stated, a cantilever spring with triangular width and uniform thickness or parabolic thickness and uniform width can make the bending stress constant throughout the spring. However, the cross sectional area at the loading end degenerates to zero in either case since either width or thickness becomes zero. With zero area, the shear stress hikes to infinity theoretically. Actually, no force can be applied perpendicular to a plane with zero area. So the above constant bending stress can only be achieved in theory.

The bending stress (σ) and shear stress (τ) acting at the same point on a cross section of a cantilever spring are perpendicular each other and can be simultaneously considered and combined into Von Mises stress (σ_v) as follows [5].

$$\sigma_v = \sqrt{\sigma^2 + 3\tau^2} \tag{4}$$

Von Mises stress measures the distortion energy level at a point and has been widely used as a criterion in determining the onset of yielding failure based on distortion energy theory (DET) for ductile materials. Cantilever springs are made of ductile materials. DET applies well in cantilever springs for yielding failure prediction.

To avoid zero area at the loading end and also improve material utilization efficiency, cantilever springs can be made to have trapezoidal width and uniform thickness or tapered thickness and uniform width or variable both width and thickness. The ideal case is Von Mises stress is uniform within cantilever springs. The variable width and thickness parameters of a designed cantilever spring can be optimized to meet the desired force-deflection requirements and also improve its material utilization efficiency [6].

The force-deflection relationship is usually limited to be linear for cantilever springs when the deflection is not large and before geometric or material nonlinearity happens to the spring. For a beam-based spring to realize a desired nonlinear force-deflection function, the topology, shape and size of the beams of the spring have to be customized or synthesized [7]. The beam configuration of a nonlinear spring is not so simple like that in a regular cantilever spring.

Based on spring rate (stiffness), nonlinear springs are usually divided into two groups: nonlinear stiffening (hardening) and nonlinear softening [8]. A nonlinear hardening spring gradually increases its spring rate when the spring deflection progresses. This group of nonlinear springs provide a progressively hardening reaction as the springs get deflected. They are also called progressive nonlinear springs. On the other hand, a nonlinear softening spring gradually decreases its spring rate when the spring deflection increases. Nonlinear softening springs are often called degressive nonlinear springs since they provide a degressive reaction.

A contact-aided cantilever spring consists of a cantilevered beam that is similar to a regular cantilever beam spring and a fixed contact support. The overall configuration of a contact-aided cantilever spring is not complicated, but it can generate nonlinear force-deflection relationship because of the contact support. When a contact-aided cantilever spring is deflected, the deflected beam contacts the fixed support and wraps itself around the support, which decreases its cantilevered length. Because of the decreased length, the spring stiffness increases since a larger force is required to deflect a shorter cantilever spring than a longer one. The more the cantilever spring is deflected, the higher its spring stiffness, which makes the contact-aided cantilever spring become a progressive nonlinear spring.

Because of the contact support and nonlinear force-deflection feature, synthesizing contact-aided cantilever springs is much more challenging than regular cantilever springs that have no contact support and generate linear force-deflection relationships. The authors of this paper are motivated to surmount the challenges facing contact-aided cantilever springs.

In this paper, the shapes of analyzed or synthesized cantilever springs and contact supports are described by circular arcs and controlled by geometric parameters. A systematic approach is introduced to synthesize contact-aided cantilever springs. Synthesizing a contact-aided cantilever spring is systematized as optimizing its geometric control parameters. Examples on analyzing and synthesizing contact-aided cantilever springs with different shapes are presented in the paper to verify the effectiveness and demonstrate the procedure of the introduced method.

The remainder of the paper is organized as follows. The analysis on contact-aided cantilever springs is presented in section II. The parameter optimization is provided in section III. Section IV is on the synthesis of contact-aided cantilever spring. Conclusions are drawn in section V.

II. ANALYSIS OF CONTACT-AIDED CANTILEVER SPRINGS

The straight cantilever beam shown in Figure 1 has uniform rectangular cross section. Its deflection (δ) at the loading end can be calculated by the following formula.

$$\delta = \frac{4PL^3}{Ebt^3} \tag{5}$$

E is equation (5) is Young's modulus of the beam material. When the cantilever beam is used as a spring, its spring stiffness or rate (k) is the ratio of P and δ .

$$k = P/\delta = \frac{Ebt^3}{4L^3} \tag{6}$$

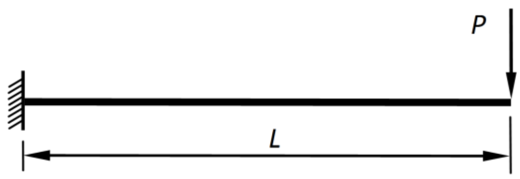

Fig. 1 A straight cantilever beam with uniform cross section.

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The cantilever beam spring with its solid model and finite element mesh shown in Figure 2 has length of 100 mm. Its rectangular cross section has width and thickness of 5 mm and 2 mm, respectively. The material of the spring is engineering plastic that has Young's modulus of 2200 MPa, Poisson' ratio of 0.35, yield strength of 71 MPa. The left end of the spring is fixed. A downward displacement of 10 mm is applied on its right end.

The solid models of analyzed cantilever beam springs are developed in the paper by the Design Modeler of ANSYS [9]. One of ANSYS Workbench applications is ANSYS Design Modeler, which is to provide modeling functions for analyses and simulations that include detailed geometry creation, CAD geometry modification, simplification and concept model creation tools.

The deflection, reaction and stress analyses of cantilever beam springs are conducted in the paper by using the Mechanical of ANSYS [10-11], which is another ANSYS Workbench application. ANSYS Workbench 15 is used here to analyze cantilever beam springs.

When a displacement of 10 mm is applied to the right end of the cantilever beam in Figure 2, the deflected cantilever beam and its Von Mises stress are shown in Figures 3 and 4, respectively. The maximum Von Mises occurs around the left fixed end, which is 6.7539 MPa. The required input force at the loading end is 0.22163 N that is from ANSYS Mechanical. The spring stiffness from ANSYS is then 0.0222 N/mm. If equation (6) is used to calculate the spring stiffness, its value is 0.0220 N/mm, which is very close to that from ANSYS simulation.

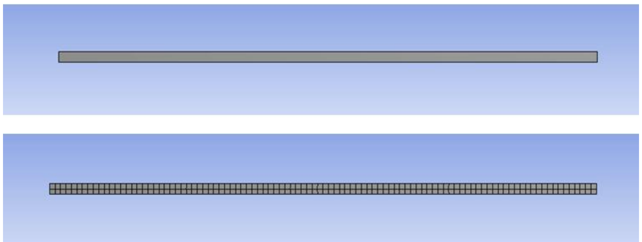

Fig. 2 The solid model and its finite element mesh of a straight cantilever beam.

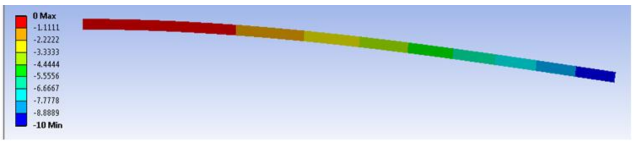

Fig. 3 The vertical deflection of the straight cantilever beam spring.

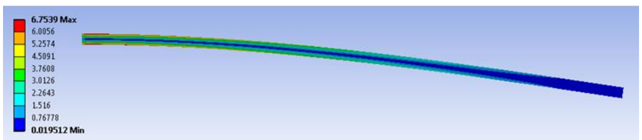

Fig. 4 The Von Mises Stress of the straight cantilever beam spring.

The above cantilever beam spring can be aided by a circular contact support to increase its stiffness [12]. As shown in Figure 5, a straight cantilever beam is aided by a fixed circular beam that has a radius of 215 mm. The two beams have the same cross section and material, which are same as those of the cantilever beam in Figure 2. When the cantilever beam is deflected, its right end region will contact the fixed support beam and make its cantilevered length decreased, and thus its spring stiffness will be increased. In order to catch contact effect accurately, the mesh fineness or density in Figure 5 is much higher than that in Figure 2, especially in the contact and target regions.

Figures 6 and 7 show the deflection and Von Mises stress of the contacted-aided cantilever beam spring. To have a downward displacement of 10 mm at the right end, the required input force is now 0.30592 N, which is 38% increase compared with the input force of 0.22163 N for the same size cantilever beam spring shown in Figure 2 that has no contact aid. The maximum Von Mises stress is 9.8463 MPa, which is 46% increase compared with that of 6.7539 MPa for the cantilever beam spring without contact support.

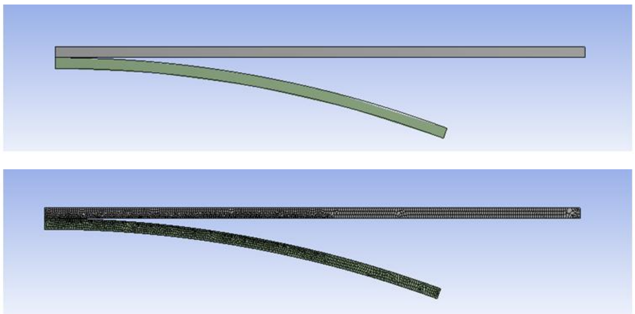

Fig. 5 The solid model and its finite element mesh of a straight cantilever beam with circular contact support.

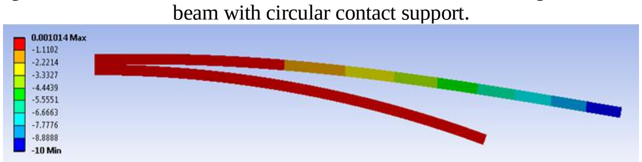

Fig. 6 The vertical deflection of the contact-aided straight cantilever beam spring with circular contact support.

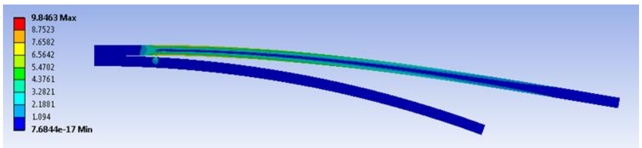

Fig. 7 The Von Mises Stress of the contact-aided straight cantilever beam spring with circular contact support.

The straight cantilever beam spring and circular contact support can be changed to circular cantilever beam spring and straight contact support [12]. The circular cantilever beam shown in Figure 8 has radius of 215 mm and horizontal length of 100 mm. The straight beam is now fixed and acts as a contact support. Both beams have the same cross section and material as those in Figure 5.

When a downward displacement of 10 mm is applied to the right end of the circular beam, its deflection and Von Mises stress are shown in Figures 9 and 10, respectively. The required input force is 0.27507 N in this case and the maximum Von Moses stress is now 10.102 MPa. The stiffness of the contacted-aided circular beam spring is softer than that of the contact-aided straight cantilever beam spring while its maximum Von Mises stress is a little above that.

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The fixed straight beam used for contact support in Figure 8 does not have to be straight. It can be also be curved. If the circular beam in Figure 8 is rotated by 180 degrees with respect to a horizontal axis and the rotated circular beam acts as a fixed contact support, the new contact-aided circular cantilever spring becomes that shown in Figure 11.

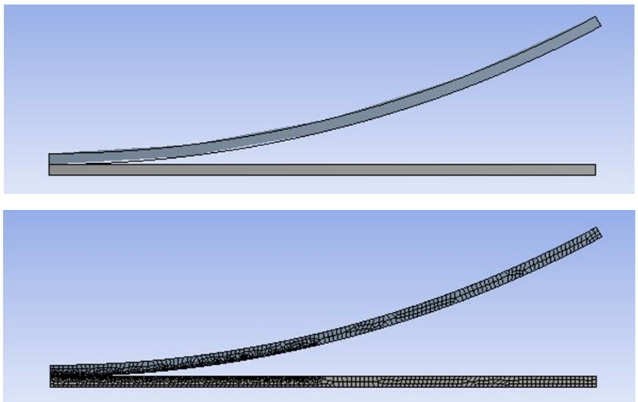

Fig. 8 The solid model and its finite element mesh of a circular cantilever beam with straight contact support.

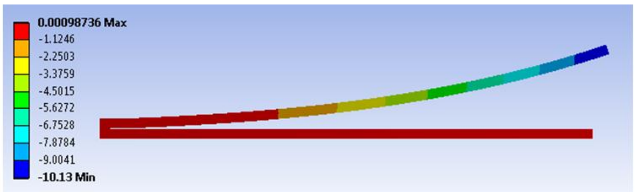

Fig. 9 The vertical deflection of the contact-aided circular cantilever beam spring with straight contact support.

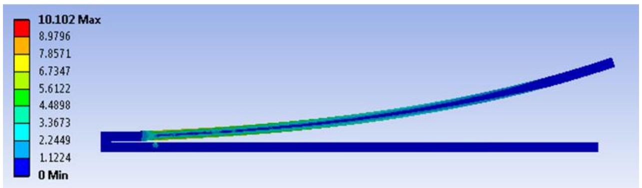

Fig. 10 The Von Mises Stress of the contact-aided circular cantilever beam spring with straight contact support.

When the downward displacement of 10 mm is applied to the right end of the upper circular beam, its deflection and Von Mises stress are shown in Figures 12 and 13, respectively. The required input force for this contact-aided circular cantilever spring is 0.24869 N and the maximum Von Moses stress is now 34.739 MPa. The maximum Von Mises stress is much higher than other cases because of the circular convex and concave contact, and so is the vertical space. The spring stiffness is lower than other cases.

The concave-convex orientation for the circular cantilever spring does not have to be opposite to that of the circular contact support. When they have the same concave-convex orientation, the vertical space will not be so large as the case in Figure 11. The circular cantilever beam in Figure 14 has the same shape and size as that in Figures 8 and 11. The fixed circular beam for contact support has a radius of 535 mm. The two circular beams have the same cross section as previous cases. To generate a downward displacement of 10 mm, the required input force is 0.30645 N, which results in a maximum Von Mises stress of 10.375 MPa. The deflection and Von Mises stress of the cantilever spring are shown in Figures 15 and 16, respectively. The stiffness for this contact-aided cantilever spring is higher than that of Figure 8 although its maximum Von Mises stress is a little bit above that of Figure 8.

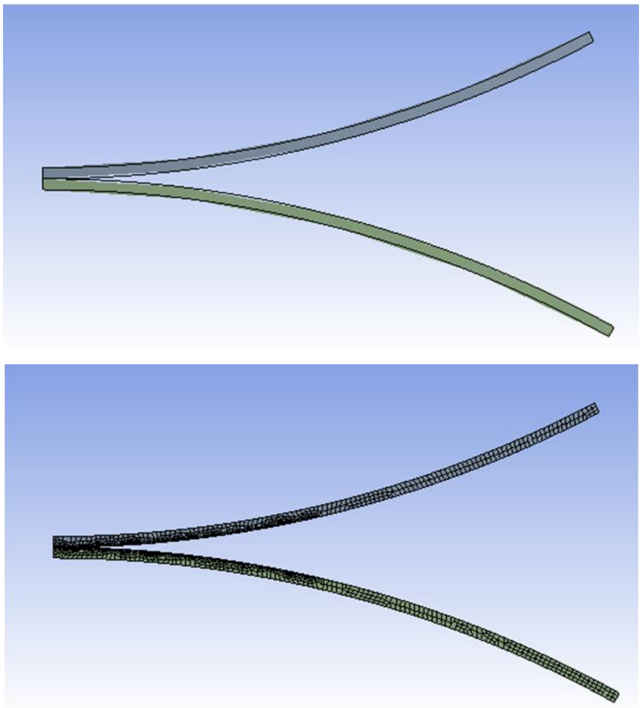

Fig. 11 The solid model and its finite element mesh of a circular cantilever beam with circular contact support.

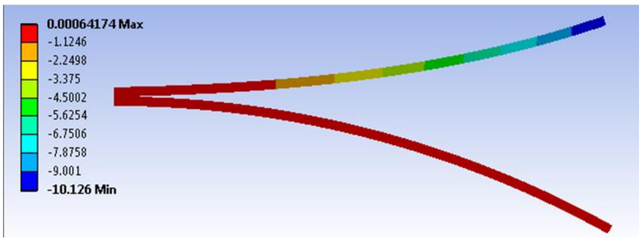

Fig. 12 The vertical deflection of the contact-aided circular cantilever beam spring with circular contact support.

Four different contact-aided cantilever beam springs are analyzed and discussed in this section. Because of the contact support, their spring stiffness values are increased compared with those without contact support, but their maximum Von Mises stress values are also more or less increased.

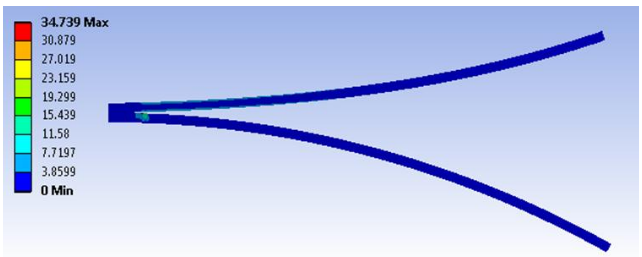

Fig. 13 The Von Mises Stress of the contact-aided circular cantilever beam spring with circular contact support.

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Four different contact-aided cantilever beam springs are analyzed and discussed in this section. Because of the contact support, their spring stiffness values are increased compared with those without contact support, but their maximum Von Mises stress values are also more or less increased.

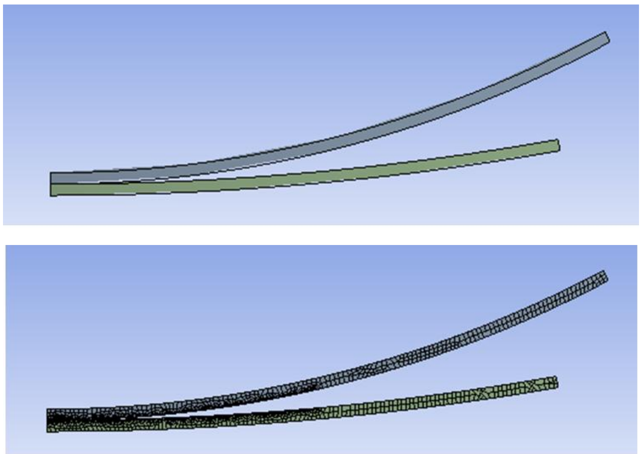

Fig. 14 The solid model and its finite element mesh of a contact-aided circular cantilever beam spring.

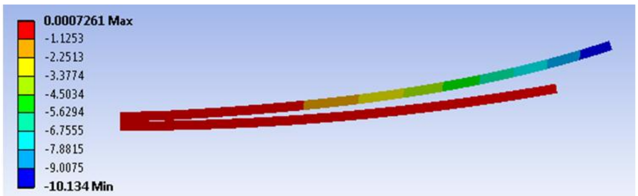

Fig. 15 The vertical deflection of the contact-aided circular cantilever beam spring.

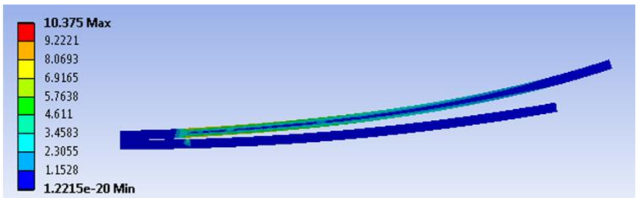

Fig. 16 The Von Mises Stress of the contact-aided circular cantilever beam spring.

III. PARAMETER OPTIMIZATION OF CONTACT-AIDED CANTILEVER SPRINGS

A contact-aided cantilever spring is composed of two critical parts: the cantilever spring and the contact support. In this paper, both parts are regarded as planar beams (either straight or circular) with rectangular cross section. Straight beams are actually special cases of circular beams that have radius of infinity. A circular beam has four independent geometric parameters to define and control its configuration: circular radius (r), circular angle (θ), cross section width (b) and thickness (t). Thus, the synthesis of a contact-aided cantilever spring is to decide the geometric control parameters for both cantilever spring and contact support beams.

The modeling of contact-aided cantilever springs is based on ANSYS Design Modeler in this paper. ANSYS Mechanical is used for analyzing the modeled contact-aided cantilever springs to derive their performance and get their deflection, reaction and Von Mises stress. Both Design Modeler and Mechanical are ANSYS Workbench applications. Design Exploration is another ANSYS Workbench application and supports design optimization through simulation results [13]. Direct Optimization is a toolbox of Design Exploration and is one of the goal driven optimization systems in ANSYS. The geometric control parameters of synthesized contact-aided cantilever springs are optimized in this paper by using the Direct Optimization Toolbox of ANSYS.

IV. SYNTHESIS OF CONTACT-AIDED CANTILEVER SPRING

A horizontal straight cantilever spring has uniform rectangular cross section with its width (b) and thickness (t) of 5 mm and 1 mm, respectively. The length (L) of the spring is 100 mm. The cantilever spring is made of spring steel that has Young's modulus (E) of 207 GPa, Poisson' ratio (v) of 0.3, yield strength (σ_y) of 450 MPa. The left end of the cantilever spring is rigidly fixed while its right end is designed to generate a reaction force (P) for a vertical deflection (δ) of 5 mm. The reaction force at the right end can be calculated according to equation (5) as 1.294 N that is perpendicular to the spring axis. With the deflection of 5 mm, the maximum bending stress calculated from equation (1) is then 155.25 MPa that occurs at the left end. The deflection , reaction force and stress can all be analyzed within ANSYS Workbench.

The synthesis of the straight cantilever spring is to keep the cantilever spring but make its reaction force increased to 1.85 N under the same input displacement of 5 mm. The approach is to introduce a circular contact support under the straight cantilever spring, which is the case shown in Figure 5. To make things simple and economical, the circular contact support beam has the same rectangular cross section and material as the cantilever spring beam. So the circular radius (r) and circular angle (θ) of the circular contact support beam are the only two geometric control parameters that need to be decided in order to generate the desired reaction force under the specified input deflection. Because the circular angle does not have so much influence as the circular radius, its value is set at a certain value during the synthesis process.

Both the straight cantilever beam and the circular contact support beam are modeled in ANSYS Design Modeler. The circular radius is defined as a design parameter. As shown in Figure 5, the bottom surface of the cantilever beam contacts the upper surface of the contact support beam. The contact area is around the left end of both beams. So a portion of the bottom surface of the cantilever beam and part of the upper surface of the contact support beam are specified as "Contact" and "Target" faces, respectively. During the analysis process, ANSYS Workbench checks the contact status for every node on the contact face against the target face [14]. The mesh density is much higher near contact and target faces in order to catch contact effect accurately.

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The synthesis of this contact-aided cantilever spring is to make the reaction force reach its desired value by optimizing the circular radius of the contact support beam. The maximum Von Mises stress within both beams is constrained to be below the yield strength of their material. The Adaptive Multiple-Objective (AMO) method in ANSYS Design Exploration is employed for parameter optimization. AMO supports multiple objectives and aims at finding the global optimal solution [14].

The optimal circular radius is found as 252.72 mm with the circular angle is set at 9 degrees. Figures 17 shows the solid model and finite element mesh of the synthesized contact-aided cantilever spring.

The generated reaction force from the synthesized contact-aided cantilever spring has its desired value of 1.85 N. The deflection and Von Mises stress of the spring are shown in Figures 18 and 19, respectively. The maximum Von Mises stress is 211.32 MPa.

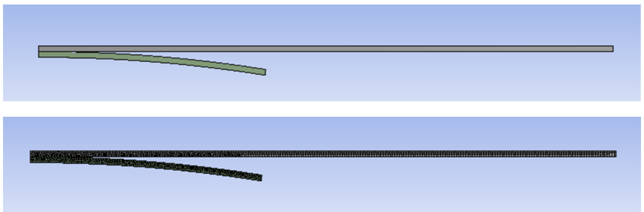

Fig. 17 The solid model and its finite element mesh of the synthesized contact-aided cantilever spring.

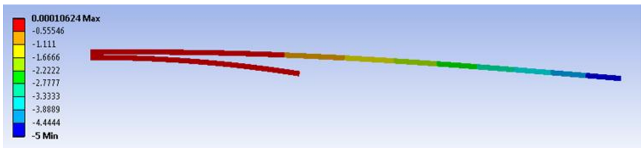

Fig. 18 The vertical deflection of the synthesized contact-aided cantilever spring.

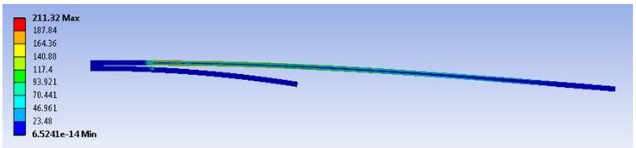

Fig. 19 The Von Mises Stress of the synthesized contact-aided cantilever spring.

To compare the results of the contact-aided cantilever spring with those of the original cantilever spring, the model, mesh, deflection and stress of the original cantilever spring that has no contact support are shown in Figure 20. The maximum Von Mises (211.32 MPa) of the contact-aided cantilever spring has a jump over that (157.91 MPa) of the original cantilever spring because of the contact.

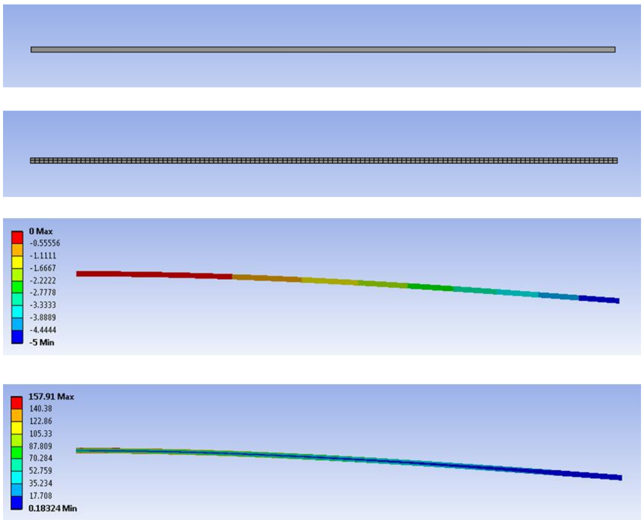

Fig. 20 The solid model, finite element mesh, deflection and Von Mises stress of the original cantilever spring.

V. CONCLUSIONS

Different from regular cantilever springs, contact-aided cantilever springs increase spring rate by decreasing cantilevered length. When a contact-aided cantilever spring is deflected, it contacts a stationary support to decrease its cantilevered length and increase its spring rate. A method for synthesizing contact-aided cantilever springs is introduced in this paper. A synthesized contact-aided cantilever spring is systematized as optimizing its geometric control parameters for the desired reaction force under the constraints of allowable stress and practical dimensions.

Analysis and synthesis of contact-aided cantilever springs are conducted within ANSYS Workbench environment in this paper. The solid model of an analyzed or synthesized contact-aided cantilever spring is first created in ANSYS Design Modeler by using its geometric control parameters. The finite element mesh is then developed in ANSYS Mechanical based on the created solid model. The deflection, reaction and stress are analyzed through the finite element mesh. ANSYS Design Exploration is employed to optimize the geometric control parameters. The optimal solution is obtained by using the AMO optimization method.

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