International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 10, October-2016

Analysing Heat Transfer in Annular Stepped Fin

(Using Theoretical & Numerical Technique)

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***Abstract* - ASFs (Annular stepped fins) require less material than ADFs (annular disc fins) while retaining the ability to produce the same cooling rate in a convection environment. A simple analysis was developed for ASFs that considered radiative heat transfer through linearization of the radiation terms. The linearized equations were solved by exact analytical method. For the particular geometry, the temperature results were obtained using this analytical approach for one dimensional steady state heat transfer. The same ASF geometry for two dimensional steady state heat transfer analysis was solved by Numerical method. The fin performance parameters for ADF and ASF in two dimensional heat transfer analysis were compared.**

1. INTRODUCTION

The rate of heat transfer in the fins is directly proportional to the extent of the wall surface, the heat transfer coefficient, and the temperature difference between the fluid and the adjacent surface.A fin with heat generation, which will have a high fin surface temperature, could be used to cool nuclear reactors, where heat is generated from a nuclear source consisting of rapidly moving neutrons and gamma rays [2]. To increase the heat-transfer rate from a fluid-carrying tube, annular fins are attached to the outer surface. This is a standard practice to augment the heat transfer from a primary cylindrical surface. However, since the cross-sectional area of annular disc fins is constant, the fin material does not effectively conduct heat near the fin tip. Hence, different tapered profiles (for example, triangular, trapezoidal, parabolic, or hyperbolic) have been proposed in the literature [1]. These profiles make better use of the fin material than a constant-thickness fin while being able to maintain the same heat-transfer rate, but may require complex fabrication processes. Alternatively, an annular disc fin with a step change in thickness both saves material and is easy to fabricate [3]. Annular fins are an important part of fin-and-tube heat exchangers. The primary concern of most investigations of such heat exchangers is the performance of the annular fins. Chambers and Somers [4] determined the performance of an annular fin with a rectangular profile for boundary conditions consisting of a constant temperature at the fin base and insulation at the fin tip. Smith and Sucec [5] calculated the efficiency of triangular fins using a power- rate per unit fin volume, the optimum fin profile or shape can be determined by minimizing the fin volume for a given heat-transfer rate. For conduction and convection conditions, a criterion for optimal fins was first reported by Schmidt [20]. Duffin [21] used a variation calculus technique to determine the optimum fin shape to yield a maximum heat-transfer rate per unit volume. Liu [22] extended the variational principle to determine the optimum profile of fins with internal heat generation. The optimization of radiating fins was addressed by Cobble [23]. Kundu and Das [24] described a generalized methodology to determine the optimum design of thin fins with uniform volumetric heat generation. Fabbri [25] proposed a genetic algorithm for fin-profile optimization, and used it to determine the optimum profile shape in a convective environment for different polynomial orders. Kundu and Lee [26] demonstrated a novel analysis based on the calculus of variation to determine the smallest envelope fin shape for wet fins with a nonlinear mode of surface transport. Hanin and Campo [27] presented an analytical formulation for the optimum profile of a straight fin by minimizing the volume for a given amount of heat transfer per unit width; according to their calculations, the optimum fin profile is a circular arc, the volume of which is on average seven times smaller than that of the optimal fins proposed by Schmidt [20]. Peng and Chen [28] employed a hybrid numerical technique based on the differential transform method and finite differences to analyze an annular disc fin for temperature-dependent thermal conductivity. They assumed that heat was dissipated from the fin surface due to convection and radiation. Sharqawy et al. [29] carried out a numerical study to determine the temperature distribution and fin efficiency of annular fins with different cross-sectional areas subject to heat and mass transfer. Moinuddin et al. [30] extended this analysis to determine the optimum dimensions. Campo and Morrone [31] used a simple computation procedure for the thermal analysis of annular fins with tapered cross-sections. Kang and Look [32] presented an

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analysis to design an annular trapezoidal fin based on a new approach to a two dimensional analytical method. Zubair et al. [33] investigated the effect of variable thermal conductivity on the optimum condition of circular fins for variable profiles. Minkler and Rouleau [34] developed analytical solutions for the temperature distribution and optimum fin parameters, and examined a convective fin with uniform internal heat generation. More recently, Torabi et al. [35] used analytical methods to establish the performance characteristics of Convective-radiative longitudinal fins with rectangular, trapezoidal, and concave parabolic profiles, with thermal conductivities, heat-transfer coefficients, and surface emissivities that varied with temperature. They used a differential transform method for the fin surface temperature. Using the same methodology, Moradi et al. [36] performed a convection radiation thermal analysis of triangular porous fins with temperature-dependent thermal conductivity. A fin must be thin to maximize the surface area to volume ratio. Tapered profiles are difficult to fabricate and may be impossible to implement. However, the fin thickness near the tip can be easily reduced in a step-wise manner by adopting a suitable stepped profile. Kundu and Das [3] analytically determined the temperature distribution of a concentric annular fin with a step change in thickness. Under dehumidifying surface conditions, Kundu [37] analyzed the profile of ASFs (annular stepped fins) to determine their performance. This profile was shown to have an improved heat-transfer rate per unit fin volume compared to a constant thickness profile. Kundu et al. [38] presented an approximate analysis of the maximum heat flow in annular fin arrays with rectangular stepped profiles and convective surface conditions. Balaram Kundu et al. proposed about the analytical technique to calculate the maximum heat transfer in annular stepped fin using modified Bessels equation[39].

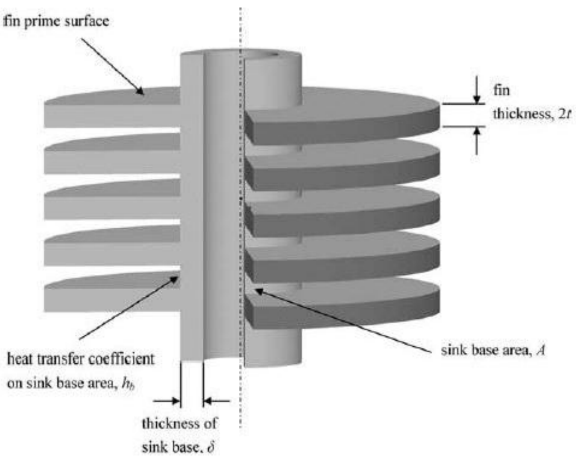

Fig 1.1 Assemble of annular disc fin

Annular fins, as shown in Fig. 1.1, are a common geometry for extended surfaces in many heat transfer applications where cooling and dehumidification processes occur, simultaneously. As with other fin geometries, weight and material cost of the extended surfaces are very important. Therefore, fin dimensions should be optimized so that the least amount of fin material can be used to dissipate a given amount of heat flow, or alternately that the highest dissipation rate be obtained from a given volume of fin material. An analytical expression was derived by Sonn and Bar- Cohen to give the optimum pin fin diameter for maximum heat dissipation rate.

LIST OF NOMENCLATURE & ABBREVATIONS

ADF	Annular disc fin
ASF	Annular stepped fin
h	Convective heat-transfer coefficient over the entire fin Surface (W m^{-2} K^{-1})
$I_m(Z)$	Modified Bessel function of first order m and Argument Z
$K_m(Z)$	Modified Bessel function of second order m and Argument Z
k	Thermal conductivity of the fin material (W m^{-1} K^{-1})
q	Actual heat-transfer rate through the fin (W)
q_i	Ideal heat-transfer rate through a fin (W)
q_0	Heat-transfer rate by the base area in the absence of a fin (W)
ξ	Dimensionless radial coordinate, r/r_3
ξ_1	Dimensionless inner radius, r_1/r_3
ξ_2	Dimensionless step radius, r_2/r_3
ξ_3	Dimensionless step radius, r_3/r_3
r	Radial coordinate starting from the center of the tube (m)
r_1	Inner radius (m)
r_2	Step radius of an ASF (m)
r_3	Outer radius (m)

2. ONE DIMENSIONAL HEAT TRANSFER ANALYSIS

This study presents a thermal analysis of an ASF. Fig. 3.1 shows a schematic diagram of the fin profile. The inner and outer radii of the fin are r_1 and r_3, respectively, and the step radius is r_2. The base and tip thicknesses of the ASF are $2t_1$ and $2t_2$, respectively. The fin is exposed to convective and radiative heat transfer with a convective heat-transfer coefficient.

Theoretical analysis

Axial conduction was neglected in one Dimensional heat transfer analysis. This analysis had completed with theoretical analysis.

This analysis includes the following steps

- Formation of Governing Differential Equations
- Formation of Suitable Boundary conditions
- Non Dimensionalization
- Rearranging GDEs as Modified Bessels equations
- Obtaining heat transfer results for specific geometry.

Assumption

- Steady state heat transfer
- No axial conduction
- No heat generation
- Constant heat transfer coefficient and thermal conductivity
- Free convection

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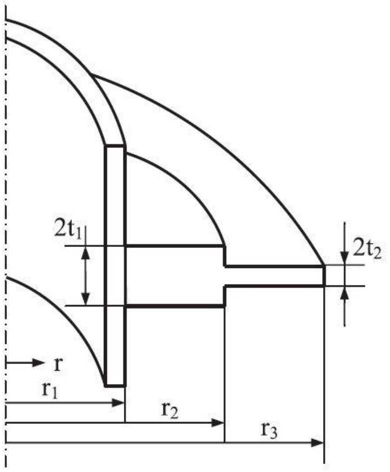

Fig 2.1. Schematic diagram of an ASF.

Formation of Governing Differential Equations

From conservation of energy equation

$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_{gen}=\dot{E}_{st}$

$\dot{E}_{gen}=\dot{E}_{st}=0$

$\dot{E}_{in}=\dot{E}_{out}$

$\dot{E}_{out,net}=0$

$q_r-q_{r+dr}-dq_{convection}-dq_{radiation}=0$

$dr-dq_{convection}-dq_{radiation}=0$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right]-\frac{hr}{kt}[T-T_\infty]-\frac{r\sigma\varepsilon}{kt}[T^4-T_\infty^4]=0 \qquad (r_b\le r\le r_t)$$

Linearised radiative Term

$T^4=4TT_\infty^3-3T_\infty^4$

Hence

$T^4-T_\infty^4=4TT_\infty^3-4T_\infty^4$

$T^4-T_\infty^4=4T_\infty^3(T-T_\infty)$

Formation of Suitable Boundary conditions

ASF is mounted on the cylinder which is maintained at constant temperature. This is the base temperature for the ASF.As tip of fin is very thin; it is considered an adiabatic tip.

The fin is surrounded by air stationary air medium with constant heat transfer coefficient.

Interface temperature

$$t_1k\frac{dT^{(1)}}{dr}=t_2k\frac{dT^{(2)}}{dr}-(t_1-t_2)h(T_i-T_\infty)-(t_1-t_2)\sigma\varepsilon(T_i^4-T_\infty^4) \qquad \text{valid at } r=r_i \text{ only}$$

Non Dimensionalization

The independent variable (r) and dependent variable (T) had replaced with appropriate non- dimensional terms as,

$\xi=\frac{r}{r_{max}}$,

$\Theta=T-T_\infty$.

$m^2=\frac{h}{kt}$

$n^2=\frac{\sigma\varepsilon}{kt}4T_\infty^3$

$j^2=[m^2+n^24T_\infty^3]r_3^2$

Boundary Conditions

(i) At $r=r_b$,

$$T^{(1)}=T_b$$

(ii) At $r=r_t$,

$$\frac{dT^{(2)}}{dr}=0$$

GDE rearranged as,

$$\xi^2\frac{d^2\Theta^{(1)}}{d\xi^2}+\xi\frac{d\Theta^{(1)}}{d\xi}-j_1^2\xi^2\Theta^{(1)}=0 \qquad \text{valid for } \xi_1\le\xi\le\xi_2$$

$$\xi^2\frac{d^2\Theta^{(2)}}{d\xi^2}+\xi\frac{d\Theta^{(2)}}{d\xi}-j_2^2\xi^2\Theta^{(2)}=0 \qquad \text{valid for } \xi_2\le\xi\le 1$$

Rearranging the GDEs as Modified Bessels equations

For solving purpose the GDEs were rearranged as Bessels Equation's Format by defining,

$z=j\,\xi$

$$z^2\frac{d^2\Theta^{(1)}}{dz^2}+z\frac{d\Theta^{(1)}}{dz}-z^2\Theta^{(1)}=0 \qquad \text{valid for } z_1\le z\le z_2$$

$$z^2\frac{d^2\Theta^{(2)}}{dz^2}+z\frac{d\Theta^{(2)}}{dz}-z^2\Theta^{(2)}=0 \qquad \text{valid for } z_1\le z\le z_2$$

Solution to Problem

Modified Bessels Function Solution

For zeroth order

$\Theta^{(1)}(z)=C_1I_0(z)+C_2K_0(z)$

$\Theta^{(2)}(z)=C_3I_0(z)+C_4K_0(z)$

Specific Geometry and heat transfer properties

Geometry:

- r_b=15mm,r_i=22.5mm,r_t=30mm,
- $2t_1$=2mm,$2t_2$=1mm,
- T_b=400K,T_∞=300K,
- h=30W/m^2K, k=40W/mK, ε=0.5

After substituting above values in equations a,b,c,d and solving simultaneously, the arbitrary constants values were

C_1 =48.8783K

C_2 =45.2642K

C_3 =44.3231K

C_4 =68.38167K

$\Theta^{(1)}(\xi)=C_1I_0(0.832\xi)+C_2K_0(0.832\xi) \qquad 0.5\le\xi\le0.75$

$\Theta^{(2)}(\xi)=C_3I_0(1.1766\xi)+C_4K_0(1.1766\xi) \qquad 0.75\le\xi\le1.0$

3. TWO DIMENSIONAL HEAT TRANSFER ANALYSIS

CFD Analysis

Axial conduction also considered in this analysis. Both ADF and ASF were analyzed by using Computational Fluid Dynamics analysis and compared.

Modeling and meshing process were done by using Gambit 2.4 software. Solving and post processing steps were done in Fluent software

Mesh models

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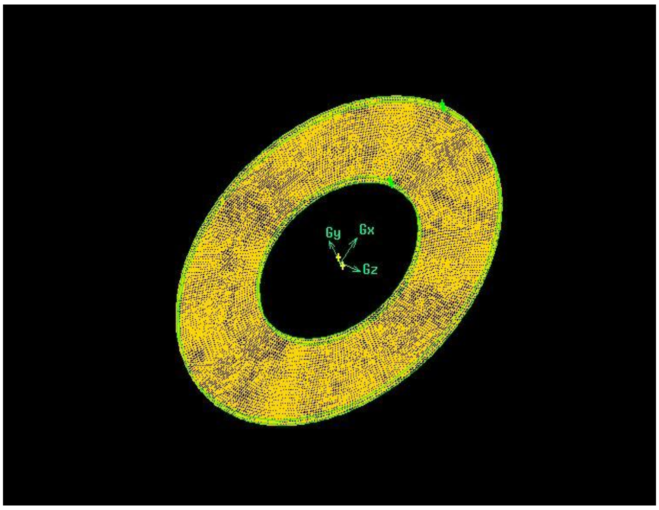

Fig 3.1 ADF Mesh model

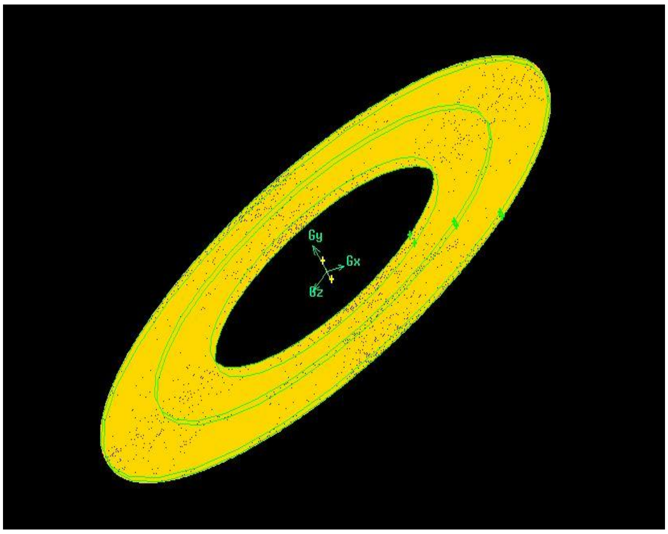

Fig 3.2 ASF Mesh model

4. RESULTS AND DISCUSSIONS

The theoretical analysis and numerical simulation was carried out for one, two dimensional heat transfer models of ASF respectively. Temperature profiles were plotted for both one and two dimensional analysis and heat transfer rates were computed. From above results following fin performance parameters were calculated.

- Efficiency of the fin
- Effectiveness of the fin.

Results of one dimensional heat transfer analysis of ASF

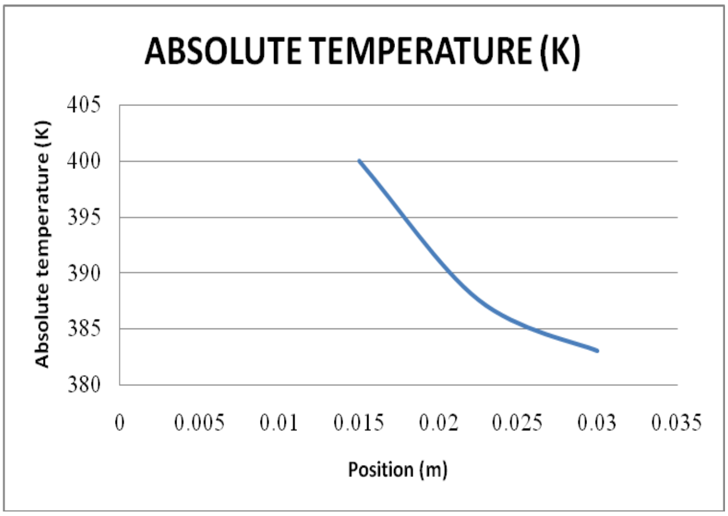

Fig 4.1 Temperature profile for one dimensional analysis

Figure 4.1 shows the temperature profile for the ASF in one dimensional heat transfer analysis.

Efficiency of ASF =79.55%

Effectiveness of ASF =18.496

Results of two dimensional heat transfer analysis of ASF& ADF for T_b =400K

The following 4.2 and 4.3 figures show the temperature distribution for the base temperature of 400K in ADF and ASF respectively.

Figure 4.4 and 4.5 shows the temperature distribution for the base temperature of 550K in ADF and ASF respectively.

Both temperature distributions were obtained from post processing results of fluent software.

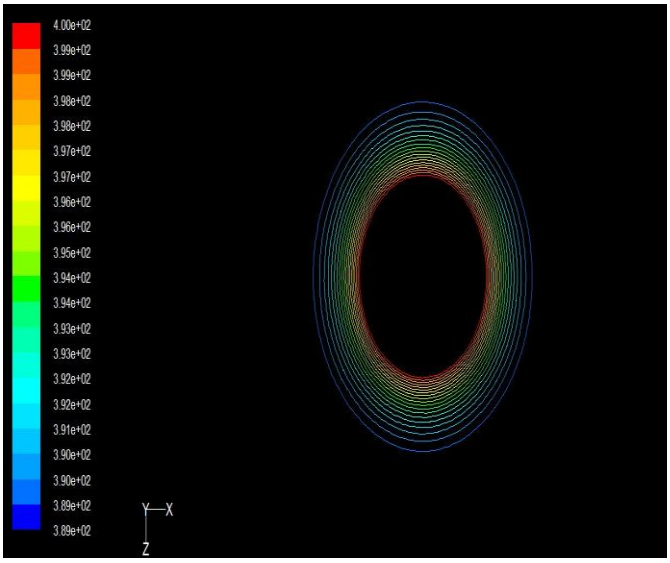

Fig 4.2 ADF Temperature Contours

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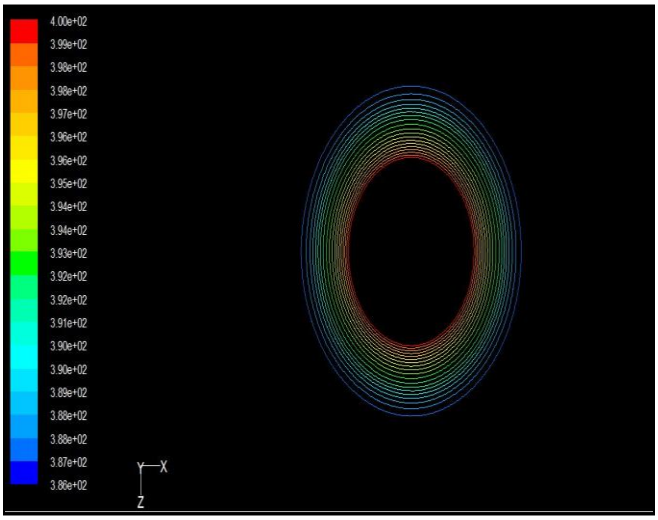

Fig 4.3 ASF Temperature Contours

Results of two dimensional heat transfer analysis of ASF & ADF for T_b =550K

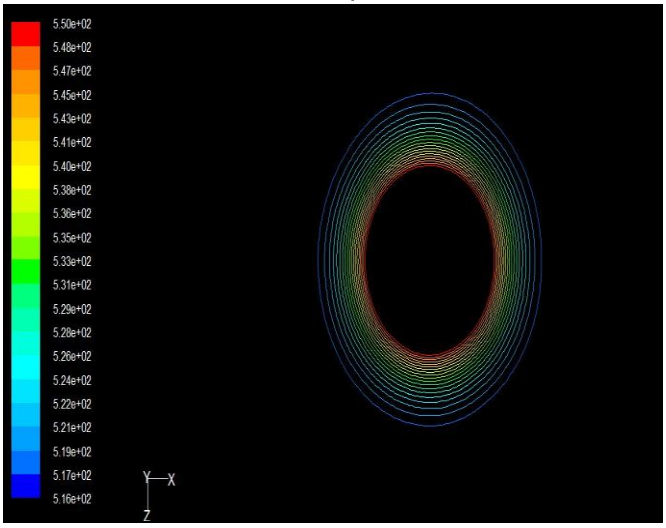

Fig 4.4 ADF Temperature Contours

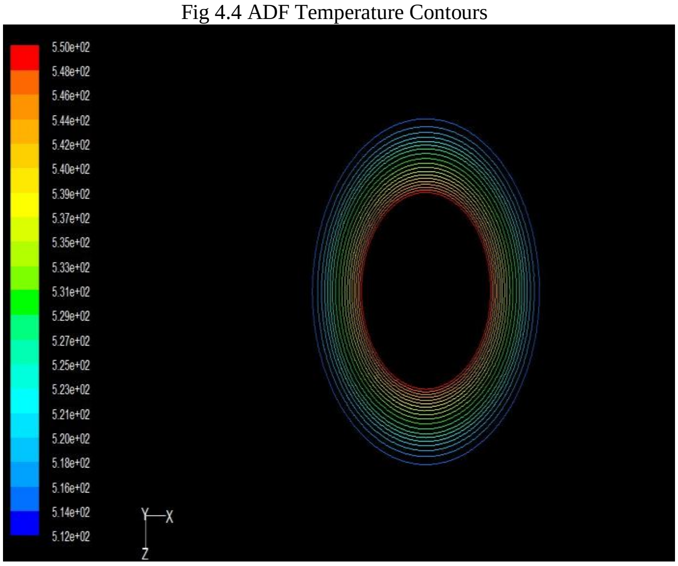

Fig 4.5 ASF Temperature Contours

Comparison of Temperature profiles of ASF & ADF for T_b =400K

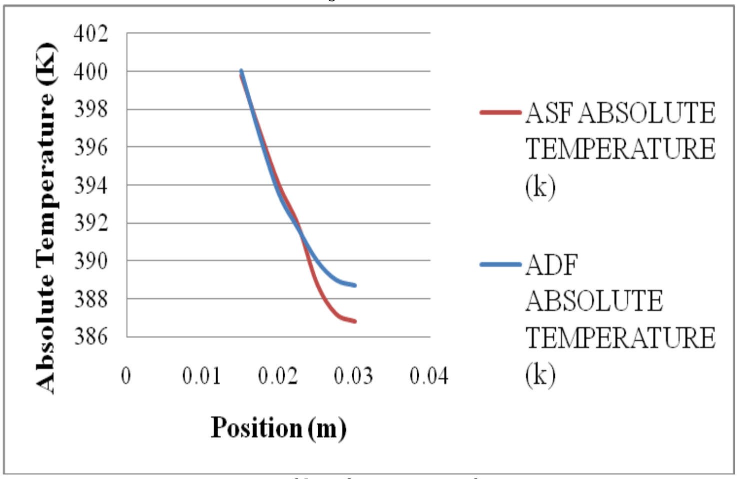

Fig 4.6 Temperature profiles of ASF & ADF for T_b =400K

Comparison of Temperature profiles of ASF & ADF for T_b =550K

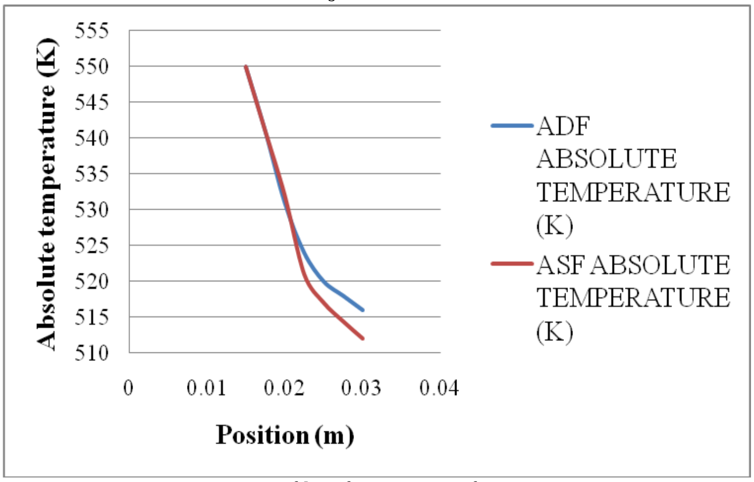

Fig 4.7 Temperature profiles of ASF & ADF for T_b =550K

Figure 4.6 and 4.7 shows the temperature profiles of ADF and ASF respectively.

From the graph, the temperature distributions were more similar to each other between base regions to just before interface region. The interface temperature of ASF was slightly more than that of ADF. But after the intersection section negative slope of ASF was suddenly increased and resulting the more temperature drop after intersection. In the ADF profile negative slope was not increased, but decreased and it causes the lesser temperature drop.

At the tip both profiles had decreased slope. Finally ASF tip temperature was two Kelvin less than that of ADF.

Comparison of performance of ASF & ADF

Table 4.1 Comparison of performance of ASF & ADF for T_b=400K

Properties	ADF	ASF	Percentage difference of ASF (%)
Heat transfer (W)	13.5	13.942	3.28 (more than ADF)
Volume(m^3)	$4.24*10^{-6}$	$3.5343*10^{-6}$	16.67 (less than ADF)
Efficiency (%)	88	91	3.7 (more than ADF)
Effectiveness	20.45	21.136	5.05 (more than ADF)

Table 4.2 Comparison of performance of ASF & ADF for T_b=550K

Properties	ADF	ASF	Percentage Difference of ASF(%)
Heat transfer (W)	18.609	19.007	2.138 (more than ADF)
Volume(m^3)	$4.24*10^{-6}$	$3.5343*10^{-6}$	16.67 (less than ADF)
Efficiency (%)	42.66	43.93	2.9 (more than ADF)
Effectiveness	10.01	10.22	2.14 (more than ADF)

From above tables ASF and ADF have the same volume up to intersection. After intersection ASF volume decreased by half but ADF volume remains the same. When compared to ADF, ASF had 16.67% less volume but surface area of both the fins was same. But ASF had 3.28% more heat transfer rate and hence performance parameters of ASF were more than ADF.

5. CONCLUSION

In theoretical one dimensional analysis to determine the heat-transfer rate through a fin, it is necessary to integrate the heat dissipation over the entire fin surface. Since radiation heat transfer takes place, the governing equations are nonlinear. However, this nonlinearity can be approximated by a linearized expression. Bessel approaches were used to solve the approximate linearized equation. But the approximation is applicable to small temperature differences only. For example T_b =400K, T_∞ =300K, Difference is 100K. For this difference percentage of radiation heat transfer error was 38%. For higher difference the error percentage will be more. From numerical analysis results the ASF had better performance results in addition to less volume for the introduction of single stepping section. Clearly ASF with multiple stepping sections will have better results than ASF with single stepping section. But in case of multiple stepping sections the location of stepping section must be optimized to get better result.

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