An Inventory Purchasing Model for Damageable Items under Trade Credit Period and Permissible Delay in Payments

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***Abstract*— This paper develops an economic lot sizing purchase model for damageable / Breakable items like glasses, electronic devices …. Etc. The demand pattern is time dependent and quadratic. Delays in payments under trade credit policy with shortage are allowed. The retailer offers a permissible delay period to his customers. The customers who get an item is allowed to pay on or before the permissible delay period which is accounted from buying time rather than from the start period of inventory sales. The retailer announces price discount to his customers who ordered breakable items. The aim of the method is to minimize the total inventory cost by finding the optimal time interval and finding the order quantity. Finally numerical example with solution is provided to demonstrate the developed model.**

Keywords- Deteriorating Items; Delay Payments; Trade Credit; Shortages;

I. INTRODUCTION

Generally deterioration plays a significant rate in many inventory systems. The process of deterioration is happened in two categories of items. The first category refers to the items that are damage, spoiled, decayed, loss of utility, evaporative or expire through the time like food grain, food stuffs and so on. While the other category refers to the items that loss their parts or their total values through time because of new technology or the alternatives like fashion and seasonal goods, electronic equipment's, computer chips and mobile phones and so on. For the first category, the items have a short natural life cycle whereas in the second category, the items have a short market life.

During the past few years, many researchers had studied inventory models for permissible delay in payments. [3] Goyal(1985) was the first proposed for developing an economic order quantity(EOQ) model under the conditions of permissible delay in payments.[1] Aggarwal and Jaggi (1995) extended Goyal's model to allow the inventory to had deteriorating items.[5] Hwang and Shinn(1997) developed a model considering exponentially deteriorating items and found decision policy for selling price and lot size.[4] Huang(2003) extended Goyal's model to the case in which the supplier offers the retailer the permissible delay period M, and the retailer in turn provides the trade credit period M to his customers.[8] Teng(2009) developed an EOQ model for a retailer who receives a full trade credit from its supplier and offers a partial trade credit to its bad credit customers or a full trade credit to its good credit customers. [2] Chen and Kang (2010) considered trade credit and imperfect quality in an integrated vendor-buyer supply chain model.[9] Thangam and Uthayakumar (2011) have built a mathematical model for a retailer under two level trade credit and two payment methods.[7]Muniappan.P (2014) explained an economic lot sizing production model for deteriorating items under two level trade credit. [10] A.Thangam (2014) extended retailer's inventory system in a two level trade credit financing with selling price discount and partial order cancellations.

Here, we extended the linear demand to quadratic demand dependent on time and deteriorating rate under trade credit policy. The comparison of linear and exponential demands and quadratic demand seems to be best approach. Because in case of linear demand there is uniform change and exponential demand leads to high rate of change, which is not found its reality, the quadratic demand is suitable for seasonal products, where the demand becomes high in the middle of the season and low towards the end.

In this paper we propose damageable/breakable items with allowing a shortage. The purchasing rate is linearly dependent on demand. To handle the risks of trade credit situations, retailer collects a higher interest from his customers when they did not settle the payment within the credit period time. Suppose the goods were damaged price discount will be allowed, we sale that item. The proposed model is based on the inventory items like fashion and seasonal goods, electronic equipment's, computer chips and mobile phones and so on. The numerical examples are illustrated in this model.

A. *NOTATIONS*

To develope the mathematical model, the following assumptions are being made.

- A - The ordering cost per order.
- Q - The order quantity per replenishment.
- Q_s - The minimum order quantity at which the delay is permitted by the retailer .
- D(t) - Demand rate at any time $t \geq 0$

m_0 - Mark up of selling price for damaged item.
T - The length of replenishment cycle.
T_s - The time interval that Q_S units are depleted to zero due to demand.
r - The price discount.
P - The purchase cost per unit item.
h - The holding cost per unit per unit time.
ɵ - The deteriorating rate.
M - Retailer's permissible delay in settling the accounts.
I_e - Interest which can be earned per year.
I_p - Interest charges in stocks per year.
B(Q)-The number of damaged units per unit of time at time t and is a function of current inventory level Q.
t_1 - The time at which the inventory reaches zero.
C_1 -The shortage cost per period.
S -The selling price per item.
TC -The total cost of the system.

B. *ASSUMPTIONS*

1. The demand rate D(t) is known and dependent on time which is given by $D(t)=a+bt+ct^2, a\geq 0, b\neq 0, c\neq 0$, a , b , c are constants.
2. There is no replenishment or repair of deteriorated items takes place in a given cycle.
3. Time horizon is infinite.
4. Lead time is zero.
5. Selling price for damaged items is a multiple of purchasing cost $S_b=m_o.P$ $0<m_o<1$.
6. The trade credit policy is adopted, the retailer allows credit period to customers.

II. MATHEMATICAL FORMULATION

The inventory system is developed as follows Q units of item arrive at the inventory system at the beginning of each cycle. During the time of interval $[0,t_1]$ the inventory level decreasing due to the demand and damageable or breakable items. Finally a shortage occurs due to the demand during the time of interval $[t_1,T]$, The behavior of the inventory model is demonstrated in fig.1.

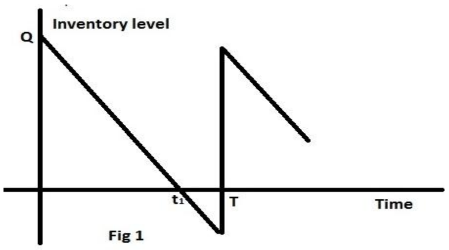

Fig 1

As the inventory reduces due to the demand rate as well as breakable items during the interval $[0, t_1]$, the differential equations representing the inventory status is

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t) \qquad 0 < t < t_1 \; --(1)$$

The solution of equation (1) with boundary condition $t=t_1$, $I(t)=0$ is as follows

$$I(t) = \left[\frac{\theta b + \theta c(t_1 + t) - 2c}{\theta^2}\right](t_1 - t) \quad --(2)$$

If the condition t=0 I(t)=Q the solution of quantity

$$Q = t_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] \qquad -----(3)$$

During the third interval $[t_1,T]$ shortage occurred, that is the inventory level at time t is governed by the following differential equation,

$$\frac{dI(t)}{dt} = -D(t) t_1 < t < T \qquad -----(4)$$

With the condition $t=t_1, I(t)=0$, the solution of (2) is

$$I(t) = a(t_1 - t) + \frac{b(t_1{}^2 - t^2)}{2} + \frac{c(t_1{}^3 - t^3)}{3} \quad --(5)$$

The holding cost HC during the period [0, T]

$$HC = h\int_0^{t_1} I(t)\,dt$$

$$HC = \frac{h}{\theta^2}\left[(b\theta - 2c)\left(\frac{t_1{}^2}{2}\right) + 2\theta c\left(\frac{t_1{}^3}{3}\right)\right] \quad --(6)$$

The total shortage cost SC during the period [0, T] is given by

$$SC = c_1\int_{t_1}^{T} I(t)\,dt$$

$$SC = c_1\left[\begin{array}{c} a\left(t_1 T - \frac{T^2}{2} - \frac{t_1{}^2}{2}\right) + \frac{b}{2}\left(t_1{}^2 T - \frac{T^3}{3} - \frac{2t_1{}^3}{3}\right) \\ + \left(\frac{c}{3}\right)\left(t_1{}^3 T - \frac{T^4}{4} - \frac{3t_1{}^4}{4}\right) \end{array}\right] - (7)$$

The purchasing cost PC during the period [0, T] is PC=P*Q

$$PC = Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right]$$

The ordering cost OC during the period [0, T] is OC=A

Breaking cost: Total number of damageable units is

$$\theta(Q) = \int_0^{t_1} B(Q)\,dt \quad = a_1(Q)^r t_1$$

Where B(Q) is breaking rate when inventory level is Q and it can be substituted as follows

$$B(Q) = a_1(Q)^r \qquad 0 < r < 1$$

As mentioned, selling price for each damaged item is $S_b=m_0$ *P, $0<m_0<1$ which is a multiple of last purchasing cost. So total selling price for damageable item is

$$= \theta(Q)^r * m_0 * P$$

$$= a_1(Q)^r t_1 * m_0 * P$$

Based on order quantity and predetermined order quantity we have the following two cases

i) $Q < Q_s$, $T < T_s$
ii) $Q \geq Q_s$, $T \geq T_s$

I.If the order quantity is less than the minimum order quantity $Q < Q_s$, $T < T_s$ Here the permissible delay in payments is not allowed, because of the retailer order quantity Q is less then Q_s.

$$IP = PI_p\int_0^{T} I(t)\,dt$$

$$IP = PI_p\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{T} I(t)\,dt\right]$$

$$IP = PI_p\left\{\begin{array}{c} \left(\frac{(b\theta - 2c)}{\theta^2} - a\right)\frac{t_1{}^2}{2} + \frac{t_1{}^3}{3}\left(\frac{2c}{3} - b\right) + \\ \left(a + \frac{bt_1}{2} + \frac{ct_1{}^3}{3}\right)t_1 T - \left(\frac{a}{2} + \frac{bT}{6}\right)T^2 - \frac{c}{4}\left(\frac{T^4}{3} + t_1{}^4\right) \end{array}\right\} --(8)$$

Hence the retailers annual total relevant cost per unit time is

$$TC_1 = \frac{1}{T}\{OC + PC + HC + SC + IP - Cost\ of\ D.Items\}$$

$$TC_1 = \frac{1}{T}\left\{\begin{array}{c} A + \frac{h}{\theta^2}\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ +PI_p\left\{\begin{array}{c}\left(\frac{(b\theta-2c)}{\theta^2} - a\right)\frac{t_1^2}{2} \\ +\frac{t_1^3}{3}\left(\frac{2c}{3} - b\right) + \left(a + \frac{bt_1}{2} + \frac{ct_1^3}{3}\right)t_1 T \\ -\left(\frac{a}{2} + \frac{bT}{6}\right)T^2 - \frac{c}{4}\left(\frac{T^4}{3} + t_1^4\right)\end{array}\right\} \\ +Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] \\ +c_1\left[\begin{array}{c} a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + \frac{b}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2t_1^3}{3}\right) \\ +\left(\frac{c}{3}\right)\left(t_1^3 T - \frac{T^4}{4} - \frac{3t_1^4}{4}\right)\end{array}\right] - a_1(Q)^r t_1 * m_0 * P \end{array}\right\}$$

For minimizing the total relevant cost per unit of time the approximate optimal value of T can be Optimal by solving the following equation

$$\frac{\partial TC_1}{\partial T} = -\frac{1}{T^2}\left\{\begin{array}{c} A + \left(\frac{h + PI_p}{\theta^2}\right)\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ -(c_1+1)\left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4}\right] \\ +Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] - a_1(Q)^r t_1 * m_0 * P \end{array}\right\}$$

$$-(c_1 + PI_p)\left(\frac{a}{2} + \frac{bT}{3} + \frac{cT^2}{4}\right) = 0 --(9)$$

This also satisfies the conditions

$$\frac{\partial^2 TC_1}{\partial T^2} > 0$$

$$\frac{\partial^2 TC_1}{\partial T^2} = \frac{2}{T^3}\left\{\begin{array}{c} A + \left(\frac{h + PI_p}{\theta^2}\right)\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ -\left[\begin{array}{c}(c_1+1)\left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4}\right] \\ +a_1(Q)^r t_1 * m_0 * P\end{array}\right] \\ +Pt_1\left[\frac{\theta c t_1 + \theta b - 2c}{\theta^2}\right] \end{array}\right\}$$

$$-(c_1 + PIp)\left(\frac{b}{3} + \frac{cT}{2}\right) > 0 ----(10)$$

Here ,

$(\mathbf{b\theta} > 2c)$

$$Pt_1\left[\frac{\theta c t_1 + \theta b - 2c}{\theta^2}\right] > \left[\begin{array}{c}(c_1+1)\left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4}\right] \\ +a_1(Q)^r t_1 * m_0 * P + (c_1 + PIp)\left(\frac{b}{3} + \frac{cT}{2}\right)\end{array}\right]$$

II. If the order quantity is greater than the minimum order quantity $Q \geq Q_s$, $T \geq T_s$

Now we consider the following three cases:

Case 1: $t_1 < M$, Case 2: $t_1 > M$, Case 3: $t_1 = M$

Case: I $t_1 < M$

In this situation, since the length of period with positive stock is less than the permissible delay period, the retailer can use the sales revenue to earn interest at an annual rate IE in (0, M)

The interest earn IE is

$$IE = SI_e\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{M} I(t)\,dt\right]$$

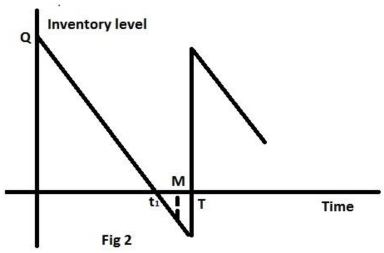

Fig 2

$$IE = SI_e\left\{\begin{array}{c} \frac{1}{\theta^2}\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ +\left[\begin{array}{c} a\left(t_1 M - \frac{M^2}{2} - \frac{t_1^2}{2}\right) + \frac{b}{2}\left(t_1^2 M - \frac{M^3}{3} - \frac{2t_1^3}{3}\right) \\ +\left(\frac{c}{3}\right)\left(t_1^3 M - \frac{M^4}{4} - \frac{3t_1^4}{4}\right)\end{array}\right]\end{array}\right\}$$

Therefore total average cost per unit time is

$$TC_2 = \frac{1}{T}\{OC + PC + HC + SC - IE - Cost\ of\ D.Items\}$$

$$TC_2 = \frac{1}{T}\left\{\begin{array}{c} A + \frac{h}{\theta^2}\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] + Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] \\ +c_1\left[\begin{array}{c} a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + \frac{b}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2t_1^3}{3}\right) \\ +\left(\frac{c}{3}\right)\left(t_1^3 T - \frac{T^4}{4} - \frac{3t_1^4}{4}\right)\end{array}\right] \\ -SI_e\left\{\begin{array}{c} \frac{1}{\theta^2}\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ +\left[\begin{array}{c} a\left(t_1 M - \frac{M^2}{2} - \frac{t_1^2}{2}\right) + \frac{b}{2}\left(t_1^2 M - \frac{M^3}{3} - \frac{2t_1^3}{3}\right) \\ +\left(\frac{c}{3}\right)\left(t_1^3 M - \frac{M^4}{4} - \frac{3t_1^4}{4}\right)\end{array}\right]\end{array}\right\} - a_1(Q)^r t_1 * m_0 * P \end{array}\right\}$$

For minimizing the total relevant cost per unit of time the approximate optimal value of T can be optimal by solving the following equation

$$\frac{\partial TC_2}{\partial T} = 0$$

$$\frac{\partial TC_2}{\partial T} = -\frac{1}{T^2}\left\{\begin{array}{c} A + \left(\frac{h}{\theta^2} + SI_e\right)\left[(b\theta - 2c)\left(\frac{t_1^2}{2}\right) + 2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ -(SI_e + c_1)\left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4}\right] \\ -SI_e\left[\begin{array}{c} a\left(t_1 M - \frac{M^2}{2}\right) + \frac{b}{2}\left(t_1^2 M - \frac{M^3}{3}\right) \\ +\left(\frac{c}{3}\right)\left(t_1^3 M - \frac{M^4}{4}\right)\end{array}\right] \\ +Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] - a_1(Q)^r t_1 * m_0 * P \end{array}\right\}$$

$$-c_1\left(\frac{a}{2} + \frac{bT}{3} + \frac{cT^2}{4}\right) = 0 ----(11)$$

This also satisfies the conditions

$$\frac{\partial^2 TC_2}{\partial T^2} > 0$$

$$\frac{\partial^2 TC_2}{\partial T^2}=\frac{2}{T^3}\left\{\begin{array}{c} A+\left(\frac{h}{\theta^2}+SI_e\right)\left[(b\theta-2c)\left(\frac{t_1^2}{2}\right)+2\theta c\left(\frac{t_1^3}{3}\right)\right] \\ -(SI_e+c_1)\left[\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{ct_1^4}{4}\right] \\ -SI_e\left[\begin{array}{c} a\left(t_1M-\frac{M^2}{2}\right)+\frac{b}{2}\left(t_1^2M-\frac{M^3}{3}\right)\\ +\left(\frac{c}{3}\right)\left(t_1^3M-\frac{M^4}{4}\right)\end{array}\right] \\ +Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]-a_1(Q)^r t_1*m_0*P\end{array}\right\}-c_1\left(\frac{b}{3}+\frac{cT}{2}\right)>0--(12)$$

Here $(b\theta > 2c)$,

$$\left[\begin{array}{c} Pt_1\left[\frac{\theta ct_1+\theta b-2c}{\theta^2}\right]>(SI_e+c_1)\left[\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{ct_1^4}{4}\right]\\ +SI_e\left[a\left(t_1M-\frac{M^2}{2}\right)+\frac{b}{2}\left(t_1^2M-\frac{M^3}{3}\right)+\left(\frac{c}{3}\right)\left(t_1^3M-\frac{M^4}{4}\right)\right]\\ +a_1(Q)^r t_1*m_0*P+c_1\left(\frac{b}{3}+\frac{cT}{2}\right)\end{array}\right]$$

Case :II $t_1 > M$

Since $t_1 > M$ the retailer pays interest at an annual rate IP during the period (M,t_1) interest pays in this case, denoted by IP is given by

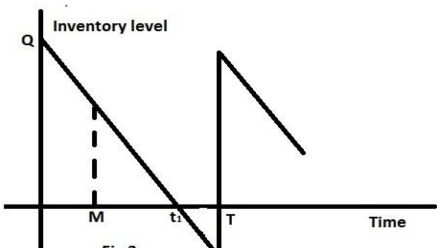

Fig 3

$$IP=PI_p(t_1-M)\int_M^{t_1} I(t)\,dt$$

$$IP=PI_p(t_1-M)\left[\frac{(\theta b-2c)}{\theta^2}\left(\frac{t_1^2}{2}-t_1M+\frac{M^2}{2}\right)+\frac{c}{\theta}\left(\frac{2t_1^3}{3}-t_1^2M+\frac{M^3}{3}\right)\right]$$

Then the total average cost per unit time is

$$TC_3=\frac{1}{T}\{OC+PC+HC+SC+IP-IE-\text{Cost of D. Items}\}$$

$$TC_3=\frac{1}{T}\left\{\begin{array}{c} A+\frac{h}{\theta^2}\left[(b\theta-2c)\left(\frac{t_1^2}{2}\right)+2\theta c\left(\frac{t_1^3}{3}\right)\right]+Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]\\ +c_1\left[a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+\frac{b}{2}\left(t_1^2T-\frac{T^3}{3}-\frac{2t_1^3}{3}\right)+\left(\frac{c}{3}\right)\left(t_1^3T-\frac{T^4}{4}-\frac{3t_1^4}{4}\right)\right]\\ +PI_p(t_1-M)\left[\begin{array}{c}\frac{(\theta b-2c)}{\theta^2}\left(\frac{t_1^2}{2}-t_1M+\frac{M^2}{2}\right)\\ +\frac{c}{\theta}\left(\frac{2t_1^3}{3}-t_1^2M+\frac{M^3}{3}\right)\end{array}\right]-a_1(Q)^r t_1*m_0*P\end{array}\right\}$$

For minimizing the total relevant cost per unit of time the approximate optimal value of T can be optimal by solving the following equation

$$\frac{\partial TC_3}{\partial T}=0$$

$$\frac{\partial TC_3}{\partial T}=-\frac{1}{T^2}\left\{\begin{array}{c} A+\frac{h+PI_p(t_1-M)}{\theta^2}\left[(b\theta-2c)\left(\frac{t_1^2}{2}\right)+2\theta c\left(\frac{t_1^3}{3}\right)\right]\\ +Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]-c_1\left[\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{ct_1^4}{4}\right]\\ +PI_p(t_1-M)\left[\begin{array}{c}\frac{(\theta b-2c)}{\theta^2}\left(\frac{M^2}{2}-t_1M\right)\\ +\frac{c}{\theta}\left(\frac{M^3}{3}-t_1^2M\right)\end{array}\right]\\ -a_1(q)^r t_1*m_0*P\end{array}\right\}-c_1\left(\frac{a}{2}+\frac{bT}{3}+\frac{cT^2}{4}\right)=0-(13)$$

This also satisfies the conditions

$$\frac{\partial^2 TC_3}{\partial T^2}>0$$

$$\frac{\partial^2 TC_3}{\partial T^2}=\frac{2}{T^3}\left\{\begin{array}{c} A+\frac{h+PI_p(t_1-M)}{\theta^2}\left[(b\theta-2c)\left(\frac{t_1^2}{2}\right)+2\theta c\left(\frac{t_1^3}{3}\right)\right]\\ +Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]-c_1\left[\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{ct_1^4}{4}\right]\\ +PI_p(t_1-M)\left[\frac{(\theta b-2c)}{\theta^2}\left(\frac{M^2}{2}-t_1M\right)+\frac{c}{\theta}\left(\frac{M^3}{3}-t_1^2M\right)\right]\\ -a_1(q)^r t_1*m_0*P\end{array}\right\}-c_1\left(\frac{b}{3}+\frac{cT}{2}\right)>0--(14)$$

Here $(b\theta > 2c)(t_1 > M)$

$$Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]>\left[\begin{array}{c} c_1\left[\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{ct_1^4}{4}\right]\\ +PI_p(t_1-M)\left[\begin{array}{c}\frac{(\theta b-2c)}{\theta^2}\left(\frac{M^2}{2}-t_1M\right)\\ +\frac{c}{\theta}\left(\frac{M^3}{3}-t_1^2M\right)\end{array}\right]\\ +a_1(q)^r t_1*m_0*P+c_1\left(\frac{b}{3}+\frac{cT}{2}\right)\end{array}\right]$$

Case: III $t_1=M$

For $t_1=M$ both the cost functions TC_2 and TC_3 become identical and the cost function is obtained on substituting $t_1=M$ either in equation (14) or (12)

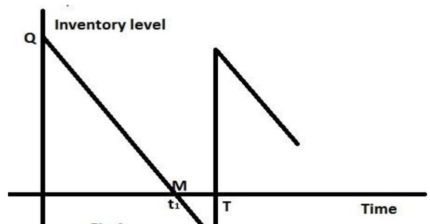

Fig 4

The total average cost per unit time

$$TC_4=\frac{1}{T}\{OC+PC+HC+SC+IP-\text{Cost of D. Items}\}$$

$$TC_4=\frac{1}{T}\left\{\begin{array}{c} A+\frac{h}{\theta^2}\left[(b\theta-2c)\left(\frac{t_1^2}{2}\right)+2\theta c\left(\frac{t_1^3}{3}\right)\right]\\ +Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]\\ +c_1\left[\begin{array}{c} a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+\frac{b}{2}\left(t_1^2T-\frac{T^3}{3}-\frac{2t_1^3}{3}\right)\\ +\left(\frac{c}{3}\right)\left(t_1^3T-\frac{T^4}{4}-\frac{3t_1^4}{4}\right)\end{array}\right]+-a_1(Q)^r t_1*m_0*P\end{array}\right\}$$

For minimizing the total relevant cost per unit of time the approximate optimal value of T is obtained by solving the following equation

$$\frac{\partial TC_4}{\partial T}=0$$

$$\frac{\partial TC_4}{\partial T}=-\frac{1}{T^2}\left\{\begin{array}{c} A+\frac{h}{\theta^2}\left[(b\theta-2c)\left(\frac{t_1^2}{2}\right)+2\theta c\left(\frac{t_1^3}{3}\right)\right]\\ +Pt_1\left[\frac{\theta b+\theta ct_1-2c}{\theta^2}\right]-c_1\left[\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{ct_1^4}{4}\right]\\ \pm a_1(Q)^r t_1*m_0*P\end{array}\right\}$$

$$-c_1\left(\frac{a}{2}+\frac{bT}{3}+\frac{cT^2}{4}\right)=0--(15)$$

This also satisfies the conditions

$$\frac{\partial^2 TC_4}{\partial T^2}>0$$

$$\frac{\partial^2 TC_4}{\partial T^2} = \frac{2}{T^3}\left\{\begin{array}{c} A + \frac{h}{\theta^2}\left[(b\theta - 2c)\left(\frac{t_1{}^2}{2}\right) + 2\theta c\left(\frac{t_1{}^3}{3}\right)\right] \\ +Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] \\ -c_1\left[\frac{a t_1{}^2}{2} + \frac{b t_1{}^3}{3} + \frac{c t_1{}^4}{4}\right] \\ -a_1(Q)^r t_1 * m_0 * P \end{array}\right\} - c_1\left(\frac{b}{3} + \frac{cT}{2}\right) > 0 - -(16)$$

Here

$(b\theta > 2c)$

$$Pt_1\left[\frac{\theta b + \theta c t_1 - 2c}{\theta^2}\right] > \left[\begin{array}{c} c_1\left[\frac{a t_1{}^2}{2} + \frac{b t_1{}^3}{3} + \frac{c t_1{}^4}{4}\right] \\ +a_1(Q)^r t_1 * m_0 * P + c_1\left(\frac{b}{3} + \frac{cT}{2}\right) \end{array}\right]$$

III.NUMERICAL EXAMPLES

Example: 1

For the numerical illustration of the developed model the values of various parameters in proper units can be taken as follows,

Table-1

S.No	Parameters	Values
1.	a	1000
2.	b	150
3.	c	15
4.	I_P	0.15
5.	I_e	0.13
6.	h	0.12
7.	P	20
8.	θ	0.28
9.	t_1	0.25
10.	c_1	4
11.	S	18
12.	r	0.02
13.	m_0	0.15
14.	A	3

Then we get optimal values as $Q^*=42$, $T^*=0.46$, $TC_1{}^*=1207$ in appropriate units.

Example:2

For the numerical illustration of the developed model the values of various parameters in proper units can be taken as follows,

Table-2 $t_1 < M$

S.No	Parameters	Values
1.	a	1000
2.	b	150
3.	c	15
4.	I_P	0.15
5.	I_e	0.13
6.	h	0.12
7.	P	20
8.	θ	0.28
9.	t_1	0.25
10.	c_1	4
11.	S	18
12.	r	0.02
13.	m_0	0.15
14.	A	3
15.	M	0.45

Then we get optimal values as $Q^*=42$, $T^*=0.57$, $TC_1{}^*=1022$ in appropriate units.

Example:3 For the numerical illustration of the developed model the values of various parameters in proper units can be taken as follows,

Table-3 $t_1 > M$

S.No	Parameters	Values
1.	a	1000
2.	b	150
3.	c	15
4.	I_P	0.15
5.	I_e	0.13
6.	h	0.12
7.	P	20
8.	θ	0.28
9.	t_1	0.25
10.	c_1	4
11.	S	18
12.	r	0.02
13.	m_0	0.15
14.	A	3
15.	M	0.15

IV. CONCLUSION

This paper introduces an inventory model for Damageable/ Breakable items with time dependent quadratic demand and delay in payments under two levels of trade credit policy, Therefore this paper present a practical solution for purchase of breakable goods to make better decision. This model is solved analytically by minimizing the total inventory cost. Finally the proposed model has been verified by the numerical example.

REFERENCES

[1] Aggarwal SP, Jaggi CK (1995) “Ordering policies of deteriorating items under permissible delay in Payments”, J Operation research scientific society 46:658-662.

[2] Chen LH, Kang (2010b) “Coordination between vender and buyer considering trade credit and Items of imperfect quality”, International journals of production Economics 123:53-61.

[3] GoyalSK(1985) “Economic order quantity under conditions of permissible delay in payments”, J Operation research scientific society 36:335-338.

[4] Huang YF (2003) “Optimal retailers ordering policies in the EOQ model under trade credit financing” ,J Operation research scientific society 54:1011-1015.

[5] Hwang H,Shinn SW (1997) “Retailers pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments”,Comp Operation Res 24:539-547.

[6] Mehdi Sajadifar& Ahmad mavoji (2014) “An inventory model with demand dependent replenishment rate for damageable item and shortage”. Industrial Engineering and Operation management 13:7-9.

[7] Muniappan.P,Uthayakumar.R&Ganesh.S(2014) “An Economic lot sizing production model for deteriorating items under two level trade credit”, Applied mathematics science 95:4737-4747.

[8] TengJT(1009) “Optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers”. Int.J prod Econ 119:415-423.

[9] Thangam A, Uthayakumar P (2011) “Two-echelon trade credit financing in a supply chain with perishable items and two different payment methods”. Int J operation research 11(4):365-382.

[10] ThangamA(2014) “Retailers Inventory system in a two level trade credit financing with selling price discount and partial order cancellations”. Journals of industrial engineering 10:3.

[11] TsaoVC(2009) “Retailers Optimal Ordering and discounting policies advance sales discount and trade credit”. Computer industrial engineering 56:208-215.-73.