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An Inventory Model With Stock Dependent Demand, Weibull Distribution Deterioration

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Abstract

An inventory problem can be solved by using several methods starting from trial and error methods to mathematical and simulation methods. Mathematical methods help in deriving certain rule and which may suggest how to minimize the total inventory cost in case of deterministic demand. Here an attempt has been made for obtaining a deterministic inventory model for stock dependent demand pattern incorporating two-parameter Weibull distribution deterioration and with reserve inventory.

Key words: EOQ, Deterioration, Stock dependent demand pattern

Introduction

A number of researchers have worked on inventory with constant demand rate, time varying demand patterns. A few of the researchers have considered the demand of the items as stock dependent demand pattern.

Datta and Pal [3] have developed an order level inventory system with power demand pattern, assuming the deterioration of items governed by a special form of Weibull density function

$$\theta(t) = \theta_0 t; 0\langle \theta_0 \langle\langle 1, t\rangle 0.$$

They used special form of Weibull density function to sidetrack the mathematical complications in deriving a compact EOQ model. Gupta and Jauhari [4] has developed an EOQ model for deteriorating items with power demand pattern with an additional feature of permissible delay in payments. They also used special form of Weibull density function for deterioration of items.

A step forward to special form of Weibull density function, here we shall develop an EOQ model with stock dependent demand pattern and using actual form of Weibull density function Z(t) = $\alpha\beta t^{\beta-1}$, where $\alpha(0\langle \alpha \le 1), \beta\rangle 0$ for deterioration of items. Despite of all mathematical intricacies, expressions for various inventory parameters are obtained.

Here we shall develop the same problem with stock dependent demand pattern with reserve inventory.

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Assumptions and Notations

Inventory model is developed under the following assumptions and notations.

Assumptions

Replenishment rate is infinite.

The lead-time is zero.

Shortages in inventory are not allowed.

The demand is given by the stock dependent demand pattern for which, demand upto time t is assumed to be , D(t) = a+bI(t),

Where D is the demand size during the fixed cycle time T, and is the demand rate at time t

The rate of deterioration at any time $t > 0$ follows the two-parameter Weibull distribution Z(t) = $\alpha\beta t^{\beta-1}$, where $\alpha(0<\alpha<1)$, is the scale parameter and $\beta(>0)$ is the shape parameter.

Notations:

T – The fixed length of each ordering/production cycle.

C_1 – The holding cost, per unit time.

C_3 – The cost of each deteriorated unit.

C_2 - The reserve inventory cost per unit time.

Development of the Model

Let Q be the number of items produced or purchased at the beginning of the cycle and (Q-S) items be delivered into the reserve inventory stock. Balance of S items as the initial inventory of the cycle. It will be the initial inventory at time t = 0 and 'd' be the demand during period t_1. Now, the inventory level S gradually falls during time period $(0, t_1)$, due to demand and deterioration. At time t = t_1 inventory level becomes zero. Shortages are fulfilled from the reserve stock (Q-S), after the period t_1.

Let I(t) be the on-hand inventory, then the various states of the system are governed by the following differential equations:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a + bI(t)) \qquad 0 \le t \le t_1 \qquad (1)$$

Where Z(t) = $\alpha\beta t^{\beta-1}$ (2)

$$\frac{dI(t)}{dt} = -a \qquad t_1 \le t \le T \qquad (3)$$

Using (2) in (1), the Solution of I(t) is,

$$e^{\int(\alpha\beta t^{\beta-1}+b)dt} = e^{(\alpha t^{\beta}+bt)}$$

$$I(t)\, e^{(\alpha t^{\beta}+bt)} = \int -a e^{(\alpha t^{\beta}+bt)} dt$$

$$I(t) = S e^{-(\alpha t^{\beta}+bt)} - a e^{-(\alpha t^{\beta}+bt)} \int e^{(\alpha t^{\beta}+bt)} dt \qquad (4)$$

Solving further, on expanding $e^{(\alpha t^{\beta}+bt)} = 1 + (\alpha t^{\beta} + bt)$, as $(\alpha \ll 1)$ gives

$$I(t) = S e^{-(\alpha t^{\beta}+bt)} - a e^{-(\alpha t^{\beta}+bt)} [t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{bt^2}{2}] \qquad (5)$$

Solution of (3) using $I(t_1) = 0$ gives, I(t) = $a(t_1 - t)$ (6)

Using I(t_1) = 0, in (5) gives,

$$S = a[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{bt_1^2}{2}] \quad (7)$$

The total amount of deteriorated units,

$$= S - \int_0^{t_1} (a + bI(t))dt$$

Using (7) in the above, the total amount of deteriorated units in $[0, t_1]$

$$= a[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{bt_1^2}{2}] - [at + \frac{bt^2}{2}]_0^{t_1}$$

$$= a[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{bt_1^2}{2}] - [at_1 + \frac{bt_1^2}{2}]$$

$$= \frac{\alpha t_1^{\beta+1}}{\beta+1} \quad (8)$$

Average total cost per unit is given by

$$C(S, T) = C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{C_1}{T}\int_0^{t_1} I(t)dt - \frac{C_2}{T}\int_{t_1}^{T} I(t)dt \quad (9)$$

Substituting the values of I(t) from (5) and (6), eliminating S using (7) and integrating yields,

$$C(S,T) = C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1} +$$

$$\frac{C_1}{T}\int_0^{t_1} (Se^{-(\alpha t^{\beta}+bt)} - ae^{-(\alpha t^{\beta}+bt)}[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{bt^2}{2}])\, dt +$$

$$\frac{C_2}{T}\int_{t_1}^{T} (a(t_1 - t))dt$$

$$= C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1}$$

$$+ \frac{C_1}{T}\int_0^{t_1} \left[\left(a[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{bt_1^2}{2}]e^{-(\alpha t^{\beta}+bt)} - ae^{-(\alpha t^{\beta}+bt)}[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{bt^2}{2}] \right) dt \right.$$

$$- \frac{C_2}{T}\int_{t_1}^{T} a(t_1 - t)\, dt$$

$$= C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1} +$$

$$\frac{C_1}{T}\int_0^{t_1} \left\{ \begin{array}{l} a[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{bt_1^2}{2}][1-(\alpha t_1^{\beta} + bt_1)] - \\ a[1-(\alpha t_1^{\beta} + bt_1)][t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{bt^2}{2}] \end{array} \right\} dt$$

$$- \frac{C_2}{T}\int_{t_1}^{T} a(t_1 - t)\, dt$$

$$= C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{C_1 a}{T}\int_0^{t_1} [t_1 - \alpha t_1^{\beta+1} - bt_1^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{\alpha b t_1^{\beta+1}}{\beta+1} + \frac{bt_1^2}{2} - \frac{\alpha b t_1^{\beta+2}}{2}] -$$

$$[t - \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{bt^2}{2} - \alpha t^{\beta+1} - \frac{bt^{\beta+2}}{2} - \frac{bt^{\beta+2}}{\beta+1} - \frac{b^2t^3}{2}]dt +$$

$$+ \frac{C_2 a}{T}[t_1 t - \frac{t^2}{2}]_{t_1}^{T}$$

$$= C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{C_1 a}{T}\left[t_1 t - \frac{bt_1^2 t}{2} - \frac{\alpha(\beta+1) - \alpha + \alpha b)}{\beta+1} t_1^{\beta+1} t - \frac{\alpha b t_1^{\beta+2} t}{2} \right]_0^{t_1}$$

$$- \left[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{bt^3}{6} - \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{bt^{\beta+3}}{2(\beta+3)} - \frac{bt^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{b^2t^4}{8} \right]_0^{t_1}$$

$$- \frac{C_2 a}{T}[t_1 t - \frac{t^2}{2}]_{t_1}^{T}$$

$$= \quad C_3 \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{C_1 a}{T}\left[\frac{t_1^2}{2} - \frac{b t_1^3}{3} - \frac{b^2 t_1^4}{8} - \left\{ \frac{\alpha(\beta+b)}{\beta+1} + \frac{\alpha}{\beta+2} + \frac{\alpha}{(\beta+1)(\beta+2)} \right\} t_1^{\beta+2} \right.$$

$$\left. - \left\{ \frac{\alpha}{2} + \frac{1}{2(\beta+3)} + \frac{1}{(\beta+1)(\beta+3)} \right\} b t_1^{\beta+3} \right] - \frac{C_2 a}{T}\{t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\} \qquad (10)$$

(neglecting higher powers of $\alpha^2, \alpha^3 ....$)

Further, for the minimization of the cost, we set

$$\frac{dC(t_1)}{dt_1} = 0$$

$$\Rightarrow C_3 \alpha t_1^{\beta} + \frac{C_1 a}{T}\left\{ t_1 - b t_1^2 - \frac{b^2 t_1^3}{2} - \left\{ \frac{((\beta+b)+1)(\beta+2)}{\beta+1} \right\} \alpha t_1^{\beta+1} \right.$$

$$\left. - \left\{ \frac{(\alpha(\beta+1)+1)(\beta+3)}{2(\beta+1)} \right\} b t_1^{\beta+2} \right\} - \frac{C_2 a}{T}[\mathrm{T} - t_1] = 0$$

On solving it, we obtain value of t_1 and let this value of t_1 be the optimum value of t_1^*

Solving the optimum value of t_1^* in (7), optimum value of S is,

$$S^* = \quad a[t^*_1 + \frac{\alpha t^{*\beta+1}_1}{\beta+1} + \frac{b t^{*2}_1}{2}]$$

Conclusion

From the above work, An inventory model is studied using Weibull distribution. With Stock dependent demand rate, we have obtained Optimal value of S, with minimum cost .

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