An Inventory Model For Deteriorating Items With Three Parameter Weibull Deterioration And Price Dependent Demand.

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Abstract

The paper presents an inventory model for deteriorating items with price dependent demand. Deterioration rate follows a three parameter Weibull distribution. Shortages are allowed and are completely backlogged. The results are illustrated with the help of numerical example. Sensitivity analyses are carried out to analyze the effect of changes in the optimal solution with respect to change in one parameter at a time.

***Keywords**: Deterioration items, holding cost, Inventory, Price – dependent demand time, Shortages.*

1. Introduction

Inventory models are classified in to three categories (1) Deterioration (2) Obsolescence (3) no deterioration/no obsolescence. Deterioration makes the product value dull.

Ajanta Roy [1] presented an inventory model for time proportional deterioration rate and demand is function of selling price. The Author discussed the model without shortage and also with shortages in which the shortages are completely backlogged. Mukesh Kumar, Anand Chauhan, Rajat Kumar [8] extended Ajanta Roy models with trade credit. Tripathy C.K and L.M. Pradhan [13] gave a model in which the demand of the product decreases with the increase of time and sale price and deterioration rate follows a three parameter Weibull distribution. Now Tripathy C.K and L.M. Pradhan [14] included salvage value and developed an EOQ model for three parameter Weilbull distribution deterioration rate under permissible delay in payments. Padmanabhan.G, Prem Vrat [9] formulated an EOQ model for perishable items under stock dependent selling rate. Sahoo.N.K., Sahoo .C.K. & Sahoo.S.K described an inventory model for price dependent demand and time varying holding cost. Vikas Sharma and Rekha Rani Chaudhary [16] explained and inventory model for two parameter Weibull deterioration rate. They found profit for their model. Sanjay JAIN and Mukesh KUMAR [12] explained an inventory model with ramp type demand and three parameter Weibull deterioration rate. The Authors also analyzed and summarized economic order quantity models done by few researchers. There are some products which start deteriorate only after some interval of time. This was explained by taking three parameter Weibull distribution deterioration rate. Anil Kumar Sharma, Manoj Kumar Sharma and Nisha Ramani [2] described an inventory model for two – parameter Weibull distribution deterioration rate and demand rate is power pattern. Manoj Kumar Meher, Gobinda Chandra Panda, Sudhir Kumar Sahu [7] adopted a two – parameter Weibull distribution deterioration to develop an inventory model under permissible delay in payments. Kun – Shan Wu [6] made an attempt in his paper to obtain the optimal ordering quantity of deteriorating items for two – parameter Weibull distribution deterioration under shortages and permissible delay in payments. P.K.Tripathy and S.Pradham [15] also define an inventory model with two – parameter Weibull distribution as demand rate and deterioration rate increases with time.

In this present paper, we have developed an inventory model for three-parameter Weibull deterioration rate and price dependent demand. Shortages are allowed and are completely backlogged. Holding cost is assumed to be constant. Our aim is to increase the profit.

2. Assumptions and notations

(i) The demand rate is a function of selling price.

(ii) Shortages are allowed and are completely backlogged

(iii) Lead time is zero.

(iv) Replenishment is instantaneous

(v) A is the set up cost

(vi) C is the unit cost of an item

(vii) p is the selling price

(viii) Demand D (t) = f (p) = a-p, where a > p.

(ix) C_2 is the shortage cost per unit time

(x) $\theta(t) = \alpha\beta(t-\gamma)^{\beta-1}$, $0 \le \alpha < 1$, $\beta > 0$ and $-\infty < \gamma < \infty$ is the deterioration rate. At time T_1 the Inventory becomes Zero and shortages start occurring.

(xi) h is the constant holding cost.
(xii) T is the length of the cycle.

3. Mathematical formulation and solution

Let I (t) be the inventory at time T ($0 \le t \le T$) the differential equation for the instantaneous state over (0,T) are given by

$$\frac{dI(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} I(t) = -(a-p),\ 0 \le t \le T_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -(a-p), \quad T_1 \le t \le T \quad (2)$$

With boundary condition I (T_1) = 0

Solving equation (1)

$$I(t) = -(a-p)\left[(t-T_1) + \frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1} - (T_1-\gamma)^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\left((t-\gamma)^{2\beta+1} - (T_1-\gamma)^{2\beta+1}\right)\right],\ 0 \le t \le T_1 \quad (3)$$

Solving equation (2)

$$= -(a-p)(t-T_1), \quad T_1 \le t \le T \quad (4)$$

Shortage Cost

$$SC = \frac{C_2}{T}\int_{T_1}^{T}(a-p)(t-T_1)\,dt$$

$$= \frac{C_2}{2T}(a-p)(T-T_1)^2 \quad (5)$$

Holding Cost

$$HC = \frac{h}{T}\int_0^{T_1} I(t)\,dt$$

$$= \frac{-h}{T}(a-p)\left[\frac{\alpha}{\beta+1}\left(\frac{(T_1-\gamma)^{\beta+2}}{\beta+2} - T_1(T_1-\gamma)^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(\frac{(T_1-\gamma)^{2\beta+2}}{2(\beta+1)} - T_1(T_1-\gamma)^{2\beta+1}\right)\right] + h(a-p)\left[\frac{T_1^2}{2} + \frac{\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2(-\gamma)^{2\beta+2}}{4(2\beta+1)(\beta+1)}\right] \quad (6)$$

Stock loss due Deterioration

$$D = (a-p)\int_0^{T_1} e^{\alpha(t-\gamma)^\beta}\,dt - (a-p)\int_0^{T_1} dt$$

$$= (a-p)\left[\frac{\alpha(T_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha^2(T_1-\gamma)^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha^2(-\gamma)^{2\beta+1}}{2(2\beta+1)}\right] \quad (7)$$

Order Quality

$$Q = \left[D + \int_0^{T}(a-p)\,dt\right]$$

$$= (a-p)\left[\frac{\alpha(T_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha^2(T_1-\gamma)^{2\beta+1}}{2(2\beta+1)} + T - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha^2(-\gamma)^{2\beta+1}}{2(2\beta+1)}\right] \quad (8)$$

Purchase cost $= \frac{CQ}{T}$

$$= (a-p)\frac{C}{T}\left[\frac{\alpha(T_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha^2(T_1-\gamma)^{2\beta+1}}{2(2\beta+1)} + T - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha^2(-\gamma)^{2\beta+1}}{2(2\beta+1)}\right] \quad (9)$$

Total Profit Per unit time is

= p (a-p) - $\frac{1}{T}$ [Ordering cost + purchase cost + shortages cost + holding cost]

$$K(p,T,T_1) = p(a-p) - \frac{1}{T}\left\{A + \frac{C_2}{2}(a-p)(T-T_1)^2 + C(a-p)\left(\frac{\alpha(T_1-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha^2(-\gamma)^{2\beta+1}}{2(2\beta+1)} + \frac{\alpha^2(T_1-\gamma)^{\beta+1}}{2(2\beta+1)} + T\right) - h(a-p)\left[\frac{\alpha}{\beta+1}\right.\right.$$

$$\left(\frac{(T_1-\gamma)^{\beta+2}}{\beta+2} - T_1 (T_1 - \gamma)^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}$$

$$\left(\frac{(T_1-\gamma)^{2\beta+2}}{2(\beta+1)} - T_1 (T_1 - \gamma)^{2\beta+1}\right)] + h\,(a-p)\left[\frac{T_1^2}{2} + \right.$$

$$\left.\frac{\alpha(-\gamma)^{\beta+2}}{(\beta+2)(\beta+1)} + \frac{\alpha^2(-\gamma)^{2\beta+2}}{4(2\beta+1)(\beta+1)}\right]\}$$

Let $T_1 = vT$, $0 < v < 1$

$$K(p,T) = p\,(a-p) - \frac{1}{T}\left\{A + \frac{C_2}{2}(a-p)(T-vT)^2 + C\,(a-p)\right.$$

$$\left(\frac{\alpha(vT-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} - \right.$$

$$\left.\frac{\alpha^2(-\gamma)^{2\beta+1}}{2(2\beta+1)} + \frac{\alpha^2(vT-\gamma)^{2\beta+1}}{2(2\beta+1)} + T\right) - h\,(a-p)\,[$$

$$\frac{\alpha}{\beta+1}\left(\frac{(Tv-\gamma)^{\beta+2}}{\beta+2} - vT\,(vT-\gamma)^{\beta+1}\right) +$$

$$\frac{\alpha^2}{2(2\beta+1)}$$

$$\left(\frac{(vT-\gamma)^{2\beta+2}}{2(\beta+1)} - vT\,(vT-\gamma)^{2\beta+1}\right)] + h(a-p)\,[$$

$$\frac{(vT)^2}{2}\;\frac{\alpha(-\gamma)^{\beta+2}}{(\beta+2)(\beta+1)} + \frac{\alpha^2(-\gamma)^{2\beta+2}}{4(2\beta+1)(\beta+1)}]\}.$$

$$\frac{\partial K(p,T)}{\partial T} = \frac{A}{T^2} - \frac{C_2}{2}(a-p)(1-v)^2 - C(a-p)\,[$$

$$\frac{\alpha T(\beta+1)v(vT-\gamma)^{\beta} - (vT-\gamma)^{\beta+1}}{T^2(\beta+1)} +$$

$$\frac{\alpha(-\gamma)^{\beta+1}}{T^2(\beta+1)} + \frac{\alpha^2(-\gamma)^{2\beta+1}}{2T^2(2\beta+1)} +$$

$$\frac{\alpha^2 T(2\beta+1)v(vT-\gamma)^{2\beta} - \alpha^2(vT-\gamma)^{2\beta+1}}{2T^2(2\beta+1)}\,]$$

$+ h\,(a-p)$

$$\left[\frac{\alpha T(\beta+2)v(vT-\gamma)^{\beta+1} - \alpha(vT-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2} - \right.$$

$$\alpha v^2(vT-\gamma)^{\beta} +$$

$$\frac{v\alpha^2 T(2\beta+2)(vT-\gamma)^{2\beta+1} - \alpha^2(vT-\gamma)^{2\beta+2}}{4(2\beta+1)(\beta+1)T^2}$$

$$-\frac{\alpha^2 v^2(vT-\gamma)^{2\beta}}{2}\,] - h\,(a-p)\left[\frac{v^2}{2} - \right.$$

$$\frac{\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2} - \frac{\alpha^2(-\gamma)^{2\beta+2}}{4T^2(2\beta+1)(\beta+1)}\,] \tag{10}$$

$$\frac{\partial K(p,T)}{\partial p} = (a-2p) - \frac{1}{T}\left\{\frac{C_2(T-Tv)^2}{2} - C\right.$$

$$\left(\frac{\alpha(Tv-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} - \right.$$

$$\left.\frac{\alpha^2(-\gamma)^{2\beta+1}}{2(2\beta+1)} + \frac{\alpha^2(Tv-\gamma)^{2\beta+1}}{2(2\beta+1)} + T\right) + h\left[\frac{\alpha}{\beta+1}\right.\left(\right.$$

$$\left.\frac{(Tv-\gamma)^{\beta+2}}{\beta+2} - Tv\,(Tv-\gamma)^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(\right.$$

$$\left.\frac{(Tv-\gamma)^{2\beta+2}}{2(\beta+1)} - Tv\,(Tv-\gamma)^{2\beta+1}\right)] - h\left[\frac{(Tv)^2}{2} + \right.$$

$$\left.\frac{\alpha(-\gamma)^{\beta+2}}{(\beta+2)(\beta+1)} + \frac{\alpha^2(-\gamma)^{2\beta+2}}{4(2\beta+1)(\beta+1)}\right]\} = 0. \tag{11}$$

For the maximization of profit we set,

$$\frac{\partial K(p,T)}{\partial T} = 0 \;\&\; \frac{\partial K(p,T)}{\partial p} = 0$$

Provided

$$\left(\frac{\partial^2 K(p,T)}{\partial T^2}\right) < 0,\; \left(\frac{\partial^2 K(p,T)}{\partial p^2}\right) < 0 \text{ and}$$

$$\left(\frac{\partial^2 K(p,T)}{\partial T^2}\right)\left(\frac{\partial^2 K(p,T)}{\partial p^2}\right)\text{-}\left(\frac{\partial^2 K(p,T)}{\partial T\partial p}\right) > 0$$

4. Numerical Example

Let us consider C = 5, v = 0.2, C_2 = 5, α = 0.4, β = 2, γ = 1, h = 1, a = 50, A = 1000 in proper units we get p = 32.3909, T = 5.9564, TP = 142.1949, Q = 107.5330

5. Sensitivity Analysis

α	p	T	TP	Q
0.5	32.4215	5.9714	141.5371	108.3591
0.6	32.4534	5.9869	140.8515	109.2179
0.7	32.4866	6.0031	140.1385	110.1126
0.8	32.5210	6.0198	139.3986	111.0397

H	p	T	TP	Q
2	32.43.03	5.9277	139.9951	106.7867
3	32.4698	5.8998	137.8099	106.0562
4	32.5093	5.8125	135.6390	105.3385
5	32.5488	5.8458	133.4822	104.6336

C_2	p	T	TP	Q
4	31.8084	6.5334	177.8724	121.6404
3	31.1694	7.3674	218.8579	141.8183
2	30.4342	8.6899	267.7428	174.0637
1	29.4950	11.0218	329.7243	234.6714

C	p	T	TP	Q
6	32.9898	6.0645	124.4512	105.7209
7	33.5930	6.1782	107.3297	103.8447
8	34.2008	6.2979	90.8342	101.8983
9	34.8138	6.4246	74.9689	99.8804

v	p	T	TP	Q
0.3	31.6714	6.3800	179.1983	121.7266
0.4	30.8940	6.1710	202.9896	130.9089
0.5	30.2681	5.4801	206.8682	129.9765
0.6	29.9177	4.7713	193.7094	123.3320

a	p	T	TP	Q
60	36.7900	5.2035	346.5236	124.2395
70	41.3870	4.6959	605.7523	138.6358
80	46.0894	4.3198	918.4367	151.5396

From this table, it can be observed that

1. p(t, p) is slightly sensitive to changes in α, h and it is highly sensitive to changes in a, C_2, C_2 and v.
2. p is slightly sensitive to changes in α, h and moderately sensitivity to changes in a, C_2, C_2 and v.
3. Q is slightly sensitive to changes in the values of α, h and C and it is highly sensitive to changes in a, C_2 and v.
4. T is slightly sensitive to changes in the values of α, a, h and C and moderately sensitive to changes in C_2 and v

6. Conclusion

A deterministic inventory model for deteriorating inventory model with three parameter Weibull distribution deterioration rate has been developed. Demand rate is function of selling price and holding cost is constant occurring shortages and completely backlogged. A numerical example is also given in support of the theory. A future research it may be consider to extend the model under permissible delay in payments.

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