An Improved Method for Efficient Power System Frequency Estimation

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***Abstract*—Accurate and efficient estimation of power system frequency is relevant to many power system applications. This paper provides a method for improving the efficiency of power system frequency estimation by combining the two techniques namely Smart Discrete Fourier Transform (SDFT) and Sliding DFT. Smart Discrete Fourier Transform (SDFT) has been proved to be a precise and easy implementable method for power system frequency estimation whereas Sliding DFT establishes a relationship between DFT of successive windowed sequence. Computational efficiency for power system frequency estimation has been analyzed.**

Keywords—Power system frequency estimation, Smart Discrete Fourier Transform (SDFT) ,Sliding DFT.

I. INTRODUCTION

The electric power system is base for modern society, a fundamental factor for the development of countries. It consists of a group of apparatuses, wires, and machines that links the power plants to costumers and their needs. The generation scheme of energy (thermal hydroelectric, solar, and wind) at power plants may differ but the power grid signal can be viewed as a single sinusoidal waveform with a fixed frequency (the power system frequency).

Mostly the power generation at the power grid is by turbines that work as generators of alternating current. The rotation velocity of these turbines determines the power system frequency having nominal value 50/60 Hz. Due to technical reasons this frequency fluctuates in close proximity of its nominal value. It is quite important that for a correct operation of the power system, frequency and phase of all power generation units should remain synchronous within narrow limits. If, for example, a generator drops 2 Hz below the nominal power system frequency, it will rapidly build up enough heat to destroy itself [1]. Therefore, a tight control is needed over operator units and so accurate and efficient estimation of power system frequency is of great importance.

Smart Discrete Fourier Transform (SDFT) has been proved as fast and precise in power system frequency estimation meeting real time specifications [4] incomparision to other methods like Prony Method [2], Discrete Fourier Transform (DFT) [3] etc. It estimates the frequency by three consecutive windowed sequences differing by one sample.

Sliding DFT is a computationally efficient way of finding DFT of a sequence if DFT of previous sequence differing by one sample is known [5][6].

The organization of this paper is as follows: Basic principle of SDFT has been described in section II and Sliding DFT in section III. Section IV describes the proposed method. Section V is comparative analysis for efficiency andfinally, conclusion is given in section VI.

II. SMART DISCRETE FOURIER TRANSFORM (SDFT)

The Smart DFT is a technique to find out the power system frequency by establishing the relation between three consecutive windowed sequences differing by one sample. Consider a sinusoidal input signal of frequency ω= 2πf as follows:

$$x(t) = Xcos(\omega t + \varphi) \tag{1}$$

Where X is the amplitude and φ is the phase angle of the voltage/current signal.

Suppose that x(t) is sampled with a sampling rate (60*N) Hz waveform to produce the sample set x(k).

$$x(k) = Xcos\left(\frac{\omega k}{60N} + \varphi\right) \qquad k = 0,1,2, \dots, N-1 \tag{2}$$

The signal x(t) is conventionally represented by a phasor (a complex number) $\bar{x}$.

$$\bar{x} = Xe^{-j\varphi} = Xcos(\varphi) + Xsin(\varphi) \tag{3}$$

Then x(t) can be expressed as

$$x(t) = \frac{\bar{x}e^{-j\omega T} + \overline{x^*}e^{-j\omega T}}{2} \tag{4}$$

where * denotes complex conjugate.

Moreover, the fundamental frequency (60Hz) component of DFT of x(k) is given by

$$\widehat{x_r} = \frac{2}{N}\sum_{k=0}^{N-1} x(k+r)e^{-\frac{j2\pi k}{N}} \tag{5}$$

Combing (4) and (5) and taking frequency deviation ω = 2π (60 +Δf) into consideration, at last, we obtain:

$$\widehat{x_r} = A_r + B_r \quad (6)$$

Where

$$A_r = \frac{\bar{x}}{N}\frac{\sin\left(\frac{N\theta_1}{2}\right)}{\sin\left(\frac{\theta_1}{2}\right)} e^{-\frac{j\pi}{60N}(\Delta f(2r+N-1)+120r)}$$

$$B_r = \frac{\overline{x^*}}{N}\frac{\sin\left(\frac{N\theta_2}{2}\right)}{\sin\left(\frac{\theta_2}{2}\right)} e^{-\frac{j\pi}{60N}(\Delta f(2r+N-1)+120(r+N-1))}$$

$$\theta_1 = 2\pi\Delta f/60N$$

$$\theta_2 = 2\pi(120+\Delta f)/60N$$

If we define

$$a = e^{-j\left(\frac{\pi}{60N}(2\Delta f+120)\right)} \quad (7)$$

We can write

$$\widehat{x_{r+1}} = A_{r+1} + B_{r+1} = A_r a^1 + B_r a^{-1} \quad (8)$$

$$\widehat{x_{r+2}} = A_{r+2} + B_{r+2} = A_r a^2 + B_r a^{-2} \quad (9)$$

Now after some manipulation with (6), (8) and (9) we get

$$a = \frac{(\widehat{x_r} + \widehat{x_{r+2}}) \pm \sqrt{(\widehat{x_r} + \widehat{x_{r+2}})^2 - 4(\widehat{x_{r+1}})^2}}{2(\widehat{x_{r+1}})} \quad (10)$$

And so

$$f = 60 + \Delta f = \cos^{-1}(Re(a)) * \frac{60}{N} \quad (11)$$

III. SLIDING DFT

Sliding DFT gives the relation between DFT of two successive time sequence differing by one sample. A discrete-time signal can be considered as a sequence

…….., x(-2),x(-1), x(0), x(1), x(2),………...x(n), x(n + 1),……

Say, x(n) is a time sequence of length N=16.Now x(15) and x(16) will be

x(15) = x(0), x(1), x(2), x(3) ……….. x(14), x(15)
x(16) = x(1), x(2), x(3), x(4) ……….. x(15), x(16)

From DFT we can write DFT of x(n)

$$X_k = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

$$= x(0) + x(1)W_N^k + .. + x(N-1)W_N^{(N-1)k}$$

And from DFT properties, if N-DFT of x[n] is X[k] then

$$x\left[((n-m))_N\right] \xleftrightarrow{N-DFT} W_N^{km} X[k]$$

From above we can observe that first element x(0) in DFT of x(n) is unmodified. Now consider

$\hat{x}$ (15) = x(16), x(1), x(2), x(3), x(4),…….x(15)

If $\hat{X}$ (15) ,X(15) and X(16) is DFT of $\hat{x}$ (15), x(15) and x(16) then we can write

$\hat{X}_k(15) = X(15) - x(0) + x(16)$

And so

$X_k(16) = \hat{X}_k(15)\, e^{j2\pi k/N}$

So, we can relate the DFTs of two successive windowed sequences, each of length N, x (n-1) and x(n) as:

$$X_k(n) = [X_k(n-1) - x(n-N) + x(n)]e^{-j2\pi k/N}$$

IV. PROPOSED METHOD

As we know Sliding DFT relates DFTs of two successive windowed sequences, we will express $\widehat{x_{r+1}}$ in terms of $\widehat{x_r}$,and $\widehat{x_{r+2}}$ in terms of $\widehat{x_{r+1}}$.

$$\widehat{x_{r+1}} = (\widehat{x_r} - x(r+1-N) + x(r+1))e^{-j2\pi/N} \quad (11)$$

$$\widehat{x_{r+2}} = (\widehat{x_{r+1}} - x(r+2-N) + x(r+2))e^{-j2\pi/N} \quad (12)$$

Now (11) can be written as

$$\widehat{x_r} = \widehat{x_{r+1}}e^{j2\pi/N} + (x(r+1-N) - x(r+1))e^{j2\pi/N} \quad (13)$$

(10)can be written as

$$a = p \pm \sqrt{p+1} \quad (14)$$

Where

$$p = \frac{(\widehat{x_r} + \widehat{x_{r+2}})}{2(\widehat{x_{r+1}})} \quad (15)$$

From (12),(13) we can write

$$p = \cos\left(\frac{2\pi}{N}\right) + \frac{C + De^{-\frac{j2\pi}{N}}}{2\widehat{x_{r+1}}} \quad (16)$$

Where C=$x(r+1-N) - x(r+1)$ and
D=$x(r+2) - x(r+2-N)$

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V. COMPARITIVE ANALYSIS

In traditional method of SDFT, first the complex computation is N complex multiplication and N-1 addition per sequence. Then for calculation of p in(15) it will be 3N complex multiplication and 3N-3 complex addition for three DFT sequences and 1 complex addition and one division additional. So total 3N complex multiplication, 3N-2 complex addition and one complex division but when we introduce Sliding DFT concept in SDFT method the complex calculation reduces to great extent. We need only 2 complex multiplication and 1 complex addition for central sequence and for calculation of p in (16) it is 1 complex multiplication, 1 complex addition and one complex division so total 3 complex multiplication, 2 complex addition and one complex division. Other important point is that the computational cost is fixed and independent of sequence length N.

VI. CONCLUSION

From comparative analysis of section 5 we can observe that introduction of Sliding DFT concept in Smart Discrete Fourier Transform (SDFT) method enhances the computational efficiency to large extent keeping all the advantages of Smart Discrete Fourier Transform (SDFT) for power system frequency estimation.

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