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An Improved Level Set Method With Two Step Splitting Evolution

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Abstract

Level Set is a deformable contour model where the user specifies a starting contour that is evolved to the image contour. As opposed to other contour models, e.g. Snakes [1], where the contour is described in a parametric manner, the Level Set method is a geometric deformable model. The contour is described as a surface developed by partial differential equations, where the contour is the zero level of the surface. In this paper we give novel pure-particle algorithm for the simulation of reaction-diffusion systems on deforming surfaces[5], which represent an important class of biological models, Because they provide explanations to complex phenomena such as pattern formation or morphogenesis. The algorithm uses an implicit Lagrangian level-set representation to track the motion of the surface, the framework of discretization-corrected PSE operators to discretize the spatial derivatives of the governing equations as well as pseudo-forces to adapt the particle distribution to local resolution requirements, which renders the use of Cartesian grids unnecessary. A diffusion term is introduced into LSE, resulting in a TSSM equation, to which a piecewise constant solution can be derived. We propose a two-step splitting method (TSSM) to iteratively solve the RD-LSE equation: first iterating the LSE equation, and then solving the diffusion equation. The second step regularizes the level set function obtained in the first step to ensure stability, and thus the complex and costly re-initialization procedure is completely eliminated from LSE.

1. Introduction

In the last twenty years, the level set method (LSM) of Osher and Sethian [11] has become a popular numerical technique for tracking moving interfaces in computational geometry, fluid mechanics, computer graphics, computer vision and material sciences. The main reasons of its success are the high flexibility of this method to adapt to different problems, the ability to deal with changes of topology (contour breaking and merging) without any extra functions and the guarantee of the existence of solutions in the class of viscosity partial differential equations (PDEs). Moreover, extensive numerical algorithms based on Hamilton-Jacobi equations have been developed, accurately handling shocks and providing stable numerical schemas.

The key idea of the LSM is to implicitly represent a contour or interface as the zero level set of a higher dimensional function, called the level set function (LSF), and formulate the evolution of the contour through the evolution of the level set function. For closed contours, signed distance functions (SDFs) were originally adopted to represent level set functions because they directly provide stability and accuracy to the LSM.

The rest of the paper is organized as follows. Section 2 illustrates the different types of methods for image segmentation Section 3 presents Level Set Method. Section 4 literature review of LSM and section-5 result of recent research on image segmentation using LSM Section 6 concludes the paper.

2. IMAGE SEGMENTATION

Segmentation is the process of partitioning an image into semantically interpretable regions. The purpose of segmentation is to decompose the image into parts that are meaningful with respect to a particular application. Image segmentation is typically used to locate objects and boundaries (lines, curves, etc.) in images. The result of image segmentation is a set of regions that collectively cover the entire image, or a set of contours extracted from the image. Each of the pixels in a region is similar with respect to some characteristic or computed property, such as color, intensity, or texture. Adjacent regions are significantly different with respect to the same characteristic.

2.1 IMAGE SEGMENTATION METHODS

Several general-purpose algorithms and techniques have been developed for image segmentation. These are listed below:

- Clustering methods
- Compression-based methods
- Histogram-based methods
- Edge detection methods
- Region growing methods
- Split-and-merge methods
- Partial differential equation-based methods
    1. Parametric methods
    2. Level set methods
    3. Fast Marching methods
- Graph partitioning methods
- Watershed transformation
- Model based segmentation
- Semi-automatic segmentation
- Segmentation Benchmarking
- Neural networks segmentation

LEVEL SET METHOD

Segmenting images with level set methods was introduced at the end of the 1980's and was based on previous work on moving curvatures. Since then several variants and improvements have come up. Some of the improvements are aimed at speeding up the processing. Other methods have strength related to specific challenges like noise and broken edges. In the level set method, the curve is represented implicitly as a level set of a 2D scalar function referred to as the level set function which is usually defined on the same domain as the image. The level set is defined as the set of points that have the same function value. Fig1 shows an example of embedding a curve as a zero level set. It is worth noting that the level set function is different from the level sets of images, which are sometimes used for image enhancement. The sole purpose of the level set function is to provide an implicit representation of the evolving curve. Instead of tracking a curve through time, the level set method evolves a curve by updating the level set function at fixed coordinates through time. This perspective is similar to that of an Eulerian formulation of motion as opposed to a Lagrangian formulation, which is analogous to the parametric deformable model. A useful property of this approach is that the level set function remains a valid function while the embedded curve can change its topology.

The level set method is a numerical technique used for tracking interfaces and shapes[1]. Level set is optimization method to extract or segment the object by its shape from an image. These interface can have sharp corners .The technique can find a wide range of application including problems in image processing, computer graphics, shape of snowflakes. Consider an image f with background and foreground. Boundaries can be detected using curve evolution. The boundary of an open domain can be represented using a curve C as the isoline of a Lipschitz continuous function:

$$\boldsymbol{f}:\Omega \rightarrow R$$
$$\phi: \Omega \rightarrow C \text{ ,where } C=\{x, t\} \in \phi(x, t) = 0$$
$$\phi(x, t) > 0 \text{ for } x \in \Omega$$
$$\phi(x, t) < 0 \text{ for } x \in \bar{\Omega}$$
$$\phi(x, t) = 0 \text{ for } x \in \partial\Omega = \Gamma(t) \qquad (1)$$

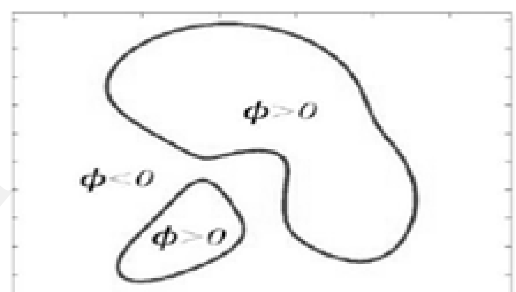


Fig -1: arbitrary active contour

In effect Φ divides the image into 3 regions, inside of the level set with positive Φ, boundary with Φ=0 and outside of the levelset with negative Φ. Then, an iterative procedure is followed, which uses an edge stopping function to decide the rate at which, the curve evolves. The evolution of curve happens in a direction normal to itself and the evolution stops when the curve meets an object or boundary. Broadly there are two approaches- Edge based and Region based.

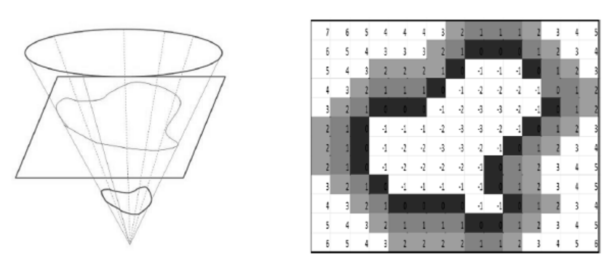


Fig -2: Level set evolution

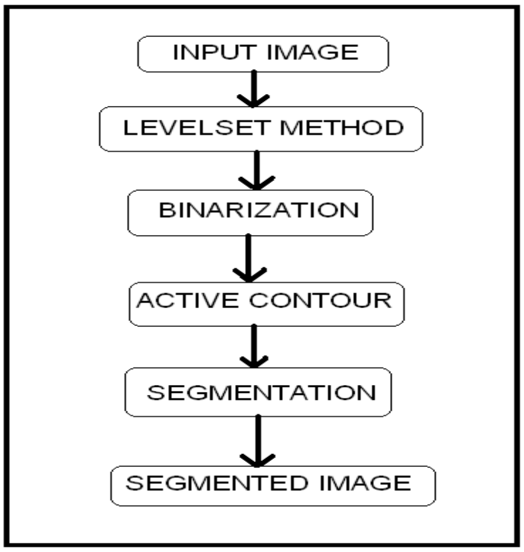


Fig -3: Flow chart of image segmentation using level set method

3. LITERATURE REVIEW

Level set methods have seen tremendously expanded applications in many areas over the past 15 years. This has been made possible by the exibility of the level set formulation in dealing with dynamic evolutions and topological changes of curves and surfaces, and by the mathematical theory and numerical tools developed in the past 15 years in studying these methods.The level set methods (LSM) can be categorized into partial differential equation (PDE) based ones and variational ones .

It was first introduced by Osher and Sethian [1] and has become a more and more popular theoretical and numerical framework within image processing, fluid mechanics, graphics, computer vision, etc. The level set method is basically used for tracking moving fronts by considering the front as the zero level set of an embedded function, called the level set function. In image processing, it is used for propagating curves in 2D or surfaces in 3D[11]. The applications of the level set method cover most fields in image processing, such as noise removal, image inpainting, image segmentation and reconstruction. In image segmentation, the level set method has some advantages compared to the active contour model. The level set method conquers the difficulties of topological transformations[2]. The level set approach is able to handle complex topological changes automatically.

The traditional level set method depends on the gradient of the given image to stop the curve evolution [3]. Therefore, it has some drawbacks. Later, some variational level set methods are developed. In a sequence of papers beginning with Chan and Vese , the authors propose a different active contour model that does not use the gradient of the image for the stopping process. The stopping term is based on the Mumford-Shah functional for segmentation. The Chan-Vese model can detect contours both with and without gradients[10]. In addition, by using this model and its level set formulation, interior contours are automatically detected, and the initial curve can be anywhere in the image. The liberty of formulation of these level set methods gives us countless possibilities.

In recent years, some variational level set formulations[3] have been proposed to regularize the LSF during evolution, and hence the re-initialization procedure can be eliminated. These variational LSMs without re-initialization have many advantages over the traditional methods, including higher efficiency and easier implementation. Chunming Lia proposed new variational formulation for geometric active contours that forces the level set function to be close to a signed distance function, and therefore completely eliminates the need of the costly re-initialization procedure. Other problems of Intensity inhomogeneities occur in real-world images and may cause considerable difficulties in image segmentation. Chunming Li a new variational level set formulation in which the regularity of the level set function such that the derived level set evolution has a unique forward-and-backward (FAB) diffusion effect, which is able to maintain a desired shape of the level set function, particularly a signed distance profile near the zero level set. This yields a new type of level set evolution called distance regularized level set evolution (DRLSE).

3.1 Comparison of different LSM method

Reference	Method	Advantage	Application
Chunning Li, Chenyang Xu [3]	New Variational Formulation	Good Performance Over Weak Boundaries	Stimulated & Real Images
M. Airouche, L. bentabet	Image Segmentation	Easy To Detect Oil Spills.	Real Satellite Images

[7]	Using Active Contour		
Chunning Le, Chenyang Xu [8]	Distance Regularized Level Set Method	Relatively Larger Timestamp To Reduce Iteration	MRI images
Li Mi, Xu Xiangmin, Qian Min [9]	Fast Level Set Method On Overall Information Of Image	More Accurate, Faster With Accuracy Of 95.2%	Prostrate Of Nucleus Cells.
Sheng Yan, Jianping Yuan, Chaohuan Hou [10]	New Extended Chan Vese Level Set Model	The Boundary Of Cyst Is Detected Even With Noise	Synthetic And Ultrasound Images
Paresh Chandra Barman, Md. Sipon Miah, Bikash Chandra Singh [11]	A New variational Level Set without re-initialization	Works on images with weak boundaries and strong noise.	MRI images
Chunning Li, Rui Huang, Zhaohua Ding [12]	A Novel Region Based Method	Much More Robust To Piecewise Constant	MRI Images And Bias Correction
Amir Fazlollahi, Nicholas Dowson, Fabrice Meriaudeau .9 [13]	Reaction Diffusion Model	Selection Of Threshold On Soft Segmentation To Be More Or Less Conservative In Their Estimation.	PET, MRI, DTI Images

Table 1: Comparison of different LSM method

4. PROPOSED METHODOLOGY DIFFUSION LEVEL SET EVOLUTION

Since the zero level is used to represent the object contour, we only need to consider the zero level set of the LSF. As pointed out in [8], with the same initial zero level set, different embedded LSFs will give the same final stable interface. Therefore, we can use a function with different phase fields as the LSF. Motivated by the phase transition theory, we propose to construct a TSSM equation by adding a diffusion term into the conventional LSE equation. Such an introduction of diffusion to LSE will make LSE stable without re-initialization.

By adding a diffusion term "ε Δ φ" into the LSE equation in Eq. (3) or Eq. (4), we have the following RD equation for LSM:

$$\varphi_{t} = \varepsilon\Delta\varphi - \frac{1}{\varepsilon}L(\varphi)\ ,\ x \in \Omega \subset R^{n}$$
$$\text{Subject to } \varphi(x, t = 0, \varepsilon) = \varphi_0(x)$$

where ε is a small positive constant, $L(\varphi)= -F|\nabla\varphi|$, for PDE-based LSM or $L(\varphi) = -F\delta(\varphi)$ for variational LSM, Δ is the Laplacian operator defined by Eq. (1) has two dynamic processes: the diffusion term "ε Δ φ" gradually regularizes the LSF to be piecewise constant in each segment domain Ωi, and the reaction term "$-\varepsilon^{-1}L(\varphi)$" forces the final stable solution of Eq. (1) to L(φ)=0, which determines Ωi. In the traditional LSMs , due to the absence of the diffusion term we have to regularize the LSF by an extra procedure, i.e., re-initialization. In the following, based on the Van der Waals-Cahn-Hilliard theory of phase transitions, we will first analyze the equilibrium solution of Eq. (1) when ε→0+ for variational LSM, and then generalize the analysis into a unified framework for both PDE-based LSM and variational LSM. Let Ω c R^n, n=2 or 3, be the domain of the level set function φ and assume that E(φ) is an energy functional w.r.t. φ, the Euler equations of E(φ) and F(φ) are the same, i.e., E ϕ (φ)=F ϕ (φ), where F(φ) E(φ). For variational LSM, assuming that the L(φ) in Eq. (1) is obtained by minimizing an energy functional E(φ), i.e., L(φ)=Eφ(φ).

Algorithm : Diffusion based level set evolution(D-LSE):

1. Initialization: $\Phi^n = \varphi 0$, n = 0
2. Compute $\varphi^{n+1/2}$ as
$$\varphi^{n+1/2} = \Phi^n - \Delta t_1 .\ L(\varphi n$$
3. Compute φ^{n+1} as

$\varphi^{n+1} = \Phi^n + \Delta t_2 \Delta \varphi n$

Where $\Phi^n = \varphi^{n+1/2}$

4. If φ_{n+1} satisfies stationary condition, stop; otherwise, n = n+1 and return to Step 2.

5.2 IMPLEMENTATION

From the analysis, we see that the equilibrium solution of Eq. (1) is piecewise constant as $\varepsilon \rightarrow 0+$, which is the characteristic of phase transition [13][14]. On the other hand, Eq. (1) has the intrinsic problem of phase transition, i.e., the stiff parameter ε–1 makes Eq. (1) difficult to implement [11][13][14]. In this section, we propose a splitting method to implement Eq. (1) to reduce the side effect of stiff parameter ε–1.

5.3 Two-Step Splitting Method (TSSM)

A TSSM algorithm to implement RD has been proposed in [11] to generate the curvature dependent motion. The reaction function is first forced to generate a binary function with values 0 and 1, and then the diffusion function is applied to the binary function to generate curvature-dependent motion. Different where the diffusion function is used to generate curvature-dependent motion, in our proposed RD based LSM, the LSE is driven by the reaction function, i.e., the LSE equation. Therefore, we propose to use the diffusion function to regularize the LSF generated by the reaction function. To this end, we propose the following TSSM to solve the RD.

Step 1: Solve the reaction term φ t=–ε–1L(φ) with φ(x,t=0)=φn till some time Tr to obtain the intermediate solution, denoted by $\varphi^{n+1/2} = \varphi(x, Tr)$;

Step 2: Solve the diffusion term φt =εΔφ, φ(x,t=0) = $\varphi^{n+1/2}$ till some time Td , and then the final level set is $\varphi^{n+1} = \varphi(x, Td)$.

5.4 Numerical Implementation

A. Numerical approximation for the spatial and time derivatives:

In implementing the traditional LSMs, the upwind scheme is often used to keep numerical stability. By introducing the diffusion term, in the proposed D-LSE the simple central difference scheme can be used to compute all the spatial partial derivatives $\partial(\cdot)/\partial x_i$, i = 1,…,n, and the simple forward difference scheme can be used to compute the temporal partial derivative φt .

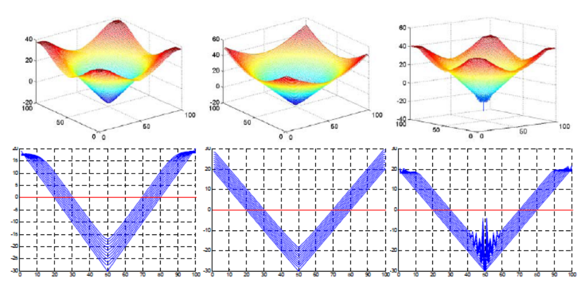
B. Setting for the time steps Δt1 and Δt2: Since Eq. (19) is a linear PDE, the standard Von Neumann analysis [19][23][24] can be used to analyze the stability for the time step Δt2.

6. EXPERIMENTAL RESULTS

In our experiments, all the competing methods use the same level set model, while the only differences are the different regularization terms used in them. We set ρ = 0.5, other parameters are set according to the different experiments.

We first apply the diffusion method to PDE-based LSM to demonstrate its superior performance to re-initialization methods; second, we apply it to edge-based variational level set models with different Dirac functionals and compare it with GDRLSE methods for images with weak boundaries; third, we apply the DIFFUSION method to classical GAC model [5] and the CV model [18] in comparison with GDRLSE and representative LSMs with re-initialization; finally, we quantitatively compare DIFFUSION with GDRLSE methods for the edge-based variational level set model, the PDE-based GAC model and the region-based CV model. The advantages of our DIFFUSION method over re-initialization methods and GDRLSE methods are summarized as follows.

A. The DIFFUSION method can keep the LSE process stable for both variational LSM and PDE-based LSM, and it is much more efficient than re-initialization method (refer to Figs. 5 and 6 in Section 5.2).



fig(4) Top row: the final LSFs. Bottom row: the middle slices of the LSFs in iterations. From left to right: results by RD method, re-initialization method and the direct implementation without re-initialization. We set Δt1=Δt2=0.1.

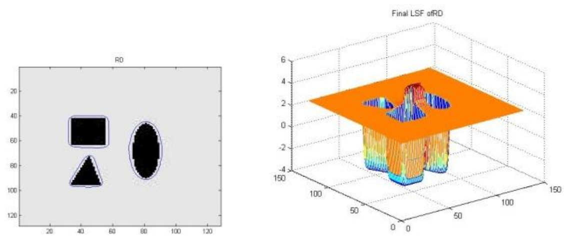


Fig (5) the final LSFs. We set Δt1=Δt2=0.1.

Table 2: Comparison of experimental results of Diffusion based LSM and DRLSE method.

	ΔT	Mu (μ)	Lamda (γ)	Alpha ($\propto$)	$\in$	(σ)	time
DIFFU-LSM	1	0.2/ ΔT	5	-0.3	1.5	0.8	4.8
DRLSE	1	0.2/ ΔT	5	-0.3	1.5	0.8	6.2

CONCLUSION

In this paper, we proposed a reaction-diffusion based level set evolution (LSE), which is completely free of the re-initialization procedure required by traditional level set methods. A two-step-splitting-method (TSSM) was then proposed to effectively solve the DIFFUSION based LSE. The proposed DIFFUSION method can be generally applied to either variational level set methods or PDE-based level set methods. It can be implemented by using the simple finite difference scheme. The DIFFUSION method has the following advantages over the traditional level set method and state-of-the-art algorithms [9][59][34]. First, the DIFFUSION method is general, which can be applied to the PDE-based level set methods and variational ones. Second, the DIFFUSION method has much better performance on weak boundary anti-leakage. Third, the implementation of the DIFFUSION equation is very simple and it does not need the upwind scheme at all. Fourth, the DIFFUSION method is robust to noise. The experiments on synthetic and real images demonstrated the promising performance of our approach.

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