An Fuzzy Inventory Model for Decaying Items with a Demand-Dependent Selling Price as a Degree n Polynomial, Linear Deterioration Rate, and Constant Holding Cost

Neha Rani
Research Scholar, Department of Mathematics
School of Applied Sciences,
Om Sterling Global University,
Hisar-125001, Haryana, India

Vinod Kumar
Professor and Head, Department of Mathematics,
School of Applied Sciences,
Om Sterling Global University,
Hisar-125001, Haryana, India

***Abstract* -- A deterministic inventory model for a general demand rate dependent on selling price is used in this work. The lead time is set to zero. The rate of degradation increases in direct proportion to the passage of time. The cost of holding, ordering, and other costs are assumed to remain constant. This model uses a triangular fuzzy number to represent the costs of shortages, degradation, and holding. The graded mean integration approach is used to defuzzify the numbers. This listing is for a single item. The shortages are permitted and fully backlogged. The lead time is presumed to be zero. The solution of the differential equation was used to calculate the optimal value in this research. The model is quantitatively validated, and convexity is shown as a two-dimensional graph using maple software.**

Keywords— Fuzzy, Selling Price Demand, Polynomial Function, Holding Cost.

I. INTRODUCTION

Inventory control is the practice of having the correct number of parts and products on hand to avoid shortages, overstocking, and other costly issues. A person can protect himself from making impulsive decisions and avoid the misery and money that comes with overstocking inventory by employing inventory control. As the name implies, inventory control helps you keep track of your inventory levels so you can make the most of the resources and avoid product deterioration and obsolescence. For small businesses, pen and paper systems and Excel inventory systems are sufficient, but they will fall behind if you have more complex inventory requirements. For firms to purchase, there are a variety of complicated and straightforward software options. There is an inventory management solution for today's companies.

Multiple methods have been created to control inventories in various firms utilizing updated software by several academics. Variable demand and degradation rates were used to create the models. A literature study on inventory models was done by several researchers, and the following are their findings:

Henri Prade and Didier Dubois (1978) in [1] established that mean value and membership function were calculated using fuzzy integers. Calculus, they came to the conclusion, is an extension of classical tolerance analysis. Chih Hsun Hsieh (2002) in [2] by varying various constraints, presented two fuzzy models: crisp model and fuzzy model. He used the graded mean integration method to get the best result for the models. He also operated the expanded Langrangean approach to solve inequality constraint issues. He concluded that the finest fuzzy manufacturing amount is a specific form of trapezoidal number.

Jershan Chiang and Jing-Shing Yao (2003) in [3] using triangular fuzzy numbers and signed distance and centroid methods for defuzzification, created an inventory model with no backorders. They also compared the results of the two methods. P.K. Kundu, Sujit Kumar and A. Goswami (2003) in [4] dealt with real-life problems. They created a more realistic EOQ model that featured a fading rate and demand rate that were both fuzzy. They also assessed the problems that emerged as a result of outdated equipment. The model was statistically validated and carefully evaluated.

L. A. Aziz and J. K Syed (2007) in [5] saw the ordering and holding costs as ambiguous variables. They utilized a hazy atmosphere with no shortages in the simulation. The model utilizes the Signed Distance method to defuzzify. P. K. De. Apurva Rawal (2011) in [6] used real-world data from Banasthali University's LPG shop to construct a model in a fuzzy environment. He looked at triangular fuzzy numbers and used the signed distance approach to defuzzify them so that EOQ (economic order quantity) could be calculated.

Tripti Chakrabarti and Suman Saha (2012) in [7] considered both the rate of deterioration and the demand to be time-dependent. Ordering cost, unit cost, and holding cost were all regarded as constants, whereas deterioration was handled as a triangular number. To validate the model, sensitivity analysis was used.

Pavan Kumar and D. Dutta (2012) in [8] used trapezoidal fuzzy numbers and a signed defuzzification approach with no flaws. They discovered that trapezoidal fuzzy numbers produce significantly better outcomes than triangular fuzzy

numbers. Pavan Kumar and D. Dutta (2013) in [9] made changes to their model which they built in 2012 in [8] to account for shortages and make it more realistic. L. S. Rajput and Sushil (2015) in [10] devised a demand model that was time-dependent. The profit function was defuzzified using both the centroid approach and the signed distance method, and the demand rate, backlog rate, and degradation rate were all handled as triangular fuzzy integers.
The study provides a deterministic inventory model in a fuzzy environment with demand as a degree n polynomial, linearly dependent degradation rate, and constant holding cost in this article. This model employs a triangular fuzzy number to describe the costs of shortages, degradation, and holding. The numbers are defuzzified using the graded mean integration method.

II. FORMULATION AND SOLUTION OF MATHEMATICAL MODEL

Assumptions and Notations:

Notations:

The assumptions used in the model are listed as follows: -

- The demand rate is selling price dependent as a polynomial function of degree n.
- $D(s) = \alpha s^n + \beta s^{n-2} + \gamma$, where s is the selling price per unit time and $\alpha \geq 0, \beta \geq 0, \gamma \leq 0$.
- The lead time is zero.
- The deterioration rate, $\theta(t) = \lambda t, 0 < \lambda < 1$.
- Holding cost per unit time constant, $H(t) = h,$ where $, h \geq 0$.
- The deterioration rate, $\theta(t) = \lambda t, 0 < \lambda < 1$
- The Order quantity per cycle is L.
- Shortages are allowed and are fully backlogged.
- The cost of an item is C.
- The shortage cost per unit time is C_s .
- The fuzzy shortages parameter is $C_S = (C_{S1}, C_{S2}, C_{S2})$.
- The deterioration cost per unit time is C_D.
- The fuzzy deterioration parameter is $C_D = (C_{D1}, C_{D2}, C_{D2})$.
- The holding cost per unit time is H(t)=h.
- The fuzzy holding cost parameter is $h = (h_1, h_2, h_3)$.
- This inventory Model deals with single item.
- υ is the time at which inventory level reaches zero, $\upsilon \geq 0$.

In interval $(0, \upsilon)$ inventory is positive and, in the interval, (υ, T) the inventory level is negative.

Notations:

- D - Demand Rate.
- A - The ordering cost.
- θ - Deterioration Rate.
- H(t) - Holding Cost.
- C -Purchase Cost per unit
- L - Order Quantity
- υ - The time when inventory level reached zero.
- T - The length of a cycle time.

Mathematical Formulation

Crisp Model:
Period $(0, \upsilon)$ includes inventory with positive level and decreasing and at time $t = \upsilon$, the inventory level approaches to zero and shortage starts and inventory became negative in the interval (υ, T).

Differential equation for inventory level at time t is as follows: -

$$\frac{dI(t)}{dt} + \lambda t I(t) = -(\alpha s^n + \beta s^{n-2} + \gamma); 0 \leq t \leq \upsilon \quad (1)$$

$$\frac{dI(t)}{dt} = -(\alpha s^n + \beta s^{n-2} + \gamma); \quad \upsilon \leq t \leq T \quad (2)$$

having conditions $I(t) = 0,$ at $t = \upsilon$

On solving equation (1) and (2) and neglecting higher powers of t, we get

$$I(t) = \begin{bmatrix} (\alpha s^n + \beta s^{n-2} + \gamma)(\upsilon - t) \\ +\lambda\left(\frac{\upsilon^3}{6} + \frac{t^3}{3} - \frac{\upsilon t^2}{2}\right) \\ +\lambda^2\left(\frac{\upsilon^5}{40} - \frac{t^5}{15} - \frac{t^2\upsilon^3}{12} + \frac{\upsilon t^4}{8}\right) \end{bmatrix} ; 0 \leq t \leq \upsilon$$

$$I(t) = -(\alpha s^n + \beta s^{n-2} + \gamma)(t - \upsilon); \upsilon \leq t \leq T$$

The stock loss due to deterioration is given as:

$$D' = (\alpha s^n + \beta s^{n-2} + \gamma)\int_0^\upsilon e^{\frac{\lambda t^2}{2}} dt - (\alpha s^n + \beta s^{n-2} + \gamma)\int_0^\upsilon dt$$

$$= (\alpha s^n + \beta s^{n-2} + \gamma)\left(\frac{\lambda \upsilon^3}{6} + \frac{\lambda^2 \upsilon^5}{40}\right)$$

$$L = D' + \int_0^T (\alpha s^n + \beta s^{n-2} + \gamma) dt$$

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$$L=\begin{bmatrix}(\alpha s^n+\beta s^{n-2}+\gamma)(\frac{\lambda\upsilon^3}{6}+\frac{\lambda^2\upsilon^5}{40})\\ +(\alpha s^n+\beta s^{n-2}+\gamma)T\end{bmatrix} \quad (3)$$

Holding Cost is given as:

$$H(t)=\int_0^{\upsilon} he^{\frac{-\lambda t^2}{2}}[\int_t^{\upsilon}(\alpha s^n+\beta s^{n-2}+\gamma)e^{\frac{\lambda u^2}{2}}du]dt$$

$$=\begin{bmatrix}(\alpha s^n+\beta s^{n-2}+\gamma)\int_0^{\upsilon} h\left(1-\frac{\lambda t^2}{2}+\frac{\lambda^2 t^4}{8}\right)\\ \times\left[\int_t^{\upsilon}\left(1+\frac{\lambda u^2}{2}+\frac{\lambda^2 u^4}{8}\right)du\right]dt\end{bmatrix} \quad (4)$$

$$H(t)=h(\alpha s^n+\beta s^{n-2}+\gamma)\left[\frac{\upsilon^2}{2}+\frac{\lambda\upsilon^4}{12}+\frac{\lambda^2\upsilon^6}{19}-\frac{\lambda^3\upsilon^8}{960}\right]$$

The shortages during the cycle are given as:

$$S=-\int_{\upsilon}^{T}(-(\alpha s^n+\beta s^{n-2}+\gamma)(t-\upsilon))dt$$

$$S=\frac{1}{2}(\alpha s^n+\beta s^{n-2}+\gamma)(T-\upsilon)^2 \quad (5)$$

Total cost per unit time is as follows:

$$P=\frac{1}{T}[A+C_D L+H+C_s S]$$

$$P(T,s)=\frac{1}{T}\begin{bmatrix}A+C_D(\alpha s^n+\beta s^{n-2}+\gamma)\\ \times\left(T+\frac{\gamma\upsilon^3}{6}+\frac{\gamma^2\upsilon^5}{40}\right)\\ +h(\alpha s^n+\beta s^{n-2}+\gamma)\\ \times\left(\frac{\upsilon^2}{2}+\frac{\lambda\upsilon^4}{12}+\frac{\lambda^2\upsilon^6}{19}-\frac{\lambda^3\upsilon^8}{960}\right)\\ +\frac{C_s}{2}(\alpha s^n+\beta s^{n-2}+\gamma)(T-\upsilon)^2\end{bmatrix} \quad (6)$$

Fuzzy Model:

The environment is in a state of flux. So, consider some parameters namely C_D, h and C_S which may change within some boundaries. Let $C_D=(C_{D1},C_{D2},C_{D2})$, $h=(h_1,h_2,h_3)$ and $C_S=(C_{S1},C_{S2},C_{S3})$ be the triangular fuzzy numbers.

The total inventory of the system per unit time in fuzzy sense is given by:

$$P(T,s)=\frac{1}{T}\begin{bmatrix}A+C_D(\alpha s^n+\beta s^{n-2}+\gamma)\left(T+\frac{\gamma\upsilon^3}{6}+\frac{\gamma^2\upsilon^5}{40}\right)\\ +h(\alpha s^n+\beta s^{n-2}+\gamma)\left(\frac{\upsilon^2}{2}+\frac{\lambda\upsilon^4}{12}+\frac{\lambda^2\upsilon^6}{19}-\frac{\lambda^3\upsilon^8}{960}\right)\\ +\frac{C_s}{2}(\alpha s^n+\beta s^{n-2}+\gamma)(T-\upsilon)^2\end{bmatrix}$$

(7)

Now, we defuzzify the total inventory cost $P(T,s)$ by using graded mean integration method, it follows:

$$d_F P(T,s)=\frac{1}{4}\left(P_1(T,s)+4P_2(T,s)+P_3(T,s)\right)$$

(8)

where,

$$P_1(T,s)=\frac{1}{T}\begin{bmatrix}A+C_{D1}(\alpha s^n+\beta s^{n-2}+\gamma)\left(T+\frac{\gamma\upsilon^3}{6}+\frac{\gamma^2\upsilon^5}{40}\right)\\ +h_1(\alpha s^n+\beta s^{n-2}+\gamma)\left(\frac{\upsilon^2}{2}+\frac{\lambda\upsilon^4}{12}+\frac{\lambda^2\upsilon^6}{19}-\frac{\lambda^3\upsilon^8}{960}\right)\\ +\frac{C_{S1}}{2}(\alpha s^n+\beta s^{n-2}+\gamma)(T-\upsilon)^2\end{bmatrix},$$

$$P_2(T,s)=\frac{1}{T}\begin{bmatrix}A+C_{D2}(\alpha s^n+\beta s^{n-2}+\gamma)\left(T+\frac{\gamma\upsilon^3}{6}+\frac{\gamma^2\upsilon^5}{40}\right)\\ +h_2(\alpha s^n+\beta s^{n-2}+\gamma)\left(\frac{\upsilon^2}{2}+\frac{\lambda\upsilon^4}{12}+\frac{\lambda^2\upsilon^6}{19}-\frac{\lambda^3\upsilon^8}{960}\right)\\ +\frac{C_{S2}}{2}(\alpha s^n+\beta s^{n-2}+\gamma)(T-\upsilon)^2\end{bmatrix}$$

and

$$P_3(T,s)=\frac{1}{T}\begin{bmatrix}A+C_{D3}(\alpha s^n+\beta s^{n-2}+\gamma)\left(T+\frac{\gamma\upsilon^3}{6}+\frac{\gamma^2\upsilon^5}{40}\right)\\ +h_3(\alpha s^n+\beta s^{n-2}+\gamma)\left(\frac{\upsilon^2}{2}+\frac{\lambda\upsilon^4}{12}+\frac{\lambda^2\upsilon^6}{19}-\frac{\lambda^3\upsilon^8}{960}\right)\\ +\frac{C_{S3}}{2}(\alpha s^n+\beta s^{n-2}+\gamma)(T-\upsilon)^2\end{bmatrix}$$

Therefore, the value of $d_F P(T,s)$ is given by:

$$d_F P(T,s) = \frac{1}{4}\left(P_1(T,s) + 4P_2(T,s) + P_3(T,s)\right)$$

$$d_F P(T,s) = \frac{1}{4}\left(\begin{array}{l} \frac{1}{T}\left[\begin{array}{l} A + C_{D1}(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times\left(T + \frac{\gamma\upsilon^3}{6} + \frac{\gamma^2\upsilon^5}{40}\right) \\ + h_1(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times\left(\frac{\upsilon^2}{2} + \frac{\lambda\upsilon^4}{12} + \frac{\lambda^2\upsilon^6}{19} - \frac{\lambda^3\upsilon^8}{960}\right) \\ + \frac{C_{S1}}{2}(\alpha s^n + \beta s^{n-2} + \gamma)(T-\upsilon)^2 \end{array} \right] \\ + 4\times\frac{1}{T}\left[\begin{array}{l} A + C_{D2}(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times\left(T + \frac{\gamma\upsilon^3}{6} + \frac{\gamma^2\upsilon^5}{40}\right) \\ + h_2(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times\left(\frac{\upsilon^2}{2} + \frac{\lambda\upsilon^4}{12} + \frac{\lambda^2\upsilon^6}{19} - \frac{\lambda^3\upsilon^8}{960}\right) \\ + \frac{C_{S2}}{2}(\alpha s^n + \beta s^{n-2} + \gamma)(T-\upsilon)^2 \end{array} \right] \\ + \frac{1}{T}\left[\begin{array}{l} A + C_{D3}(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times\left(T + \frac{\gamma\upsilon^3}{6} + \frac{\gamma^2\upsilon^5}{40}\right) \\ + h_3(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times\left(\frac{\upsilon^2}{2} + \frac{\lambda\upsilon^4}{12} + \frac{\lambda^2\upsilon^6}{19} - \frac{\lambda^3\upsilon^8}{960}\right) \\ + \frac{C_{S3}}{2}(\alpha s^n + \beta s^{n-2} + \gamma) \\ \times(T-\upsilon)^2 \end{array} \right] \end{array} \right) \quad (9)$$

For optimal value of s and T, we have

- $$\frac{\partial}{\partial s}\left(d_F P(T,s)\right) = 0 \quad (10)$$

and

- $$\frac{\partial}{\partial T}\left(d_F P(T,s)\right) = 0 \quad (11)$$

The total minimum cost per unit time P(T,s) satisfy by sufficient condition:

$$\frac{\partial^2}{\partial T^2}\left(d_F P(T,s)\right) > 0 \quad (12)$$

and

$$\frac{\partial^2}{\partial s^2}\left(d_F P(T,s)\right) > 0 \quad (13)$$

and

$$\frac{\partial^2\left(d_F P(T,s)\right)}{\partial T^2} \times \frac{\partial^2\left(d_F P(T,s)\right)}{\partial s^2} - \frac{\partial^2\left(d_F P(T,s)\right)}{\partial s \partial T} > 0 \quad (14)$$

We get the value of s and T by solving equation (10) and (11) and putting these values in equation (9), we obtain the minimum value for $d_F P(T,s)$ which satisfy the necessary conditions (12), (13) and (14).

III. NUMERICAL ILLUSTRATION:

Crisp Model:

PARAMETERS	EXAMPLE	PARAMETERS	EXAMPLE
a	99999.001	λ	1.5
b	999.01	υ	2
c	-9999	n	5
A(ordering cost)	9878.005	Optimal Value of T	23.80541497
C_S	109	Optimal Value of s	0.6277914890
C_D	936.001	Total optimal cost per unit time	414.9434073
h	7	-	-

FUZZY MODEL:

PARAMETERS	EXAMPLE	PARAMETERS	EXAMPLE	
C_{S1}	109	h_1	7	
C_{S2}	108	h_2	6	
C_{S3}	110	h_3	5	
b	999	Λ	1.2	
c	-999	n	5	
A(ordering cost)	9878	Optimal Value of T	19.96359913	
a	9999999	Optimal Value of s	0.1583173361	
C_{D1}	935	Total optimal cost per unit time	494.7995614	
C_{D2}	936			
C_{D3}	937			
υ	1.2	-	-	

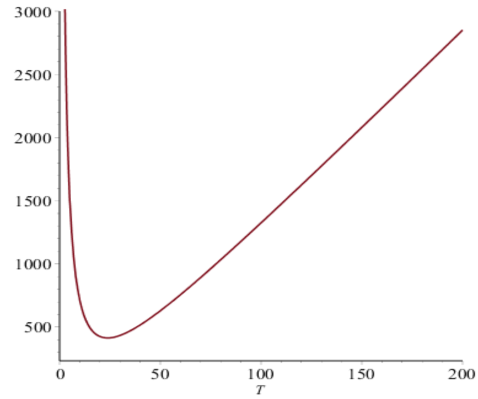

Figure.1- Total cost function verses T for crisp model

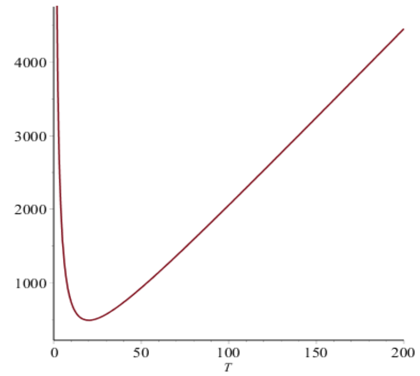

Figure.2- Total cost function verses T for fuzzy model

IV. CONCLUDING REMARKS

A deterministic inventory model based on selling price raise to power n, linear decline rate, and constant holding cost is presented in this work. For the values of T, the total optimal cost has been determined, which meets the essential requirement. As a triangular fuzzy number, this model included shortages, degradation, and holding costs. The graded mean integration approach is used for defuzzification. The model has been validated using numerical and graphical representations. This model fits nicely with the concept where the demand rate is based on increasing powers of the selling price as product prices grow day by day. As a consequence, future communities and businesses will benefit greatly from the concept.

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