An Evolution of New Methodology for the Detection of Isomorphism and Inversions of Planar Kinematic Chain

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***Abstrat*-Topological characteristics had been keen area of research. There are numerous methodologies for the detection of isomorphism between the kinematic chains but each method have its own inadequacy. This paper presents theoretical approach, an easy method to compute and is reliable for the detection of isomorphism and inversion of kinematic chains based on topological characteristic. In this approach, kinematic chains are represented by Path Matrix(PM) and element of matrix are used for detection of Isomorphism and Inversion. The above method have been applied on the kinematic chains of 1-D.O.F. , 6-links and 8-links and are in agreement to the available journal.**

Keywords- Isomorphism, Inversion, Path Matrix.

I. INTRODUCTION

Topological characteristics was found to be quite difficult to setup between kinematic chains. One of the major characteristics is ISOMORPHISM. A deep study about isomorphism is needed otherwise it would result in copied solution and needless effort. For distinct planar kinematic chain structural analysis and synthesis is very important in design of mechanism.

Kinematicians around the world approached various methods to specify the non-isomorphic kinematic chains, but the outcomes had inconsistency in results. In early 70's researchers proposed graph theory but it rose isomorphism problem and the problems was that time termed as "The graph Isomorphism disease". The papers given in Reference [1] and [2] Crossley and J.J. Uicker introduced method based on characteristic polynomials have complex graph and large numerical. In 80's the method presented by [3] and [4] Yan & Hall and Mruthyunjaya was simple but the reliability of this method is still a question . The basis for most of the methods was adjacency matrix (Uicher & Raciu 1975) or a distance matrix (A.C.Rao 1988) but the methods are complex , difficult to apply and its reliability has been failed in detecting uniqueness,

and is also a time taking method for determination of isomorphism of a kinematic chain.

Mruthyunjaya and Balasubramanian [7] proposed a vertex-vertex degree matrix whose ijth term is sum of degrees of link i and j if i and j are adjacent and is equal to 1 otherwise. In Ambekar and Agrawal [9] two canonical forms of any adjacency are possible – one yielding a maximum code - Max Code and other yields minimum code – Min Code. The Min Code is used as canonical number to identify the kinematic chains. Huafeng Ding et al [12] proposed new concept, such as the perimeter loop, the maximum perimeter degree sequence, the perimeter topological graph and the method for obtaining the perimeter loop. It is based on perimeter topological graph and some rule for relabeling its vertices canonically, a one-to-one descriptive method, the canonical adjacency matrix set of kinematic chain is proposed. Ashok kumar Sharma and Arvind kumar Shukla [25] develop a method for detection of isomorphism of the planar kinematic chain based on structural properties. In this approach, kinematic chains are represented by matrices and the elements of matrices are used for detection of isomorphism.

II. TERMINOLOGIES

The following terminologies have been defined to understand the terms before applying the method. The various definitionswith their abbreviations are-

- *Degree of Link*- It is defined as the numerical value assigned to a link based on its connectivity. Therefore, binary link has degree equals to two and ternary link has degree equals to three.
- *Nodal Value (NV)* - It is an arbitrary value which is equals to degree of link.
- *Joint Value (JV)* - It is the inverse of summation of nodal value.
- *Link Value (LV)* - It is defined as sum of all elements of a specific row or a column of a matrix.
- *Chain Value (CV)* – It is the addition of all the link values of a kinematic chain.
- *Shortest Path Value (SPV)* – It is the least sum of all joint values between two links under consideration.

III. Path Matrix

For an n-link kinematic chain it is defined as an nxn square matrix,

$$PM=\{P_{ij}\}_{nxn}$$

The element of matrix are defined as $\mathbf{P_{ij}}$ which is the least sum of all joint value between two link i and j, and is equals to zero if i is equals to j. Of course, all diagonal element $\mathbf{P_{ij} = 0}$.

$$PM = \begin{pmatrix} 0 & P_{12} & P_{13} & \ldots & P_{1n} \\ P_{21} & 0 & P_{23} & \ldots & P_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ P_{n1} & P_{n2} & P_{n3} & \ldots & 0 \end{pmatrix}$$

IV. METHODOLOGY

In this method the value of a specific element of the matrix for a kinematic chain is computed by the shortest path value. As mentioned above the matrix has same number of rows and columns i.e. nxn, so the matrix used is square and symmetric as well. The summation of the elements of a row gives us the Link Value and the addition of link value gives Chain value.

For example taking in view a kinematic chain with six bar , seven joints, and single degree of freedom, considering the figure (1) the links viz A,B,C,D,E and F, where A,B,D and E are binary links whereas C and F are ternary links. Taking two binary links A and B, since their nodal value is 2 for each, by this very method their joint value will be [1/(2+2)] =1/4 or (15/60).Now considering a binary and a ternary link viz A and F, their joint value [1/(2+3)] =1/5 or (12/60). And lastly ternary and ternary link viz C and F, their joint value [1/(3+3)] = 1/6 or (10/60).

Similarly, all the joint values are mentioned in the figure given below-

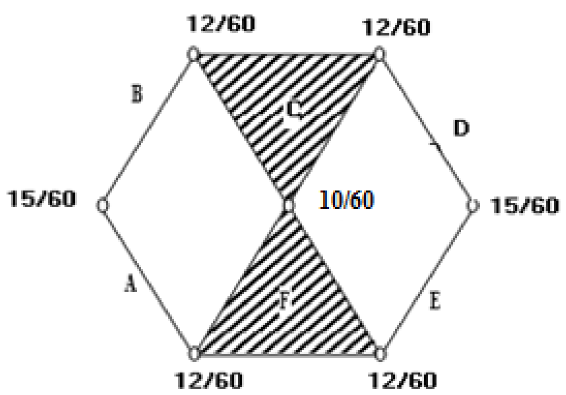

FIGURE 1 (WATT CHAIN)

Shortest Path Value (SPV) is the least sum of all joint values between two links under consideration. For example considering the link A and C, the different paths which can be opted.

Path NO.	Various path	Value of Path
1	**A**-F-**C**	1/60(10+12)=**22/60**
2	**A**-B-**C**	1/60(15+12)=**27/60**
3	**A**-F-E-D-**C**	1/60(12+12+15+12)=**51/60**

The shortest path value between the links A and C is 22/60 i.e. Path no. 1 from the above table. And the value is represented by element P_{13} of the matrix. Similarly, shortest path value between the links A and B i.e. 15/60, represented by P_{12} of the matrix. Since it is a square and symmetric matrix, therefore, $P_{13} = P_{31}$ and $P_{12} = P_{21}$, and so on. And hence, with same method the matrix will be filled completely given by PM_1.

$$PM_1 = 1/60 \begin{pmatrix} 0 & 15 & 22 & 39 & 24 & 12 \\ 15 & 0 & 12 & 24 & 39 & 22 \\ 22 & 12 & 0 & 12 & 22 & 10 \\ 39 & 24 & 12 & 0 & 15 & 22 \\ 24 & 39 & 22 & 15 & 0 & 12 \\ 12 & 22 & 10 & 22 & 12 & 0 \end{pmatrix}$$

In the six-bar chain used as in figure (1) – the value of link A is :- ($P_{11}+P_{12}+P_{13}+P_{14}+P_{15}+P_{16}$ =112/60) and so on the values of links B,C,D,E and F are 112/60,78/60,112/60,112/60 and 78/60 respectively. Now, the value for the Watt chain is the 'summation of the values of all the link values' i.e." 604/60".

1/60[112,112,78,112,112,78]
Or, 1/60 [2(78),4(112)]

In the above two lines, the first one is the descriptive one whereas the one in the very next line is precise notation of Numerical String of the Watt Chain.

For six bar chain, Figure 1
Chain Value = 604/60
Numerical String for chain = 1/60[2(78),4(112)]

The second example, considering the Stephenson Chain and applying same methodology as used in the Watt Chain.

Various Links	Link Value	Numerical String Of distinct link
(A,B,D,E)	112/60	1/60[(12),(15),(22),(24),(39)]
(C,F)	78/60	1/60[(10),2(12),2(22)]

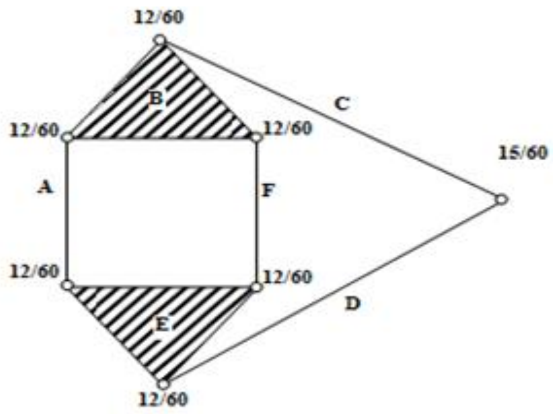

FIGURE 2 (STEPHENSON CHAIN)

$$PM_2 = 1/60 \begin{pmatrix} 0 & 12 & 24 & 24 & 12 & 24 \\ 12 & 0 & 12 & 27 & 24 & 12 \\ 24 & 12 & 0 & 15 & 27 & 24 \\ 24 & 27 & 15 & 0 & 12 & 24 \\ 12 & 24 & 27 & 12 & 0 & 12 \\ 24 & 12 & 24 & 24 & 12 & 0 \end{pmatrix}$$

Chain Value = 570/60
Numerical String of chain=1/60[2(87),2(96),2(102)]

V. INVERSION

The inversion of a kinematic chain is obtained by grounding different links and observing the motion of various links of the kinematic chain and so the inversion of a chain is established. According to the work done before, the number of inversions for six-bar, eight-bar and ten-bar link with degree of freedom 'one' are 5,71 and 1834 inversions respectively.

Mathematically, inversions are calculated in two steps, first is to check for the links having same Link Value followed by the checking of similarity of one to one correspondence among their Numerical String among the links having equal link value in the second step. If the element values are identical then it will be considered as inversions of a kinematic chain.

Now the result obtained in six link chain :

Links	Link Values	Numerical Strings
A	112/60	1/60[(12),(15),(22),(24),(39)]
B	112/60	1/60[(12),(15),(22),(24),(39)]
C	78/60	1/60[(10),2(12),2(22)]
D	112/60	1/60[(12),(15),(22),(24),(39)]
E	112/60	1/60[(12),(15),(22),(24),(39)]
F	78/60	1/60[(10),2(12),2(22)]

TABLE-1

Therefore, a six link WATT CHAIN has 2 distinct inversion

Links	Link Values	Numerical Strings
A	96/60	1/60[2(12),3(24)]
B	87/60	1/60[3(12),24,27]
C	102/60	1/60[(12),(15),2(24),(27)]
D	102/60	1/60[(12),(15),2(24),(27)]
E	87/60	1/60[3(12),24,27]
F	96/60	1/60[2(12),3(24)]

Various Links	Link Value	Numerical String Of distinct link
(A,F)	96/60	1/60[2(12),3(24)]
(B,E)	87/60	1/60[3(12),(24),(27)]
(C,D)	102/60	1/60[(12),(15),2(24),(27)]

TABLE-2

Therefore, a six link STEPHENSON CHAIN has3 distinct inversion

Similarly, the Link values and the Numerical string of distinct links of 16 eight-link chains having 'one' degree of freedom with 71 inversions are tabulated in APPENDIX-A.

VI. ISOMORPHISM

It is defined as similarity of characteristics among chains on the basis of their connectivity, number of link ,type of link and link assortment. The checking criteria for detection of isomorphism is equivalent of Link value and one to one correspondence of Numerical string.

As per the above examples considered, the chain values are 604/60 and 570/60 for Watt chain and Stephenson chain respectively. If the resultant Link values and their Numerical string were equal then the two chain have been isomorphic. But the values and other characteristics are contradicting as per the basic categorisation and hence, the two considered chains aren't isomorphic.

The **counter examples** Ten link one DOF appeared in reference [23] the chain shown in Figure [3,4] were isomorphic. When these chains are checked by proposed method for detection of isomorphism, the result obtained are same that was reported in reference [23].

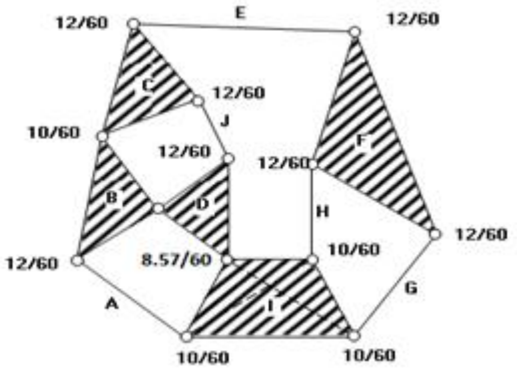

FIGURE 3

For kinematic chain(Figure.3)
Chain Value = 1966.82/60
Numerical String of chain =1/60[(162.28),(168.55)(185.71)(199.14),2(199.71),(200.57)(214.57),(218),(218.28)]

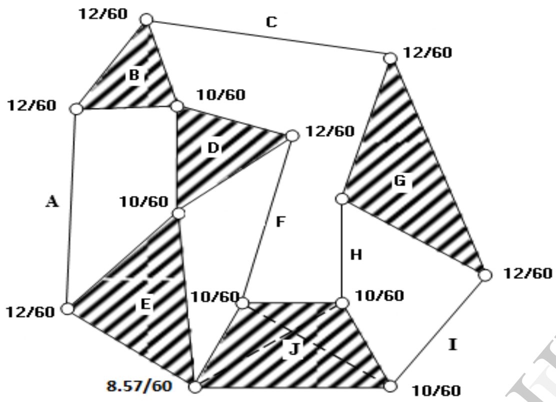

FIGURE 4

For kinematic chain(Figure.4)
Chain Value = 1966.82/60
Numerical String of chain =1/60[(162.28),(168.55),(185.71)(199.14),2(199.71) 200.57),(214.57),(218),(218.28)]

Proposed method shows that kinematic chain 3 and 4 are isomorphic as the Link value and Numerical string are same for both kinematic chain.

The **counter examples** Ten link one DOF, appeared in [7] the chain in Fig 5 and 6 are said to be non- isomorphic as these two chains have distinct Link value and Numerical string.

For kinematic chain(Figure.6)
Chain Value = 1929/60
Numerical String of chain =1/60[(139),(160.5),(168),(176.5),2(192.5),(213.5)(214.5),(234),(238)]

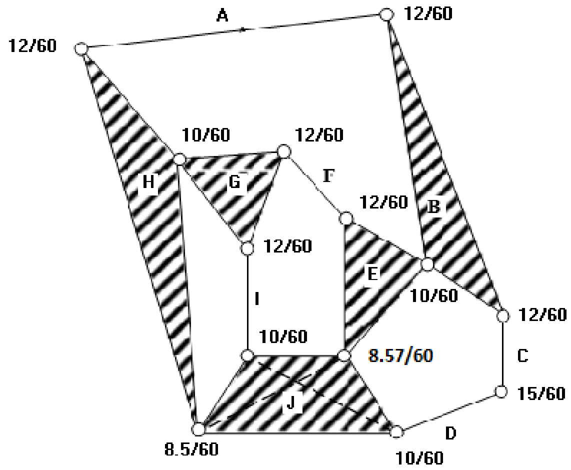

FIGURE 5

For kinematic chain(Figure.5)
Chain Value = 1933.52/60
Numerical String of chain =1/60[(143.85),(152.85)(164.42),(185.14),(202.28)(203.85),(207.71),(211.57)(217.71),(244.14)]

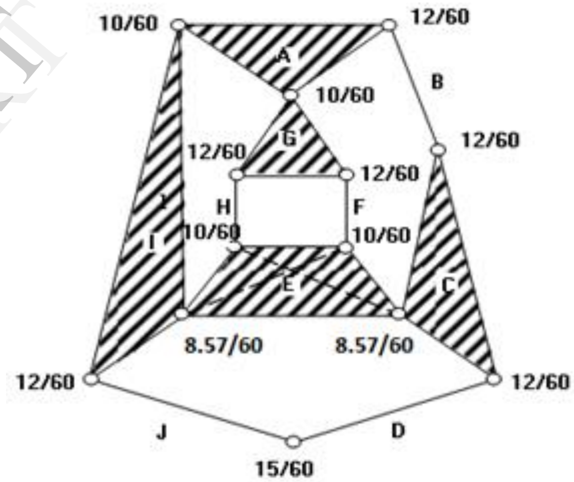

FIGURE 6

Proposed method report that kinematic chain 5 and 6 are non-isomorphic because both the Link value and Numerical string are different.

The **counter examples** in Reference [11] the chains shown in 7,8 are said to be non-isomorphic

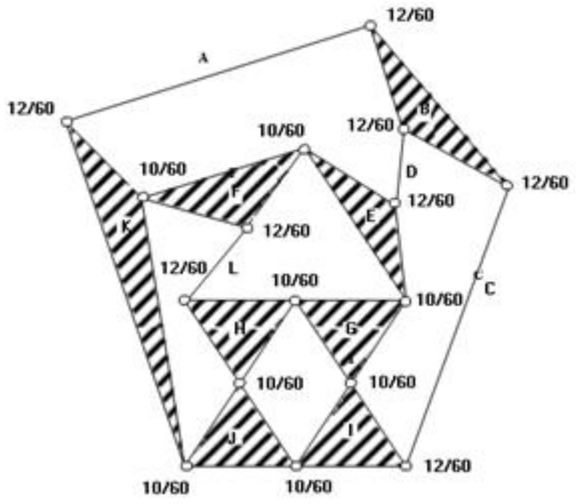

FIGURE 7

For kinematic chain(Figur e.7)
Chain value = 2556/60
Numerical String of chain
=1/60[2(188),3(196),(200),(208),(220),2(236)
(244),(248)]

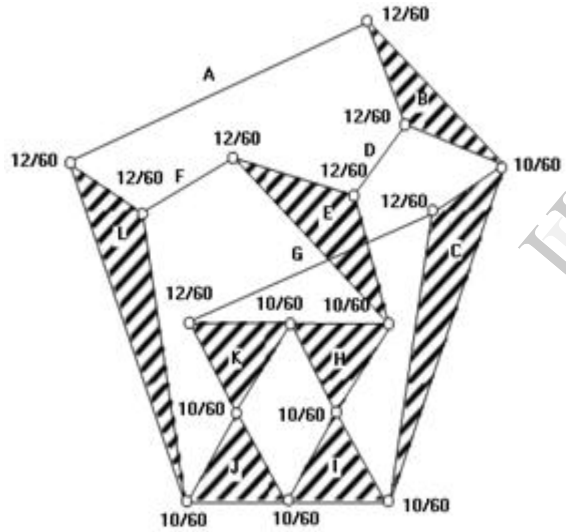

FIGURE 8

For kinematic chain(Figure.8)
Chain Value = 3174/60
Numerical String of chain =
1/60[(228) (232),(238),(248),2(250),(262),(272)
(290),(292),(302),(310)]

Proposed method report that kinematic chain 7 and 8 are non-isomorphic as Link value and Numerical string are different.

RESULT

The method introduced is reliable and precise to detect isomorphism among kinematic chain and inversion within kinematic chain. All single degree of freedom 6-link 2 kinematic chain, 8-link 16 kinematic chain, 10-link 4 kinematic chain, 12-link 2 kinematic chain have been analysed under this method and results are in agreement with available journal.

CONCLUSION

In this literature, an unique method has been evolved to identify mechanisms of planar kinematic chain. Synthesis of simple joined kinematic chain with specific number of link and degree of freedom can be carried out by reported method to reveal characteristics i.e isomorphism and inversion of kinematic chains. This method is easy to compute and reliable and involve less mathematical calculations.

Further, author strongly believe that this method is unique and applicable to planar chain of any size and complexity and does not require conversion of planar chain to their graphs. An important aspect of this method is, it is neither effected by link length nor by relabeling.

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APPENDIX-A

RESULTS OBTAINED BY THIS METHOD WHEN APPLIED TO EIGHT LINK, SINGLE DEGREE OF FREEDOM KINEMATIC CHAIN

CHAIN NO.	DISTINT LINK	LINK VALUE	NUMERICAL STRING	DISTINCT INVERSION
1	(A,D,E,H) (B,C,F,G)	120/60 181/60	1/60[2(10),(12),(20),2(22),32] 1/60[(12),(15),2(22),(32),(34),(44)]	2
2	(A,B,G,H) (C,D,E,F)	166/60 120/60	1/60[(12),(15),2(22),(27),2(34)] 1/60[2(10),(12),(20),2(22),(27)]	2
3	(A) (B) (C) (D) (E) (F) (G) (H)	122/60 166/60 170/60 127/60 150/60 136/60 118/60 160/60	1/60[2(10),(12),(20),2(22),(27)] 1/60[(12),(15),2(22),(27),2(34)] 1/60[(12),(15),(22),2(27),(34),(36)] 1/60[(10),2(12),(20),(22),(24),(27)] 1/60[2(12),2(22),2(24),(34)] 1/60[(10),2(12),(20),(22),(24),(36)] 1/60[2(10),(12),(20),3(22)] 1/60[2(12),2(22),(24),2(34)]	8
4	(A,H) (B,E) (C,D) (F,G)	160/60 133/60 166/60 128/60	1/60[2(12),2(22),(24),2(34)] 1/60[10,2(12),(20),(22),(27),(30)] 1/60[(12),(15),(22),(24),(27),(32),(34)] 1/60[2(10),(12),(20),(22),(32)]	4
5	(A,H) (B,G) (C,F) (D,E)	160/60 132/60 118/60 171/60	1/60[2(12),2(22),(24),2(34)] 1/60[(10),2(12),(20),(22),(24),(32)] 1/60[2(10),(12),(20),3(22)] 1/60[(12),(15),2(22),(32),2(34)]	4
6	(A,F) (B,E) (C,D) (G) (H)	165/60 133/60 171/60 138/60 118/60	1/60[2(12),(22),2(24),(32),(39)] 1/60[(10),2(12),2(20),(27),(32)] 1/60[(12),(15),(22),(24),(27),(32),(39)] 1/60[(10),2(12),2(20),2(32)] 1/60[3(10),4(22)]	5
7	(A) (B,F) (C,E) (D) (G) (H)	164/60 132/60 150/60 124/60 122/60 150/60	1/60[2(12),(22),2(24),(34),(36)] 1/60[(10),2(12),(20),(22),(24),(32)] 1/60[2(12),(22),3(24),(32)] 1/60[3(12),3(24),(32)] 1/60[2(10),(12),3(22),(24)] 1/60[2(12),2(22),2(24),(34)]	6
8	(A,D,G,H) (B,C,E,F)	160/60 136/60	1/60[2(12),2(22),(24),2(34)] 1/60[(10),2(12),2(22),(24),(32)]	2
9	(A,B,D,G) (C,E,F,H)	126/60 150/60	1/60[(10),2(12),2(22),2(24)] 1/60[2(12),2(22),2(24),(34)]	2
10	(A) (B,F,G,H) (C,E) (D)	118/60 138/60 146/60 154 /60	1/60[4(10),2(22),(34)] 1/60[(10),(12),3(20),(24),(32)] 1/60[3(12),(22),(24),2(32)] 1/60[2(12),4(24),(34)]	4
11	(A) (B) (C) (D) (E,H) (F) (G)	127/60 166/60 144/60 109/60 138/60 132/60 128/60	1/60[(10),2(12),3(22),(27)] 1/60[(12),(15),(22),(24),(25),(34)] 1/60[(10),(15),3(20),(27),(32)] 1/60[4(10),2(22),(25)] 1/60[(10),(12),3(20),(22),(34)] 1/60[(10),2(12),3(22),(32)] 1/60[(10),(12),3(20),(22),(24)]	7
12	(A) (B,H) (C) (D) (E) (F) (G)	150/60 135.14/60 106.14/60 146.14/60 168.71/60 112.28/60 147.14/60	1/60[3(12),(22),(24),(32),(36)] 1/60[(10),(12),2(20),(18.57),(24),(30.57)] 1/60[3(10),(8.57),(20.57),(22),(25)] 1/60[(10),(15),(18.57),2(20),(30.57),(36)] 1/60[(12),(15),(20.57),(24),2(30.57),(36)] 1/60[2(12),(8.57),3(18.57),(24)] 1/60[2(12),(20.57),3(24),(30.57)]	7
13	(A) (B) (C) (D) (E) (F) (G) (H)	108.28/60 164.71/60 147.14/60 92.71/60 141.14/60 167/60 130.14/60 135.14/60	1/60[(8.57),(10),(12),3(18.57),(22)] 1/60[(12),(15),(20.57),(22),2(30.57),(34)] 1/60[(10),(15),(18.57),(28.57),(35)] 1/60[(8.57),2(10),(18.57),(20),(20.57),(25)] 1/60[(10),(15),(18.57),(22),(27),(28.57)] 1/60[(12),(15),(22),(24),(25),(34),(35)] 1/60[(10),2(12),(18.57),(22),(27),(28.57)] 1/60[(10),(12),(18.57),2(20),(24),(30.57)]	8
14	(A,E) (B,D) (C) (F,H) (G)	172.42/60 143.28/60 98.85/60 115.99/60 153.71/60	1/60[(12),(15),(24),(20.57),(29.14),(30.57),(41.14)] 1/60[(10),(15),2(18.57),(20),2(30.57)] 1/60[2(8.57),2(10),3(20.57)] 1/60[(8.57),2(12),(17.14),2(18.57),(29.14)] 1/60[(2(12),(20.57),2(24),2(30.57)]	5
15	(A,D) (B,C,E,F,G,H)	90/60 137.5/60	1/60[(7.5)4(10),3(17.5)] 1/60[(10),(15),(17.5),2(20),2(27.5)]	2
16	(A,B,D,E) (C,F) (G,H)	145/60 110/60 120/60	1/60[(10),(15),3(20),(25),(35)] 1/60[4(10),(20),2(25)] 1/60[2(10),5(20)]	3
			TOTAL INVERSION	71

APPENDIX-B

EIGHT LINK, SINGLE DEGREE OF FREEDOM KINEMATIC CHAIN

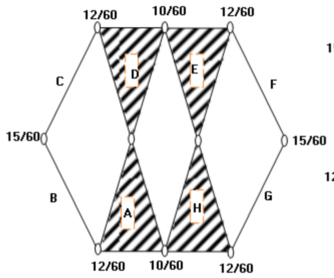

FIGURE 1

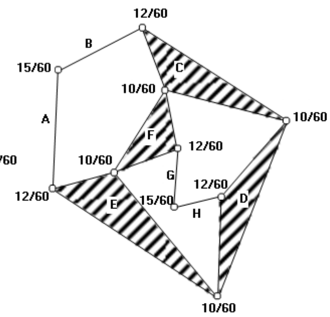

FIGURE 2

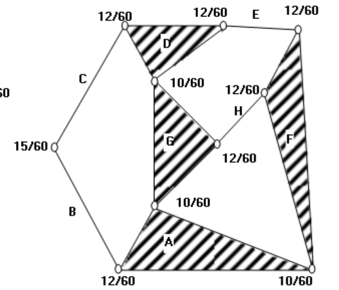

FIGURE 3

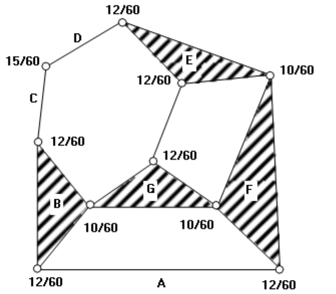

FIGURE 4

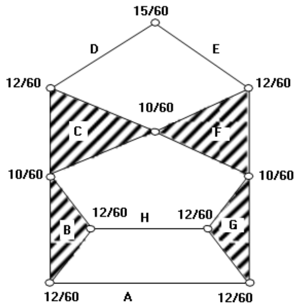

FIGURE 5

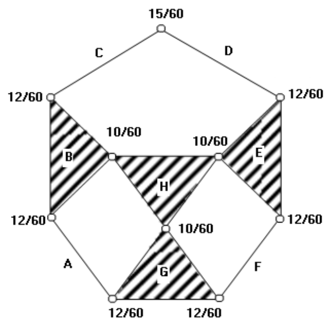

FIGURE 6

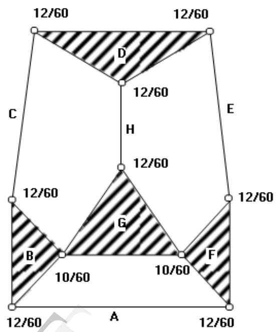

FIGURE 7

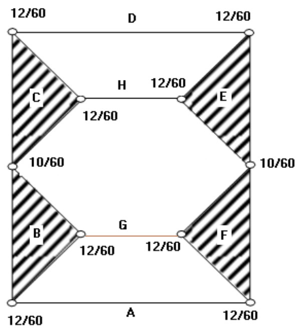

FIGURE 8

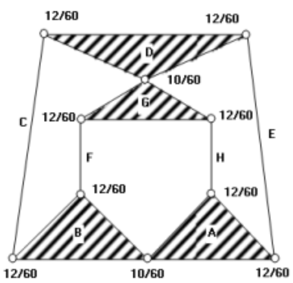

FIGURE 9

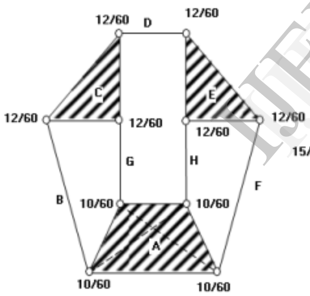

FIGURE 10

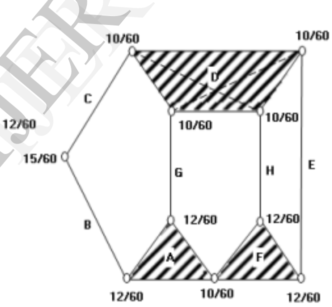

FIGURE11

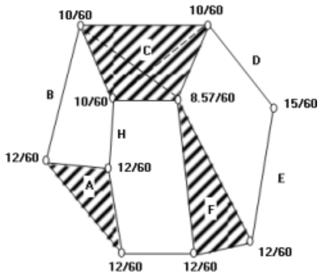

FIGURE 12

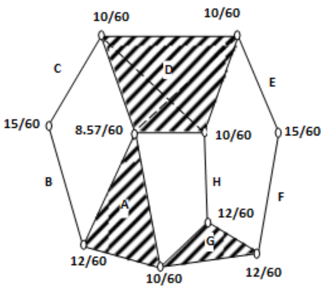

FIGURE 13

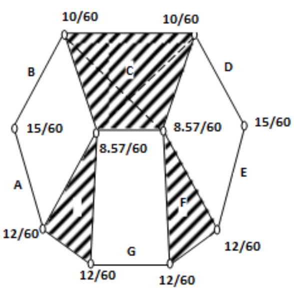

FIGURE 14

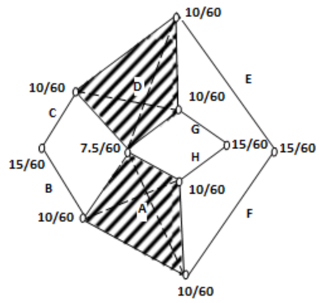

FIGURE 15

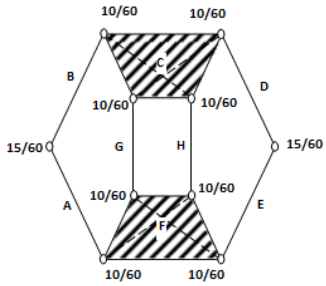

FIGURE16