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# "An EPQ Model with Stock Dependent Demand under Imprecise and Inflationary Environment using Genetic Algorithm"

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## Abstract

In this paper a production inventory model for the newly launched product is developed incorporating the effect of inflation and time value of money under imprecise environment. It is common phenomenon in the supermarket that the stock level has a motivational effect on the customers; i.e. the demand rate may go up or down if the on-hand inventory level increases or decreases. To deal with such kind of situations, demand is considered as stock dependent. Items are produced at the rate, which is constant proportional to demand rate. Production is stopped when the stock-level reached to level Q and $Q_0$ is the fixed stock-level below which inventory has no effect on demand. Model is formulated to maximize the total profit. It is assumed that all the cost parameters involved in the model are imprecise in nature and represented by triangular fuzzy numbers. Centroid Method is being used to defuzzified total fuzzy profit. A genetic algorithm with varying population size is used to solve the model. In this GA a subset of better children is included with the parent population for next generation and size of this subset is a percentage of the size of its parent set. Numerical example along with sensitivity analysis is presented to illustrate the model. Mathematica7.0 is being used to reach the optimal policies.

Keywords: Inflation, Stock-dependent demand

## 1. Introduction

It is well known that the stock level has a motivational effect on the customers in a supermarket; i.e. the demand rate may go up or down if the on-hand inventory level increases or decreases. In corporate world such a situation is known as the stock-dependent demand. It generally arises for a consumer-goods type inventory. In this area a large number of mathematical models have been reported in the existing literature. Among them, to get the idea of the trends of recent research, one may refer to the works of Teng and Chang (1995), Sarkar et. al. (1997), Datta et al.(1998), Balki and Benkherouf (2004), Teng and Chang (2005), Wu et. al. (2006), Singh et al. (2007) and Singh et al. (2010).

In classical inventory, models are developed on the basis of that all the parameters in total cost function are fixed and have the certain values. But in day-by-day changing market scenario there may be increase or decrease within a range of the values of the parameters. To deal with such type of irregularities, these parameters are considered as fuzzy in nature. When some parameters are fuzzy in nature, the resultant objective function also becomes fuzzy. To get an idea or trends in recent research one may refer to the work of Yao and Lee (1999), Yao et al. (2000), Chang et al. (2004), Maiti and Maiti (2007) and Singh and Singh (2010).

Most of the inventory models unrealistically ignore the influence of inflation. This was due to the belief that inflation would not influence the inventory policy to any significant degree. This belief is unrealistic since the resource of an enterprise is highly correlated to the return on investment. The concept of the inflation should be considered especially for long-term investment and forecasting. To get an idea trends in recent research one may refer to the work of Buzacott (1975), Misra (1979), Chandra and Bahner (1985), Lieo et al. (2000), Mehta and Shah (2003) and Singh and Singh (2010).

Use of Genetic Algorithm in complex decision making problem is well established by Michalewicz (1992). Extensive research work has been made to improve the performance of GA for single/multi-objective continuous/discrete optimization problems during last two decades.

Michalewich (1992) proposed a GA named contractive mapping genetic algorithm (CMGA) where movement from old population to new population takes place only when average fitness of new population is better than the old one and proved the asymptotic convergence of the algorithm by Banach fixed point theorem. Bessaou and Siarry (2001) proposed a GA where initially more than one population of solutions are generated. Genetic operations are done on every population a finite number of times to find a promising zone of optimum solution. last and Eyal (2005) developed a GA with varying population size, where chromosomes are classified into young, middle-age and old according to their age and lifetime. Pezzellaa. Morgantia, and Ciaschettib (2008) developed a GA for the flexible Job-shop scheduling problem, which integrates different strategies for generating the initial population, selecting the individuals for reproduction and reproducing new individuals.

In this paper an EPQ model of an item is developed considering the power form stock-dependent demand under imprecise and inflationary environment. It is assumed that the production rate is demand dependent. Two situations were discussed in this paper (I) $Q \leq Q_0$ and (II) $Q > Q_0$, where Q is the stock-level at the time production is stopped and $Q_0$ is the fixed stock-level. Model is formulated to maximize the total profit. A genetic algorithm with varying population size is used to solve the model.

## 2. Genetic Algorithm

Genetic Algorithm is exhaustive search algorithm based on the mechanics of natural selection and genesis (crossover, mutation, etc.). It was developed by Holland, his colleagues and students at the University of Michigan. Because of its generality and other advantages over conventional optimization methods, it has been successfully allied to different decision making problems.

In natural genesis, we know that chromosomes are the main carriers of hereditary factors. At the time of reproduction, crossover and mutation take place among the chromosomes of parents. In this way, hereditary factors of parents are mixed-up and carried over to their offspring. Again, Darwinian principle states that only the fittest animals can survive in nature. So, a pair of parents normally reproduces a better offspring.

The above-mentioned phenomenon is followed to create a genetic algorithm for an optimization problem. Here, potential solutions of the problem are analogous with the chromosomes, and the chromosome of better offspring with the better solution of the problem. Crossover and mutation among a set of potential solutions to get a new set of solutions are made, and it continues until terminating conditions are encountered. Michalewich proposed a genetic algorithm named Contractive Mapping Genetic Algorithm (CMGA) and proved the asymptotic convergence of the algorithm by Banach fixed point theorem. In CMGA, a movement from the old population to a new one takes place only if an average fitness of the new population is better than the fitness of the old one. In the algorithm, $p_c$, $p_m$ are probability of crossover and probability of mutation respectively, $T$ is the generation counter and $P(T)$ is the population of potential solutions for the generation $T$ . $M$ is an iteration counter in each generation to improve $P(T)$ and $M_0$ is the upper limit of $M$ . Initialize ($P(1)$) function generate the initial population $P(1)$ (initial guess of solution set) at the time of initialization. Objective function value due to each solution is taken as fitness of the solution. Evaluate ($P(T)$) function evaluates fitness of each member of $P(T)$ .

### 2.1. GA Algorithm:

1. Set generation counter $T = 0$ and maximum generation $M = 0$
2. Initialize probability of crossover $p_c$, probability of mutation $p_m$ , upper limit of iteration counter $M_0$, population size $N$ .
3. Initialize ($P(T)$) .
4. Evaluate ($P(T)$) .
5. While ($M < M_0$) .
6. Select $N$ solutions from $P(T)$ for mating pool using Roulette-Wheel process.
7. Select solutions from $P(T)$ , for crossover depending on $p_c$.
8. Make crossover on selected solutions.
9. Select solutions from $P(T)$ , for mutation depending on $p_m$.
10. Make mutation on selected solutions for mutation to get population $P_1(T)$ .
11. Evaluate ($P_1(T)$)
12. Set $M = M + 1$
13. If average fitness of $P_1(T)$ > average fitness of $P(T)$ then
14. Set $P(T+1) = P_1(T)$
15. Set $T = T + 1$
16. Set $M = 0$
17. End if
18. End While
19. Output: Best solution of $P(T)$
20. End algorithm.

### 3. 1. Triangular fuzzy number:

Let $\tilde{A} = (k_1, k_2, k_3)$ is a triangular fuzzy number, where $k_1 = k - \Delta_1, k_2 = k \; and \; k_3 = k + \Delta_2$. Such that $0 < \Delta_1 < k, 0 < \Delta_2 \; and \; \Delta_1, \Delta_2$ are determined by the decision maker based on the uncertainty of the problem. And then $\tilde{A}$ can be represented as $\tilde{A} = \left[A_l^L(\alpha), A_l^U(\alpha)\right] \cup \left[A_r^L(\alpha), A_r^U(\alpha)\right]$ subject to the constraint $0 < k_1 \le k_2 \le k_3$. And the membership function of the triangular fuzzy numbers is defined as follows.

$$\mu_{\tilde{A}}(x) : R \rightarrow [0,1]$$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x < k_1, x > k_3 \\ L(x) = \dfrac{x - k_1}{k_2 - k_1} & , k_1 \le x < k_2 \\ R(x) = \dfrac{k_3 - x}{k_3 - k_2} & , k_2 \le x < k_3 \end{cases}$$

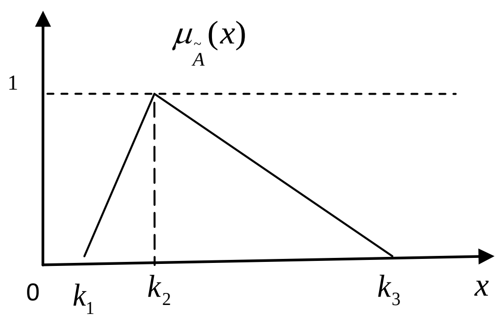


**Figure 1: Membership Function**

## 3.2. Centroid Method:

This procedure (also called center of area, center of gravity) is the most prevalent and physically appealing of all the defuzzification methods it is given by the algebraic expression as follows:

$$M_{\tilde{A}} = \frac{\int \mu_{\tilde{A}}(x).x dx}{\int \mu_{\tilde{A}}(x) dx}$$

Where $\int$ denotes an algebraic integration. Hence the centroid for $\tilde{A}$ is given as

$$M_{\tilde{k}} = \frac{k_1 + k_2 + k_3}{3} = k + \frac{1}{3}(\Delta_2 - \Delta_1)$$

## 3.3. ASSUMPTIONS AND NOTATIONS

The mathematical model in this paper is developed on the basis of the following assumptions and notation.

**3.3.1. Notations:** Following notations have been used in this paper

$\tilde{c}_1$ fuzzy holding cost of the inventory item, \$ / per unit / per unit time

$\tilde{c}_2$ fuzzy deterioration cost, \$ / per unit

$\tilde{c}_3$ fuzzy ordering cost, \$ / per order

$\tilde{c}_{1r}$ fuzzy holding cost for raw material inventory, \$ / per unit / per unit

$\tilde{c}_{2r}$ fuzzy deterioration cost for raw material inventory, \$ / per unit

$\tilde{c}_{3r}$ fuzzy ordering cost for raw material inventory, \$ / per order

$\tilde{c}_r$ fuzzy purchasing cost for raw material inventory, \$ / per unit

$\tilde{p}$ fuzzy production cost, \$ / per unit

$\tilde{s}$ fuzzy selling price, \$ / per unit

$\theta$ deterioration rate

i inflation rate

d discount rate

r = d - i

Q maximum inventory level of the production cycle

$Q_0$ constant inventory level

$T$ length of the ordering cycle

$I(t)$ inventory level at time $t \in [0, T]$

**3.3.2. Assumptions:** Model is developed under the following assumptions

(1) The inventory system involves only one item and the planning horizon is infinite.

(2) Production rate is demand dependent i.e. $P(t) = a\,D(t)$

(3) The demand rate $D(t)$ is deterministic and its functional form is given by

$$D(t) = \begin{cases} \alpha\, I(t)^{\beta}, & Q > Q_0, \\ D, & 0 \le Q \le Q_0 \end{cases}$$

Where $\alpha > 0$, $0 < \beta < 1$, $D = \alpha Q_0^{\beta}$, and both $\alpha$ and $\beta$ are known as scale and shape parameters, respectively.

(4) Shortages are not allowed.

## 4. Model Formulation

Let the production is stopped when the stock-level reached to level $Q$, $Q_0$ is the Constant stock-level. Then there may arise the following two cases

(I) $Q \le Q_0$ (II) $Q > Q_0$

### 4.1. Case I: $Q \le Q_0$

This is the classical EPQ model for deteriorating items with constant demand rate. Production is started at the time $t = 0$, with zero inventory level and stopped at the time $t = T_1$ when the inventory level reaches to the level Q. After that inventory level decreases due to the combined effect of demand and deterioration up to the time T, at which inventory level reaches to the zero level. Inventory level at any time can be described by the following differential equation and graphically represented in the figure-1.

$$I_1'(t) + \theta I_1(t) = (a-1)D, \qquad 0 \le t \le T_1 \qquad (1)$$

$$I_2'(t) + \theta I_2(t) = -D, \qquad T_1 \le t \le T \qquad (2)$$

With the boundary conditions $I_1(0) = 0$ and $I_2(T) = 0$. Solution of equation (1) and (2) are as follows

$$I_1(t) = \frac{(a-1)D}{\theta}\left(1 - e^{-\theta t}\right), \quad 0 \le t \le T_1 \qquad (3)$$

$$I_2(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right), \quad T_1 \le t \le T \qquad (4)$$

From equation (3) and (4), using the condition $I_1(T_1) = I_2(T_1)$, we have

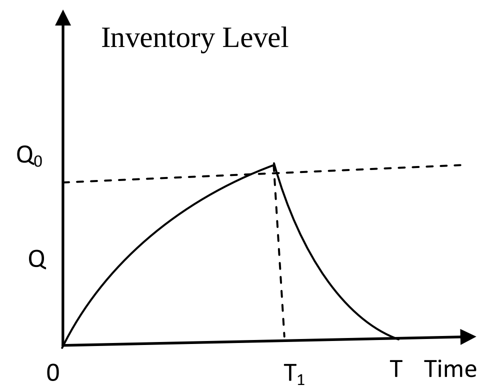


Figure 2: Case I ($Q \le Q_0$)

$$T = \frac{1}{\theta}\ln\left[a\left(e^{\theta T_1} - 1\right) + 1\right] \qquad (5)$$

Maximum inventory level is

$$Q = I_1(T_1) = \frac{(a-1)D}{\theta}\left(1 - e^{-\theta T_1}\right) \qquad (6)$$

Present worth of fuzzy holding cost of the inventory held is

$$\tilde{H}\tilde{C} = \frac{\tilde{c}_1 D}{\theta}\left[\frac{(a-1)}{r}\left(1-e^{-rT_1}\right) + \frac{(a-1)}{(\theta+r)}\left(e^{-(\theta+r)T_1} - 1\right) + \frac{e^{\theta T}}{(\theta+r)}\left(e^{-(\theta+r)T_1} - e^{-(\theta+r)T}\right) + \frac{1}{r}\left(e^{-rT} - e^{-rT_1}\right)\right] \qquad (7)$$

Present worth of fuzzy production cost is

$$\tilde{P}\tilde{C} = \frac{\tilde{p}aD}{r}\left(1-e^{-rT_1}\right) \tag{8}$$

Present worth of fuzzy deterioration cost is

$$\tilde{D}\tilde{C} = \tilde{c}_2 D\left[\frac{(a-1)}{r}\left(1-e^{-rT_1}\right)+\frac{(a-1)}{(\theta+r)}\left(e^{-(\theta+r)T_1}-1\right)\right.$$

$$\left.+\frac{e^{\theta T}}{(\theta+r)}\left(e^{-(\theta+r)T_1}-e^{-(\theta+r)T}\right)+\frac{1}{r}\left(e^{-rT}-e^{-rT_1}\right)\right] \tag{9}$$

Present worth of fuzzy sales revenue is

$$\tilde{S}\tilde{R} = \frac{\tilde{s}D}{r}\left(1-e^{-rT}\right) \tag{10}$$

Present worth of fuzzy ordering cost is

$$\tilde{O}\tilde{C} = \tilde{c}_3 \tag{11}$$

### 4.1.1. Raw Material Inventory Model

Initially, vendor purchases the raw material in lots and produces the finished goods. Vendor starts production at time (t = 0), and the raw material reaches the zero level at the time ($t = T_1$) due to the combined effect of production and deterioration. The vendor's raw materials inventory system at any time t can be represented by the following differential equation and has shown in figure 1.

$$I_r'(t)+\theta I_r(t) = -a\alpha Q_0^{\beta}, \quad 0 \le t \le T_1 \tag{12}$$

On solving the above equation using the boundary condition $I(T_1) = 0$, we have

$$I_r(t) = \frac{a\alpha Q_0^{\beta}}{\theta}\left(e^{-\theta(t-T_1)}-1\right), \; 0 \le t \le T_1 \tag{13}$$

The maximum inventory level of the raw material, i.e. the order quantity per order from outside supplier'

$$Q_r = I_r(0) = \frac{a\alpha Q_0^{\beta}}{\theta}\left(e^{\theta T_1}-1\right) \tag{14}$$

Since the order is done at the time t = 0, thus the present worth of fuzzy ordering cost is

$$\tilde{O}\tilde{C}_r = \tilde{c}_{3r} \tag{15}$$

Present worth of fuzzy holding cost for the raw material is

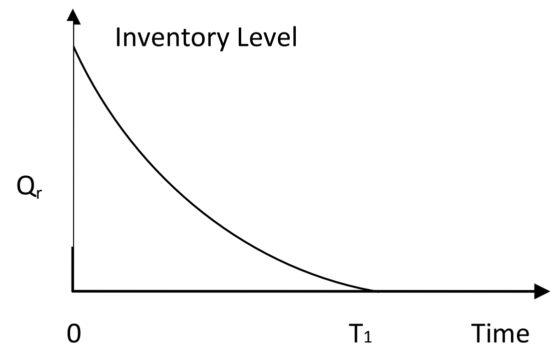


Figure 3: Raw material inventory

$$\tilde{H}\tilde{C}_r = \frac{\tilde{c}_{1r} a\alpha Q_0^{\beta}}{\theta}\left\{\frac{e^{\theta T_1}}{(\theta+r)}\left(1-e^{-(\theta+r)T_1}\right)-\frac{1}{r}\left(1-e^{-rT_1}\right)\right\} \tag{16}$$

Present worth of fuzzy deterioration cost for the raw material is

$$\tilde{D}\tilde{C}_r = \tilde{c}_{2r} a\alpha Q_0^{\beta}\left\{\frac{e^{\theta T_1}}{(\theta+r)}\left(1-e^{-(\theta+r)T_1}\right)-\frac{1}{r}\left(1-e^{-rT_1}\right)\right\} \tag{17}$$

The item cost includes the loss due to deterioration as well as the cost of the item sold. Because the order is done at t = 0. The present worth of fuzzy item cost is

$$\tilde{I}\tilde{C}_r = \tilde{c}_r Q_r = \frac{\tilde{c}_r a\alpha Q_0^{\beta}}{\theta}\left(e^{\theta T_1}-1\right) \tag{18}$$

The present worth of total cost during the cycle is the sum of fuzzy ordering cost $\tilde{O}\tilde{C}_r$, fuzzy holding cost $\tilde{H}\tilde{C}_r$, fuzzy deterioration cost $\tilde{D}\tilde{C}_r$ and fuzzy item cost $\tilde{I}\tilde{C}_r$. Thus for the raw materials, the present worth of total fuzzy cost is

$$\tilde{T}\tilde{C}_{1r} = \tilde{O}\tilde{C}_r + \tilde{H}\tilde{C}_r + \tilde{D}\tilde{C}_r + \tilde{I}\tilde{C}_r \tag{19}$$

Present worth of total fuzzy profit is

$$\tilde{T}\tilde{P}_1 = \tilde{S}\tilde{R} - \tilde{P}\tilde{C} - \tilde{H}\tilde{C} - \tilde{D}\tilde{C} - \tilde{O}\tilde{C} - \tilde{T}\tilde{C}_{1r} \tag{20}$$

**Remark (1):** $\tilde{T}\tilde{P}_1$ is the function of $T_1$ only. Now our problem is to find the optimal value of $T_1$ in order to maximize the total profit $\tilde{T}\tilde{P}_1(T_1)$ subject to the inequality constraint $Q \le Q_0$. Mathematically we have

Maximizing $\tilde{T}\tilde{P}_1(T_1)$

Subject to $Q_0 - Q \ge 0$, (21)

## 4.2. Case II: $Q > Q_0$

In this case production is started at time t = 0, with zero inventory level and stopped at time $t = T_2$ when the inventory level reached the level Q, where $Q > Q_0$. Initially the demand and the production rate are constant up to the time $t = T_1$ at which inventory level reaches to the level $Q_o$ after that demand becomes stock dependent so as the production rate up to the time $t = T_2$. after that inventory level decreases due to the combined effect of the demand and the deterioration and reaches the zero level at the time t = T. Inventory level at any time can be described by the following differential equation and graphically represented in the figure-2.

$$I_1'(t) + \theta I_1(t) = (a-1)D,\ 0 \le t \le T_1 \quad (22)$$

$$I_2'(t) + \theta I_2(t) = (a-1)\alpha\left[I_2(t)\right]^{\beta}, T_1 \le t \le T_2 \quad (23)$$

$$I_3'(t) + \theta I_3(t) = -\alpha\left[I_3(t)\right]^{\beta}, T_2 \le t \le T_3 \quad (24)$$

$$I_4'(t) + \theta I_4(t) = -D, T_3 \le t \le T \quad (25)$$

With the boundary conditions $I_1(0) = 0,\ I_2(T_1) = Q_0, I_3(T_3) = Q_0$ and $I_4(T) = 0$. Solution of equations (22), (23), (24) and (25) are as follows

$$I_1(t) = \frac{(a-1)D}{\theta}\left(1 - e^{-\theta t}\right),\ 0 \le t \le T_1 \quad (26)$$

$$I_2(t) = \left[\frac{(a-1)\alpha}{\theta} + \left\{Q_0^{(1-\beta)} - \frac{(a-1)\alpha}{\theta}\right\} e^{\theta(1-\beta)(T_1-t)}\right]^{1/(1-\beta)}, T_1 \le t \le T_2 \quad (27)$$

$$I_3(t) = \left[\left\{Q_0^{(1-\beta)} + \frac{\alpha}{\theta}\right\} e^{\theta(1-\beta)(T_3-t)} - \frac{\alpha}{\theta}\right]^{1/(1-\beta)}, T_2 \le t \le T_3 \quad (28)$$

$$I_4(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right), T_3 \le t \le T \quad (29)$$

From equation (18), using the condition $I_1(T_1) = Q_0$, we have

$$T_1 = \frac{1}{\theta}\ln\left[\frac{(a-1)D}{(a-1)D - Q_0\theta}\right] \quad (30)$$

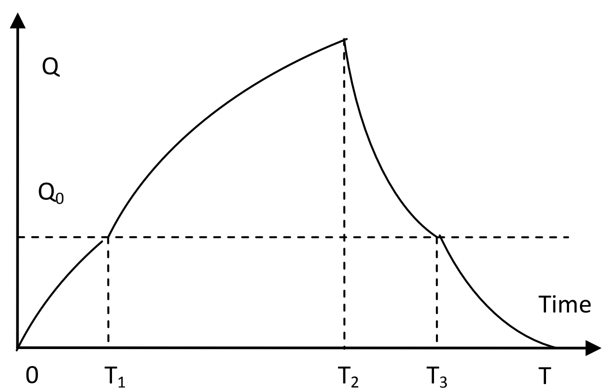


Figure 4: Case II ($Q > Q_0$)

From equation (19) and (20), using the condition $I_2(T_2) = I_3(T_2)$, we have

$$T_3 = \frac{1}{\theta(1-\beta)}\ln\left[\frac{1}{(\alpha + \theta Q_0^{(1-\beta)})}\left(a\alpha e^{\theta(1-\beta)T_2} + \left(\theta Q_0^{(1-\beta)} - (a-1)\alpha\right) e^{\theta(1-\beta)T_1}\right)\right] \quad (31)$$

From equation (21) using the condition $I_4(T_3) = Q_0$, we have

$$T_4 = \frac{1}{\theta(1-\beta)} \ln\left[ \frac{1}{(\alpha+\theta Q_0^{(1-\beta)})} \; a\alpha e^{\theta(1-\beta)T_2} + \theta Q_0^{(1-\beta)} - (a-1)\alpha \; e^{\theta(1-\beta)T_1} \right] + \frac{1}{\theta}\ln\left[1+\frac{\theta}{\alpha}Q_0^{(1-\beta)}\right] \quad (32)$$

Maximum inventory levels is

$$S = \left[ \frac{(a-1)\alpha}{\theta} + \left\{ Q_0^{(1-\beta)} - \frac{(a-1)\alpha}{\theta} \right\} e^{\theta(1-\beta)(T_1-T_2)} \right]^{1/(1-\beta)} \quad (33)$$

Present worth of fuzzy holding cost is

$$\tilde{H}\tilde{C} = \frac{\tilde{c}_1(a-1)D}{\theta}\left[ \frac{1}{r} \; 1-e^{-rT_1} \; + \frac{1}{(\theta+r)} \; e^{-(\theta+r)T_1} - 1 \right] + \tilde{c}_1\left(\frac{(a-1)\alpha}{\theta}\right)^{1/(1-\beta)} \left[ \frac{1}{r} \; e^{-rT_1} - e^{-rT_2} \; + \frac{1}{(1-\beta)(\theta+r-\theta\beta)} \left\{ \frac{\theta Q_0^{(1-\beta)}}{(a-1)\alpha} - 1 \right\} e^{\theta(1-\beta)T_1} \; e^{-(\theta+r-\theta\beta)T_1} - e^{-(\theta+r-\theta\beta)T_2} \right]$$

$$+ \tilde{c}_1\left( Q_0^{(1-\beta)} + \frac{\alpha}{\theta} \right)^{1/(1-\beta)} \left[ \frac{e^{\theta T_3}}{(\theta+r)} \; e^{-(\theta+r)T_2} - e^{-(\theta+r)T_3} + \frac{1}{(1-\beta)(\theta\beta+r)} \frac{\alpha e^{\theta\beta T_3}}{\alpha+\theta Q_0^{(1-\beta)}} \; e^{-(\theta\beta+r)T_3} - e^{-(\theta\beta+r)T_2} \; \right] + \frac{\tilde{c}_1 D}{\theta} \left[ \frac{e^{\theta T}}{(\theta+r)} \; e^{-(\theta+r)T_3} - e^{-(\theta+r)T} \; + \frac{1}{r} \; e^{-rT} - e^{-rT_3} \right] \quad (34)$$

Present worth of fuzzy deterioration cost is

$$\tilde{D}\tilde{C} = \tilde{c}_2(a-1)D\left[ \frac{1}{r} \; 1-e^{-rT_1} \; + \frac{1}{(\theta+r)} \; e^{-(\theta+r)T_1} - 1 \right] + \tilde{c}_2\theta\left( \frac{(a-1)\alpha}{\theta} \right)^{1/(1-\beta)} \left[ \frac{1}{r} \; e^{-rT_1} - e^{-rT_2} \; + \frac{1}{(1-\beta)(\theta+r-\theta\beta)} \left\{ \frac{\theta Q_0^{(1-\beta)}}{(a-1)\alpha} - 1 \right\} e^{\theta(1-\beta)T_1} \; e^{-(\theta+r-\theta\beta)T_1} - e^{-(\theta+r-\theta\beta)T_2} \; \right]$$

$$+ \tilde{c}_2\theta\left( Q_0^{(1-\beta)} + \frac{\alpha}{\theta} \right)^{1/(1-\beta)} \left[ \frac{e^{\theta T_3}}{(\theta+r)} \; e^{-(\theta+r)T_2} - e^{-(\theta+r)T_3} \; + \frac{1}{(\theta\beta+r)} \frac{1}{(1-\beta)} \frac{\alpha e^{\theta\beta T_3}}{\alpha+\theta Q_0^{(1-\beta)}} \; e^{-(\theta\beta+r)T_3} - e^{-(\theta\beta+r)T_2} \right] + \tilde{c}_2 D\left[ \frac{e^{\theta T}}{(\theta+r)} \; e^{-(\theta+r)T_3} - e^{-(\theta+r)T} \; + \frac{1}{r} \; e^{-rT} - e^{-rT_3} \; \right] \quad (35)$$

Present worth of fuzzy production cost is

$$\tilde{P}\tilde{C} = \frac{\tilde{p}aD}{r} \; 1-e^{-rT_1} \; + \tilde{p}a\alpha\left( \frac{(a-1)\alpha}{\theta} \right)^{\beta/(1-\beta)} \left[ \frac{1}{r} \; e^{-rT_1} - e^{-rT_2} \; + \frac{\beta}{(1-\beta)} \frac{1}{(\theta+r-\theta\beta)} \left\{ \frac{\theta Q_0^{(1-\beta)}}{(a-1)\alpha} - 1 \right\} e^{\theta(1-\beta)T_1} \; e^{-(\theta+r-\theta\beta)T_1} - e^{-(\theta+r-\theta\beta)T_2} \; \right] \quad (36)$$

Present worth of fuzzy sales revenue is

$$\tilde{S}\tilde{R} = \frac{\tilde{s}D}{r} \; 1-e^{-rT_1} \; + \tilde{s}\alpha\left( \frac{(a-1)\alpha}{\theta} \right)^{\beta/(1-\beta)} \left[ \frac{1}{r} \; e^{-rT_1} - e^{-rT_2} \; + \frac{\beta}{(1-\beta)} \frac{1}{(\theta+r-\theta\beta)} \right.$$

$$\left\{\frac{\theta Q_0^{(1-\beta)}}{(a-1)\alpha}-1\right\}e^{\theta(1-\beta)T_1}\left(e^{-(\theta+r-\theta\beta)T_1}-e^{-(\theta+r-\theta\beta)T_2}\right)\Bigg]$$

$$+\tilde{s}\alpha\left(Q_0^{(1-\beta)}+\frac{\alpha}{\theta}\right)^{\beta/(1-\beta)}\left[\frac{e^{\theta\beta T_3}}{(\theta\beta+r)}\left(e^{-(\theta\beta+r)T_2}\right.\right.$$

$$\left.-e^{-(\theta\beta+r)T_3}\right)-\frac{1}{(1-\beta)}\frac{\alpha\beta e^{-\theta(1-2\beta)T_3}}{(\theta-2\theta\beta-r)\left(\alpha+\theta Q_0^{(1-\beta)}\right)}$$

$$\left.\left(e^{(\theta-2\theta\beta-r)T_3}-e^{(\theta-2\theta\beta-r)T_2}\right)\right]+\frac{\tilde{s}D}{r}\left(e^{-rT_3}-e^{-rT}\right) \quad (37)$$

Present worth of fuzzy ordering cost is

$$\tilde{O}\tilde{C}=\tilde{c}_3 \quad (38)$$

### 4.2.1. Raw Material Inventory Model

Initially, vendor purchases the raw material in lots and produces the finished goods. Vendor starts production at time (t = 0), and the raw material reaches the zero level at the time ($t = T_2$) due to the combined effect of production and deterioration. The vendor's raw materials inventory system at any time t can be represented by the following differential equation and has shown in figure 1.

$$I_{r1}'(t)+\theta I_{r1}(t)=-a\alpha Q_0^{\beta},\quad 0\le t\le T_1 \quad (39)$$

$$I_{r2}'(t)+\theta I_{r2}(t)=-a\alpha\left(I_2(t)\right)^{\beta},T_1\le t\le T_2$$

$$I_{r2}'(t)+\theta I_{r2}(t)=-a\alpha\left[\frac{(a-1)\alpha}{\theta}+\left\{Q_0^{(1-\beta)}-\frac{(a-1)\alpha}{\theta}\right\}e^{\theta(1-\beta)(T_1-t)}\right]^{\beta/(1-\beta)},T_1\le t\le T_2 \quad (40)$$

On solving the above equation using the boundary condition $I_{r1}(0)=Q_r$ and $I_{r2}(T_2)=0$, we have

$$I_{r1}(t)=\left\{Q_r+\frac{a\alpha Q_0^{\beta}}{\theta}\right\}e^{-\theta t}-\frac{a\alpha Q_0^{\beta}}{\theta},\ 0\le t\le T_1 \quad (41)$$

$$I_{r2}(t)=\frac{a\alpha}{\theta}\left(\frac{(a-1)\alpha}{\theta}\right)^{\beta/(1-\beta)}\left[\left(e^{\theta(T_2-t)}-1\right)\right.$$

$$+\left\{\frac{\theta Q_0^{(1-\beta)}-(a-1)\alpha}{(1-\beta)(a-1)\alpha}\right\}e^{\theta(1-\beta)T_1}\left(e^{\theta\beta T_2}e^{-\theta t}\right.$$

$$\left.\left.-e^{-\theta(1-\beta)t}\right)\right],\ T_1\le t\le T_2 \quad (42)$$

Using the condition $I_{r1}(T_1)=I_{r2}(T_1)$, maximum inventory level of the raw material, i.e. the order quantity per order from outside supplier'

$$Q_r=\frac{a\alpha Q_0^{\beta}}{\theta}\left(e^{\theta T_1}-1\right)+\frac{a\alpha}{\theta}\left(\frac{(a-1)\alpha}{\theta}\right)^{\beta/(1-\beta)}$$

$$\left[\left(e^{\theta T_2}-e^{\theta T_1}\right)+\left\{\frac{\theta Q_0^{(1-\beta)}-(a-1)\alpha}{(1-\beta)(a-1)\alpha}\right\}e^{\theta(1-\beta)T_1}\right.$$

$$\left.\left(e^{\theta\beta T_2}-e^{\theta\beta T_1}\right)\right] \quad (43)$$

Since the order is done at the time t = 0, thus the present worth of fuzzy ordering cost is

$$\tilde{O}\tilde{C}_r=\tilde{c}_{3r} \quad (44)$$

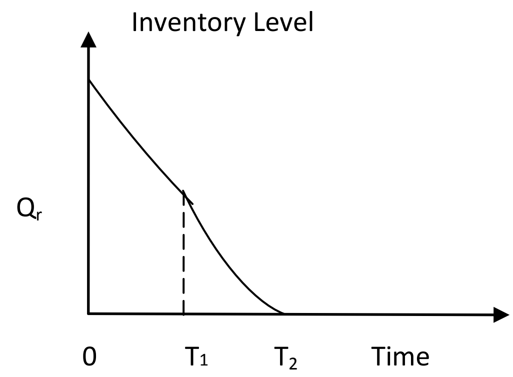


Figure 5: Raw material inventory

Present worth of fuzzy holding cost for the raw material is

$$\tilde{H}\tilde{C}_r = \tilde{c}_{1r}\Bigg[\Bigg[\left\{Q_r + \frac{a\alpha Q_0^{\beta}}{\theta}\right\}\frac{1}{(\theta+r)}\left(1-e^{-(\theta+r)T_1}\right) - \frac{a\alpha Q_0^{\beta}}{\theta r}\left(1-e^{-rT_1}\right) + \frac{a\alpha}{\theta}\left(\frac{(a-1)\alpha}{\theta}\right)^{\beta/(1-\beta)}\Bigg\{\frac{e^{\theta T_2}}{(\theta+r)}\left(e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}\right) - \frac{1}{r}\left(e^{-rT_1} - e^{-rT_2}\right) + \left\{\frac{\theta Q_0^{(1-\beta)} - (a-1)\alpha}{(1-\beta)(a-1)\alpha}\right\}e^{\theta(1-\beta)T_1}\Bigg\{\frac{e^{\theta T_2}}{(\theta+r)}\left(e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}\right) - \frac{1}{(\theta+r-\theta\beta)}\left(e^{-(\theta+r-\theta\beta)T_1} - e^{-(\theta+r-\theta\beta)T_2}\right)\Bigg\}\Bigg\}\Bigg]\Bigg] \quad (45)$$

Present worth of fuzzy deterioration cost for the raw material is

$$\tilde{D}\tilde{C}_r = \tilde{c}_{2r}\Bigg[\Bigg[\left\{Q_r + \frac{a\alpha Q_0^{\beta}}{\theta}\right\}\frac{\theta}{(\theta+r)}\left(1-e^{-(\theta+r)T_1}\right) - \frac{a\alpha Q_0^{\beta}}{r}\left(1-e^{-rT_1}\right) + a\alpha\left(\frac{(a-1)\alpha}{\theta}\right)^{\beta/(1-\beta)}\Bigg\{\frac{e^{\theta T_2}}{(\theta+r)}\left(e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}\right) - \frac{1}{r}\left(e^{-rT_1} - e^{-rT_2}\right) + \left\{\frac{\theta Q_0^{(1-\beta)} - (a-1)\alpha}{(1-\beta)(a-1)\alpha}\right\}e^{\theta(1-\beta)T_1}\Bigg\{\frac{e^{\theta T_2}}{(\theta+r)}\left(e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}\right) - \frac{1}{(\theta+r-\theta\beta)}\left(e^{-(\theta+r-\theta\beta)T_1} - e^{-(\theta+r-\theta\beta)T_2}\right)\Bigg\}\Bigg\}\Bigg]\Bigg] \quad (46)$$

The item cost includes the loss due to deterioration as well as the cost of the item sold. Because the order is done at t = 0. The present worth of fuzzy item cost is

$$\tilde{I}\tilde{C}_r = \tilde{c}_r Q_r \quad (47)$$

The present worth of total cost during the cycle is the sum of fuzzy ordering cost $\tilde{O}\tilde{C}_r$, fuzzy holding cost $\tilde{H}\tilde{C}_r$, fuzzy deterioration cost $\tilde{D}\tilde{C}_r$ and fuzzy item cost $\tilde{I}\tilde{C}_r$. Thus for the raw materials, the present worth of total fuzzy cost is

$$\tilde{T}\tilde{C}_{2r} = \tilde{O}\tilde{C}_r + \tilde{H}\tilde{C}_r + \tilde{D}\tilde{C}_r + \tilde{I}\tilde{C}_r \quad (48)$$

Present worth of total fuzzy profit is

$$\tilde{T}\tilde{P}_2 = \tilde{S}\tilde{R} - \tilde{P}\tilde{C} - \tilde{H}\tilde{C} - \tilde{D}\tilde{C} - \tilde{O}\tilde{C} - \tilde{T}\tilde{C}_{2r} \quad (49)$$

**Remark (2):** $\tilde{T}\tilde{P}_2$ is the function of $T_2$ only. Now our problem is to find the optimal value of $T_2$ in order to maximize the total profit $\tilde{T}\tilde{P}_2(T_2)$ subject to the inequality constraint $Q > Q_0$. Mathematically we have

Maximizing $\tilde{T}\tilde{P}_2(T_2)$

Subject to $Q - Q_0 > 0$, (50)

## 5. Numerical Illustrations

### 5.1. Case I: $Q_0 \geq Q$ :

The following numerical data are used to illustrate the model.

$$a = 1.2, \alpha = 25, \beta = 0.25, Q_0 = 150, \tilde{c}_1 = (0.45, 0.50, 0.60), \tilde{c}_2 = (0.35, 0.40, 0.50), \tilde{c}_3 = (130, 200, 300),$$

$\tilde{c}_{1r} = (0.45, 0.50, 0.60), \tilde{c}_{2r} = (0.35, 0.40, 0.50), \tilde{p} = (4, 5, 7), \tilde{c}_{3r} = (80, 100, 125),$
$\tilde{c}_r = (0.8, 1.0, 1.2), \tilde{s} = (10, 12, 15), d = 0.1, i = 0.05, r = 0.05, \theta = 0.01.$

For the above assumed parametric values the results are obtained using GA and results are presented in Table 1.

**Table 1:** Results for above assumed numeric data

| $T_1$ | T | $Q_r$ | Q | $TP_1(T_1)$ |
|---|---|---|---|---|
| 4.14 | 4.95 | 401.20 | 64.15 | 823.67 |

It is found that optimal cycle length T = 4.95, maximum inventory level Q = 64.15 units, production time $T_1$ = 4.14 and the present value of the optimal profit is 823.67. At the beginning of each cycle the manufacturer initially purchases 401.20 units of the raw material and produce the finished goods items at a rate P for the period $T_1$ = 4.14. During this period inventory of raw material decreases and reaches to the zero level at time t = 4.14, on the other hand inventory of finished goods items build up to the time t = 4.14 and inventory level reaches 64.15 units. At this time the manufacturer stops production and inventory depleted due to the combined effect of demand and deterioration. Inventory level reaches zero level at time t = 4.95, then production for next cycle starts.

**5.1.1. Sensitivity analysis:** variation of the total profit w.r.t. different parameters.

**Table 2:** Present value of total profits of the model due to different production rates

| A | 1.1 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 |
|---|---|---|---|---|---|---|
| $TP_1$ | 813.53 | 832.52 | 827.17 | 781.53 | 729.71 | 682.16 |

**Table 3:** Present value of total profits of the model due to different demand parameter “α”

| α | 22 | 23 | 24 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|
| $TP_1$ | 687.43 | 732.84 | 778.25 | 869.08 | 914.50 | 959.91 |

**Table 4:** Present value of total profits of the model due to different demand parameter “β”

| β | 0.22 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 |
|---|---|---|---|---|---|---|
| $TP_1$ | 677.17 | 723.77 | 772.57 | 877.17 | 933.20 | 991.87 |

**Table 5:** Present value of total profits of the model due to different ‘r’

| r | 0.047 | 0.048 | 0.049 | 0.051 | 0.052 | 0.053 |
|---|---|---|---|---|---|---|
| $TP_1$ | 839.24 | 834.00 | 828.81 | 818.58 | 813.54 | 808.54 |

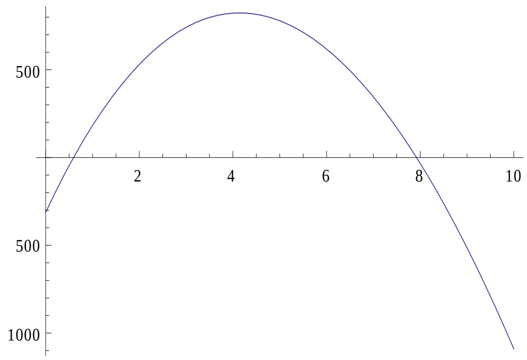


**Figure 6: Total profit with respect to $T_1$**

**6.1. Observations:**

For the above parametric values optimal profit due to different production rates are obtained and presented in Table 2. It is observed that optimal profits increases with production rate and attains a maximum limit and then decreases as production increases. It happens because increase of production rate initially increases stock level and as demand is stock dependent, and hence increases the demand of the items, which in turn increases the profit. But increase of stock level is also increases the holding cost and deterioration cost. Initially profit due to increased demand dominates the loss due to increases holding cost and deterioration cost. But after certain level of production rate if production rate, the increases holding cost and the deterioration cost dominate the profit due to increased demand and after that level of production rate if production rate increases then profit decreases.

Results are obtained for the different values of demand parameters and presented in the table 3 and 4. It is observed that profit increases with increase in demand parameters, which agrees with reality.

Results are obtained for the above parametric values and different values of 'resultant effect of inflation and discount rate' r, and presented in the table 5. It is observed that profit decreases with increase of r, which agrees with reality.

**5.2. Case II:** $Q > Q_0$

The following numerical data are used to illustrate the model.

$$a = 1.2, \alpha = 25, \beta = 0.25, Q_0 = 150, \tilde{c}_1 = \; 0.45, 0.50, 0.60 \;, \tilde{c}_2 = \; 0.35, 0.40, 0.50 \;,$$

$$\tilde{c}_3 = \; 130, 200, 300 \;, \tilde{c}_{1r} = \; 0.45, 0.50, 0.60 \;, \tilde{c}_{2r} = \; 0.35, 0.40, 0.50 \;, \tilde{p} = \; 4, 5, 7 \;,$$

$$\tilde{c}_{3r} = \; 80, 100, 125 \;, \tilde{c}_r = \; 0.8, 1.0, 1.2 \;, \tilde{s} = \; 10, 12, 15 \;, d = 0.1, i = 0.05, r = 0.05, \theta = 0.01.$$

For the above assumed parametric values the results are obtained using GA and results are presented in Table 6.

**Table 6:** Results for above assumed numeric data

| $T_1$ | $T_2$ | $T_3$ | $T$ | $Q_r$ | $Q$ | $TP_2(T_2)$ |
|---|---|---|---|---|---|---|
| 6.53 | 51.66 | 58.56 | 59.82 | 11345.10 | 913.08 | 42221.80 |

It is found that optimal cycle length T = 59.82, maximum inventory level Q = 913.08 units, production time $T_2$ = 51.66 and the present value of the optimal profit is 42221.80. At the beginning of each cycle the manufacturer initially purchases 11345.10 units of the raw material and produce the finished goods items at a rate P for the period t = 51.66. During this period inventory of raw material decreases and reaches to the zero level at time t = 51.66, on the other hand inventory of finished goods items build up to the time t = 51.66 and inventory level reaches 913.08 units. At this time production is stopped and then inventory depleted due to the combined effect of demand and deterioration. Inventory level of finished goods items reaches zero level at time t = 59.82, then production for next cycle starts.

**5.2.1. Sensitivity analysis:** variation of the total profit w.r.t. different parameters.

**Table 7:** Present value of total profits of the model due to different production rates

| a | 1.1 | 1.15 | 1.25 | 1.3 | 1.35 |
|---|---|---|---|---|---|
| $TP_1$ | -691.62 | 18453.80 | 70792.70 | 111206.00 | 720900.00 |

**Table 8:** Present value of total profits of the model due to different demand parameter "α"

| α | 22 | 23 | 24 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|
| $TP_1$ | 27117.00 | 31785.70 | 36816.90 | 48014.20 | 54210.80 | 60832.20 |

**Table 9:** Present value of total profits of the model due to different demand parameter "β"

| β | 0.22 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 |
|---|---|---|---|---|---|---|
| $TP_1$ | 17305.30 | 23494.90 | 31579.90 | 56463.60 | 76058.30 | 104557.00 |

**Table 10:** Present value of total profits of the model due to different 'r'

| r | 0.047 | 0.048 | 0.049 | 0.051 | 0.052 | 0.053 |
|---|---|---|---|---|---|---|
| $TP_1$ | 50566.50 | 47555.40 | 44781.70 | 39854.90 | 37662.90 | 35629.60 |

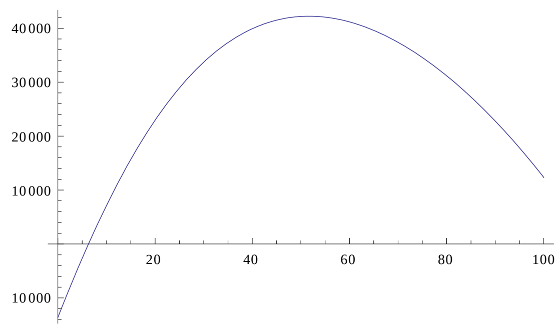


**Figure 7: Total profit with respect to $T_2$**

### 6.2. Observations:

For the above parametric values optimal profit due to different production rates are obtained and presented in Table 7. It is observed that optimal profits increase as production rate increases. It happens because increase in production rate increases stock level and as demand is stock dependent, increased stock level increases the demand of the item, which in turn increases the profit. But increase in stock level increases the holding cost. Profit due to increased demand dominates the loss due to increased holding cost. Thus increase in the production rate increases optimal profit.Results are obtained for the different values of demand parameters and presented in the table 8 and 9. It is observed that profit increases with increase in demand parameters, which agrees with reality.

Results are obtained for the above parametric values and different values of 'resultant effect of inflation and discount rate' r, and presented in the table 10. It is observed that profit decreases with increase of r, which agrees with reality.

### 7. Conclusions and Future Research

In this paper, an EPQ model has been considered under imprecise and inflationary environment and time discounting over an infinite horizon. Some interesting observations are presented. Demand is taken as the power form stock dependent and the production is taken as the demand dependent. In this paper we discussed the following two cases (I) $Q \le Q_0$ and (II) $Q > Q_0$, where Q is the stock-level at the time production is stopped and $Q_0$ is the fixed stock-level. Model is formulated to maximize the total profit. Also a genetic algorithm with varying population size is used to solve a production inventory model. It is found that this GA is efficient in solving the proposed inventory model. This GA can also be used to solve different decision making problems in different fields of science and technology. This inventory model can be extend incorporating price dependent demand, trade credit policy, two warehouse, etc.

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