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An EOQ Model with Time Dependent Demand and Weibull Distributed Deterioration

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***Abstract* — In this paper, we developed an inventory model for deteriorating items under time dependent demand function. Inventory holding cost is a linear function of time. For deterioration of units we considered two parameters Weibull distribution. In this study, a more realistic scenario is assumed where the part of shortages was backordered and the rest was lost with time. The backordering rate is variable and it is considered exponential function depending on waiting time for the next replenishment. Solution procedure of the developed model is presented with numerical example and its sensitivity analysis.**

Keywords* — *EOQ model; deterioration; time dependent demand; weibull distribution; shortages; inventory

I. INTRODUCTION

Deterioration of produced items is very important factor in inventory management and in any production. In the recent time more research work has been carried out in the inventory management to develop inventory models with deterioration of items and shortages are permitted. Commodities like vegetables, fruits and food items from depletion by direct spoilage while kept in store. It has been observed that the failure of many items may be expressed by Weibull distribution.

Ghare and Scharder [5] first formulated a mathematical model with a constant deterioration rate. Khemlnitsky and Gerchak [7] developed optimal control approach to production systems with inventory level dependent demand. Ruxian , Hongjie Lan and John Mawhinney [12] reviewed on deteriorating inventory study. Vipin Kumar, Singh and Sanjay Sharma [21] developed inventory model for profit maximization production with time dependent demand and partial backlogging. Papachristos and Skouri [10] gave an inventory model with deteriorating items, quantity discount, pricing and time-dependent partial backlogging. Samanta and Ajanta Roy [14] presented a production inventory model with deteriorating items and shortages. Inventory model with price dependent demand and time varying holding cost was given by Ajantha Roy [1]. Vinod Kumar Mishra and Lal Sahab Singh [20] have given deteriorating inventory model for time dependent demand and holding cost with partial backlogging. Kirtan Parmar, Indu Aggarwal and Gothi [9] have formulated an order level inventory model for deteriorating items under varying demand condition.

Analysis of inventory-production systems with weibull distributed deterioration is given by Azizul Baten and Anton Abdulbasah Kamil [3]. Chen and Lin [4] developed optimal replenishment scheduling for inventory items with weibull distributed deterioration and time varying demand. Sanni [13] studied an economic order quantity inventory model with time dependent weibull deterioration and trended demand.

An EOQ Model for varying items with weibull distribution deterioration and price-dependent demand was given by Begum, Sahoo, Sahu and Mishra [11]. One more condition was imposed in the form of shortage by Wu and Lee [22] in their work, an EOQ inventory model for items with weibull distributed deterioration, shortages and time varying demand. Vikas Sharma and Rekha Rani Chaudhary [19] have also developed an inventory model for deteriorating items with Weibull deterioration with time dependent demand and shortages. Tripathy & Pradhan [17] formulated an integrated partial backlogging inventory model having weibull demand and variable deterioration rate with the effect of trade credit. Amutha and Dr. Chandrasekaran [2] developed an inventory model for deteriorating products with weibull distributed deterioration, time varying demand and partial backlogging. Tripathy and Pradhan [15] have given an EOQ model for weibull deteriorating items with power demand and partial backlogging.

Some more conditions were applied on inventory holding cost by Ghosh and Chaudhuri [6] in their research in an order level inventory model for a deteriorating item with weibull distribution deterioration, time-quadratic demand and shortages. Tripathy and Mishra [16] developed an inventory model for Weibull deteriorating items with price dependent demand and time varying holding cost. Kirtan Parmar & U. B. Gothi [8] developed an EOQ model of deteriorating items using three parameter weibull distribution with constant production rate and time varying holding cost.

Vashistha & Gupta [18] has developed an inventory model with weibull distribution deterioration and time dependent demand. We have modified the same inventory model by considering inventory holding cost as a linear function of time and also including production cost to the total cost function to make the model more realistic. The sensitivity analysis is also carried out by changing the values of all the parameters one by one. We have used different formulae to find costs like IHC, DC, SC and LSC.

II. NOTATIONS

The following notations are used to develop the mathematical model:

- $Q(t)$: Inventory level of the product at time t (≥ 0).
- $R(t)$: Demand rate varying over time.
- $\theta(t)$: Deterioration rate
- A : Ordering cost per order during the cycle period.
- C_h : Inventory holding cost per unit.
- C_d : Deterioration cost per unit.
- C_s : Shortage cost per unit.
- p_c : Production cost per unit.
- l : Opportunity cost due to lost sale per unit.
- S : Initial stock level at the beginning of every cycle.
- S_1 : Inventory level at t = μ.
- S_2 : The maximum inventory level during shortage period.
- T : Duration of a cycle.
- $TC(t_1, T)$: Total cost per unit time.

III. ASSUMPTIONS

The following assumptions are considered to develop this model.

1. Replenishment rate is infinite.
2. Lead-time is zero.
3. A single item is considered over the prescribed period of time.
4. No repair or replacement of the deteriorated items takes place during a given cycle.
5. We consider finite time horizon period.
6. Demand rate R(t) is assumed to be a function of time such that $R(t) = a + b\, t + c\{t - (t-\mu) H\,(t-\mu)\}t$, where $H\,(t-\mu)$ is the Heaviside's function defined as $H(t-\mu) = \begin{cases} 1, & if \quad t \geq \mu \\ 0, & if \quad t < \mu \end{cases}$ and a is the initial rate of demand, b is the rate with which the demand rate increases. The rate of change in demand rate itself increases at a rate c. a, b and c are positive constants.
7. Holding cost is linear function of time and it is $C_h = h + r\,t$ (h, r > 0).
8. $\theta(t) = \alpha\,\beta\,t^{\beta-1}$ is the two parameter Weibull distributed deterioration rate, where α is scale parameter and β is shape parameter and $(0 < \alpha << 1)$.
9. Shortages are allowed and unsatisfied demand is backlogged at a rate $e^{-\delta(T-t)}$, where the backlogging parameter δ is a positive constant.
10. Total inventory cost is a real and continuous function which is convex to the origin.

IV. MATHEMATICAL MODEL AND ANALYSIS

The initial stock is S at time t = 0, then inventory level decreases mainly due to meet up demand with rate $-\left(a + b\,t + c\,t^2\right)$ and partly from deterioration with rate $\alpha\,\beta\,t^{\beta-1}$ and reaches to S_1 at t = μ. The stock reduces to zero at t = t_1 due to demand with rate $-(a + m\,t)$ where $m = (b + c\,\mu)$ and the deterioration with rate $\alpha\,\beta\,t^{\beta-1}$. Thereafter, shortages are allowed to occur during the time interval $[t_1, T]$ and the demand during the period $[t_1, T]$ is partially backlogged.

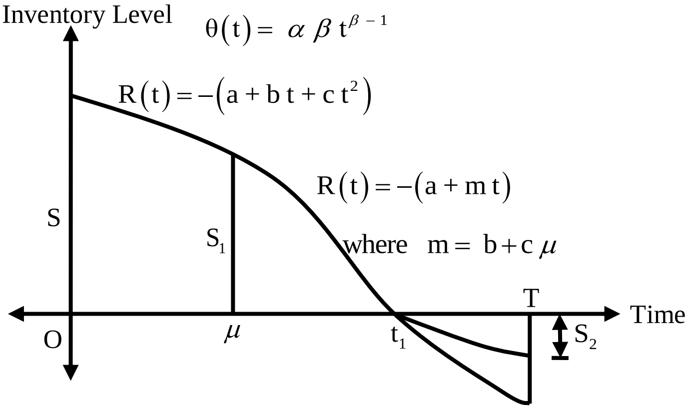

Fig. Graphical representation of inventory system

The rate of change of inventory during the intervals $(0, \mu)$, (μ, t_1) and (t_1, T) is governed by the following differential equations

$$\frac{d\,Q(t)}{d\,t} + \alpha\,\beta\,t^{\beta-1}\,Q(t) = -\left(a + b\,t + c\,t^2\right) \qquad 0 \leq t \leq \mu \tag{1}$$

$$\frac{d\,Q(t)}{d\,t} + \alpha\,\beta\,t^{\beta-1}\,Q(t) = -\left(a + m\,t\right) \text{ where } m = (b + c\,\mu) \qquad \mu \leq t \leq t_1 \tag{2}$$

$$\frac{d\,Q(t)}{d\,t} = -\left(a + m\,t\right) e^{-\delta(T-t)} \qquad t_1 \leq t \leq T \tag{3}$$

Now solving equation (1), (2) and (3) with boundary condition Q(0) = S and $Q(t_1) = 0$ we get,

$$Q(t) = S\left(1 - \alpha t^{\beta}\right) - \left(a t + \frac{b t^2}{2} + \frac{c t^3}{3}\right) + \frac{a \alpha \beta t^{\beta+1}}{\beta+1} + \frac{b \alpha \beta t^{\beta+2}}{2(\beta+2)} + \frac{c \alpha \beta t^{\beta+3}}{3(\beta+3)} \qquad 0 \le t \le \mu$$

(4)

$$Q(t) = -\Bigg[a(t - t_1) + \frac{m}{2}\left(t^2 - t_1^2\right) + \frac{a \alpha}{\beta+1}\left(t^{\beta+1} - t_1^{\beta+1}\right) + \frac{m \alpha}{(\beta+2)}\left(t^{\beta+2} - t_1^{\beta+2}\right) - a \alpha\left(t^{\beta+1} - t_1 t^{\beta}\right) - \frac{m \alpha}{2}\left(t^{\beta+2} - t_1^2 t^{\beta}\right)\Bigg] \qquad \mu \le t \le t_1$$

(5)

$$Q(t) = -\left[a(1-\delta T)(t - t_1) + \left(a \delta + m(1-\delta T)\right)\left(\frac{t^2 - t_1^2}{2}\right) + m \delta\left(\frac{t^3 - t_1^3}{3}\right)\right] \qquad t_1 \le t \le T$$

(6)

In equation (5) , $Q(\mu) = S_1$ and so

$$S_1 = \Bigg[a(t_1 - \mu) + \frac{m}{2}\left(t_1^2 - \mu^2\right) + \frac{a \alpha}{\beta+1}\left(t_1^{\beta+1} - \mu^{\beta+1}\right) + \frac{m \alpha}{(\beta+2)}\left(t_1^{\beta+2} - \mu^{\beta+2}\right) - a \alpha(t_1 - \mu)\mu^{\beta} - \frac{m \alpha}{2}\left(t_1^2 - \mu^2\right)\mu^{\beta}\Bigg]$$

(7)

In equation (4), $Q(\mu) = S_1$ which gives

$$S = \frac{1}{\left(1 - \alpha \mu^{\beta}\right)}\left[S_1 + \left(a \mu + \frac{b \mu^2}{2} + \frac{c \mu^3}{3}\right) - \frac{a \alpha \beta \mu^{\beta+1}}{\beta+1} - \frac{b \alpha \beta \mu^{\beta+2}}{2(\beta+2)} - \frac{c \alpha \beta \mu^{\beta+3}}{3(\beta+3)}\right]$$

(8)

In equation (6), $Q(T) = -S_2$ and hence

$$S_2 = a(1 - \delta T)(T - t_1) + \left(a \delta + m(1 - \delta T)\right)\left(\frac{T^2 - t_1^2}{2}\right) + \frac{m \delta}{3}\left(T^3 - t_1^3\right)$$

(9)

The total cost can be obtained by adding the following costs.

Ordering cost

$$OC = A$$

(10)

The deterioration cost during the period $[0, t_1]$

$$DC = C_d\left[\int_0^{\mu} \alpha \beta t^{\beta-1} Q(t)\,dt + \int_{\mu}^{t_1} \alpha \beta t^{\beta-1} Q(t)\,dt\right]$$

$$= C_d\,\alpha \beta\Bigg[\frac{S \mu^{\beta}}{\beta} - \left(\frac{a \mu^{\beta+1}}{\beta+1} + \frac{b \mu^{\beta+2}}{2(\beta+2)} + \frac{c \mu^{\beta+3}}{3(\beta+3)}\right) + \left(a t_1 + \frac{m t_1^2}{2}\right)\left(\frac{t_1^{\beta} - \mu^{\beta}}{\beta}\right) - a\left(\frac{t_1^{\beta+1} - \mu^{\beta+1}}{\beta+1}\right) - m\left(\frac{t_1^{\beta+2} - \mu^{\beta+2}}{\beta+2}\right)\Bigg]$$

(11)

The shortage cost during the period $[t_1, T]$

$$SC = -C_s\int_{t_1}^{T} Q(t)\,dt$$

$$= C_s\Bigg[a(1-\delta T)\frac{(T - t_1)^2}{2} + \left(\frac{a \delta + m(1-\delta T)}{2}\right)\left(\left(\frac{(T - t_1)^3}{3}\right) + t_1(T - t_1)^2\right) + \frac{m \delta}{3}\left(\frac{(T - t_1)^4}{4} + t_1(T - t_1)^3 + \frac{3 t_1^2}{2}(T - t_1)^2\right)\Bigg]$$

(12)

The opportunity cost due to lost sales during the period $[t_1, T]$

$$LSC = l\int_{t_1}^{T}(a + m t)\left(1 - e^{-\delta(T-t)}\right)dt$$

$$= l\,\delta\left[a T(T - t_1) + \left(\frac{m T - a}{2}\right)\left(T^2 - t_1^2\right) - \frac{m}{3}\left(T^3 - t_1^3\right)\right]$$

(13)

The holding cost during the period $[0, t_1]$

$$
\begin{aligned}
IHC &= \int_0^{\mu} (h + r t)\, Q(t)\, dt + \int_{\mu}^{t_1} (h + r t)\, Q(t)\, dt \\
&= h\Bigg[\left(S\mu - \frac{S\alpha\mu^{\beta+1}}{\beta+1}\right) - \left(\frac{a\mu^2}{2} + \frac{b\mu^3}{6} + \frac{c\mu^4}{12}\right) \\
&\quad + \alpha\beta\left(\frac{a\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\mu^{\beta+3}}{(\beta+2)(\beta+3)} + \frac{c\mu^{\beta+4}}{(\beta+3)(\beta+4)}\right) \\
&\quad + \left(-a t_1 - \frac{m t_1^2}{2} - \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{m\alpha t_1^{\beta+2}}{\beta+2}\right)(\mu - t_1) \\
&\quad + \left(\frac{a}{2}(\mu^2 - t_1^2) + \frac{m}{6}(\mu^3 - t_1^3)\right) - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}\left(\mu^{\beta+2} - t_1^{\beta+2}\right) \\
&\quad - \frac{m\alpha\beta}{2(\beta+2)(\beta+3)}\left(\mu^{\beta+3} - t_1^{\beta+3}\right) \\
&\quad + \left(a\alpha t_1 + \frac{m\alpha t_1^2}{2}\right)\left(\frac{\mu^{\beta+1} - t_1^{\beta+1}}{\beta+1}\right)\Bigg] \\
&\quad + r\Bigg[\left(\frac{S\mu^2}{2} - \frac{S\alpha\mu^{\beta+2}}{\beta+2}\right) - \left(\frac{a\mu^3}{3} + \frac{b\mu^4}{8} + \frac{c\mu^5}{15}\right) \\
&\quad + \alpha\beta\left(\frac{a\mu^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{b\mu^{\beta+4}}{(\beta+2)(\beta+4)} + \frac{c\mu^{\beta+5}}{(\beta+3)(\beta+5)}\right) \\
&\quad + \left(-a t_1 - \frac{m t_1^2}{2} - \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{m\alpha t_1^{\beta+2}}{\beta+2}\right)\left(\frac{\mu^2 - t_1^2}{2}\right) \\
&\quad + \left(\frac{a}{3}(\mu^3 - t_1^3) + \frac{m}{8}(\mu^4 - t_1^4)\right) - \frac{a\alpha\beta}{(\beta+1)(\beta+3)}\left(\mu^{\beta+3} - t_1^{\beta+3}\right) \\
&\quad - \frac{m\alpha\beta}{2(\beta+2)(\beta+4)}\left(\mu^{\beta+4} - t_1^{\beta+4}\right) \\
&\quad + \left(a\alpha t_1 + \frac{m\alpha t_1^2}{2}\right)\left(\frac{\mu^{\beta+2} - t_1^{\beta+2}}{\beta+2}\right)\Bigg]
\end{aligned}
\tag{14}
$$

Production cost

$$PC = p_c\,(S + S_2) \tag{15}$$

The total cost per unit time is given by

$$TC(t_1, T) = \frac{1}{T}\left(OC + IHC + SC + DC + LSC + PC\right)$$

$$
TC(t_1, T) = \frac{1}{T}\left\{
\begin{aligned}
&A + h\Bigg[\left(S\mu - \frac{S\alpha\mu^{\beta+1}}{\beta+1}\right) - \left(\frac{a\mu^2}{2} + \frac{b\mu^3}{6} + \frac{c\mu^4}{12}\right) \\
&+ \alpha\beta\left(\frac{a\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\mu^{\beta+3}}{(\beta+2)(\beta+3)} + \frac{c\mu^{\beta+4}}{(\beta+3)(\beta+4)}\right) \\
&+ \left(-a t_1 - \frac{m t_1^2}{2} - \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{m\alpha t_1^{\beta+2}}{\beta+2}\right)(\mu - t_1) \\
&+ \left(\frac{a}{2}(\mu^2 - t_1^2) + \frac{m}{6}(\mu^3 - t_1^3)\right) - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}\left(\mu^{\beta+2} - t_1^{\beta+2}\right) \\
&- \frac{m\alpha\beta}{2(\beta+2)(\beta+3)}\left(\mu^{\beta+3} - t_1^{\beta+3}\right) \\
&+ \left(a\alpha t_1 + \frac{m\alpha t_1^2}{2}\right)\left(\frac{\mu^{\beta+1} - t_1^{\beta+1}}{\beta+1}\right)\Bigg] \\
&+ r\Bigg[\left(\frac{S\mu^2}{2} - \frac{S\alpha\mu^{\beta+2}}{\beta+2}\right) - \left(\frac{a\mu^3}{3} + \frac{b\mu^4}{8} + \frac{c\mu^5}{15}\right) \\
&+ \alpha\beta\left(\frac{a\mu^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{b\mu^{\beta+4}}{(\beta+2)(\beta+4)} + \frac{c\mu^{\beta+5}}{(\beta+3)(\beta+5)}\right) \\
&+ \left(-a t_1 - \frac{m t_1^2}{2} - \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{m\alpha t_1^{\beta+2}}{\beta+2}\right)\left(\frac{\mu^2 - t_1^2}{2}\right) \\
&+ \left(\frac{a}{3}(\mu^3 - t_1^3) + \frac{m}{8}(\mu^4 - t_1^4)\right) - \frac{a\alpha\beta}{(\beta+1)(\beta+3)}\left(\mu^{\beta+3} - t_1^{\beta+3}\right) \\
&- \frac{m\alpha\beta}{2(\beta+2)(\beta+4)}\left(\mu^{\beta+4} - t_1^{\beta+4}\right) \\
&+ \left(a\alpha t_1 + \frac{m\alpha t_1^2}{2}\right)\left(\frac{\mu^{\beta+2} - t_1^{\beta+2}}{\beta+2}\right)\Bigg] \\
&+ C_s\Bigg[a(1 - \delta T)\frac{(T - t_1)^2}{2} + \left(\frac{a\delta + m(1 - \delta T)}{2}\right) \\
&\left(\left(\frac{(T - t_1)^3}{3}\right) + t_1 (T - t_1)^2\right) \\
&+ \frac{m\delta}{3}\left(\frac{(T - t_1)^4}{4} + t_1 (T - t_1)^3 + \frac{3 t_1^2}{2}(T - t_1)^2\right)\Bigg] \\
&+ C_d\,\alpha\beta\Bigg[\frac{S\mu^{\beta}}{\beta} - \left(\frac{a\mu^{\beta+1}}{\beta+1} + \frac{b\mu^{\beta+2}}{2(\beta+2)} + \frac{c\mu^{\beta+3}}{3(\beta+3)}\right) \\
&+ \left(a t_1 + \frac{m t_1^2}{2}\right)\left(\frac{t_1^{\beta} - \mu^{\beta}}{\beta}\right) \\
&- a\left(\frac{t_1^{\beta+1} - \mu^{\beta+1}}{\beta+1}\right) - m\left(\frac{t_1^{\beta+2} - \mu^{\beta+2}}{\beta+2}\right)\Bigg] \\
&+ l\,\delta\left[a T (T - t_1) + \left(\frac{m T - a}{2}\right)(T^2 - t_1^2) - \frac{m}{3}(T^3 - t_1^3)\right] \\
&+ p_c\,(S + S_2)
\end{aligned}
\right\}
$$

Our objective is to determine optimum values t_1^* and T^* such that $TC(t_1,T)$ is minimum. Note that t_1^* and T^* are the solutions of the equations

$$\frac{\partial TC(t_1,T)}{\partial t_1}=0 \quad \& \quad \frac{\partial TC(t_1,T)}{\partial T}=0 \qquad (17)$$

such that

$$\left.\begin{array}{l}\left[\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1\,\partial T}\right)^2\right]_{t_1=t_1^*,\,T=T^*}>0 \\ \left[\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right]_{t_1=t_1^*,\,T=T^*}>0\end{array}\right\} \qquad (18)$$

The optimal solution of the equations in (17) can be obtained by using the software. This has been illustrated by the following numerical example.

V. NUMERICAL EXAMPLE

Let us consider the following example to illustrate the above developed model. We consider the following values of the parameters A = 300, h = 2, r = 16, μ = 5, C_d = 18, l = 10,
α = 0.0001, β = 5, a = 2, b = 3, c = 5, m = 28, C_s = 17, δ = 0.0002 & p_c = 15 (with appropriate units of measurement). We obtain the optimal values T^* = 7.448264056 units, t_1^* = 5.2905425919 units and optimal total cost $TC(t_1,T)$ = 9770.406510 units by using appropriate software.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis helps in identifying the effect of optimal solution of the model by the changes in its parameter values. In this section, we study the sensitivity of total cost per time unit $TC(t_1,T)$ with respect to the changes in the values of the parameters A, α, μ, a, b, c, δ, h, p_c, C_s, C_d, l , m and r.

The sensitivity analysis is performed by considering 10% and 20% increase and decrease in each one of the above parameters keeping all other remaining parameter as fixed. The results are presented in Table – 1. The last column of table shows the % change in $TC(t_1,T)$ as compared to the original solution corresponding to the change in parameters values.

TABLE - I SENSITIVITY ANALYSIS

Parameter	% change	T	t_1	$TC(t_1,T)$	% changes in $TC(t_1,T)$
	– 20	7.445406199	5.29016578	9759.947791	– 0.107044875
	– 10	7.446835483	5.290354243	9765.177951	– 0.053514251
A	10	7.449691919	5.290730829	9775.633473	0.053497897
	20	7.451119074	5.290918953	9780.858839	0.106979459
	– 20	7.342798114	5.300616593	9325.835862	– 4.550175556
	– 10	7.394243966	5.295717103	9542.005314	– 2.337683668
α	10	7.505103902	5.285035598	10012.36947	2.476488128
	20	7.565040232	5.279127689	10269.41498	5.107346055
	– 20	5.048259926	4.210974646	4023.445977	– 58.82007598
	– 10	6.14066536	4.752816003	6253.2881	– 35.99766711
μ	10	9.176093149	5.811514762	15991.71499	63.67502185
	20	12.12255666	6.206800713	31100.4103	218.3123472
	– 20	7.442946559	5.289841412	9730.31566	– 0.410329403
	– 10	7.445608265	5.290192425	9750.362325	– 0.205152018
a	10	7.450913956	5.290891916	9790.44823	0.205126771
	20	7.453557987	5.291240402	9810.487494	0.410228412
	– 20	7.39193315	5.287053749	9554.925649	– 2.205444184
	– 10	7.42019919	5.288813518	9662.84006	– 1.100941408
b	10	7.476130652	5.292241495	9877.627337	1.097403945
	20	7.503801812	5.29391074	9984.504834	2.191293912
	– 20	7.046694019	5.270130313	8255.748149	– 15.50251118
	– 10	7.252724663	5.281124747	9021.998138	– 7.659951222
c	10	7.634206651	5.298550067	10501.30021	7.480688706
	20	7.811326283	5.305288724	11215.01677	14.78556963
	– 20	7.448155361	5.290556183	9770.554284	0.001512462
	– 10	7.448209705	5.290549388	9770.480401	0.000756263
δ	10	7.448318413	5.290535795	9770.332615	– 0.000756326
	20	7.448372776	5.290528998	9770.258712	– 0.001512716

Parameter	% change	T	t_1	$TC(t_1,T)$	% changes in $TC(t_1,T)$
	– 20	7.423514172	5.288219977	9677.559835	–0.950284678
	– 10	7.435909423	5.289386344	9724.01529	–0.474813625
h	10	7.460578348	5.291688823	9816.733642	0.474157667
	20	7.472852573	5.292825141	9862.996829	0.947660856
	– 20	7.610933659	5.326173188	9690.57666	–0.817057616
	– 10	7.529209709	5.308272694	9731.07186	–0.402589707
p_c	10	7.368092895	5.272982062	9808.5799	0.390704208
	20	7.288692191	5.255590224	9845.591279	0.769515246
	– 20	7.70899727	5.260068445	9378.29687	–4.013237731
	– 10	7.56889612	5.276149679	9584.207555	–1.905744205
C_s	10	7.343048417	5.303527912	9939.95053	1.735281115
	20	7.250290241	5.315322778	10095.23773	3.324643822
	– 20	7.446451526	5.293180369	9756.435631	–0.142991795
	– 10	7.447344685	5.291852949	9763.390681	–0.071806942
C_d	10	7.449209062	5.289248958	9777.48175	0.072414993
	20	7.450179142	5.287971718	9784.615065	0.145424388
	– 20	7.448290504	5.290539379	9770.364749	–0.000427436
	– 10	7.44827728	5.290540986	9770.38563	–0.000213717
l	10	7.448250832	5.290544198	9770.427392	0.000213715
	20	7.448237609	5.290545805	9770.448272	0.000427427
	– 20	7.966953092	5.312540139	9504.831551	–2.718156703
	– 10	7.687810269	5.30157837	9652.993137	–1.201724553
m	10	7.240214212	5.279778227	9864.625505	0.964330334
	20	7.057659597	5.269453198	9940.922426	1.745228462
	– 20	7.065941013	5.277231694	8311.034745	–14.9366535
	– 10	7.263975282	5.28540387	9055.049867	–7.321667149
r	10	7.62108883	5.293563685	10460.48512	7.062946719
	20	7.784171635	5.295077033	11128.07187	13.89568957

VII. CONCLUSION

From the above sensitivity analysis we may conclude that the total cost per time unit $TC(t_1,T)$ is highly sensitive to changes in the values of the parameters μ, c and r moderately sensitive to changes in the values of the parameters α, b, C_s, m and less sensitive to changes in the values of the parameters A, a, δ, h, p_c, C_d, and l.

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