An EOQ Model With Exponential Demand Rate Under Cash Discount And Permissible Delay In Payment

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Abstract

In the present paper we developed an EOQ model with exponential demand rate together with an optimal ordering policy under cash discount and permissible delay in payment to the customer. Sensitivity analysis is given with the effect of parameters in the optimal solution, since in the classical inventory models it is assumed that the customer pays to the supplier as soon as he received the items and in such cases the supplier offer a cash discount or a permissible delay to the customer.

Keywords Inventory, Cash discount, Payment Delay

"1. Introduction"

In the classical inventory models payment for the items paid by the suppliers depend on the payment paid by the customers and in such cases the supplier provides a fixed credit period to the customers during which no interest will be charged from the customers, after this credit period up to the end of a period interest charged paid by the customers. In such situations the customer starts to accumulate revenue on his sale and earn interest on his revenue. Goyal [1985] developed an EOQ model together with the condition of permissible delay in payments. Goyals model was extended by Aggarwal and Jaggi [1995] with the consideration of deteriorating items. Further this model was generalized by Jamal et al [1997] by allowing shortages also some interesting results on this study are given by Chung [1998], Sarkar et al [2000] and Teng [2001].Soni and Shah [2005] developed a mathematical model with constant rate of deterioration and the scenario of progressive credit periods, further this model was extended by Soni et al [2006] under the effect of inflation. Soni and Shah [2008] studied the Levin et al [1972] model. Nita H. Shah, Poonam Mishra [2010] developed an EOQ model for deteriorating items under supplier credits when demand is stock dependent. In the earlier inventory problems discussed under the conditions of permissible delay in payment, the supplier provides not only a fixed credit period to settle the account but also gives the cash discount offer to the customers in the business market. In his model Goyal assume that the unit purchase cost of an item is equal to the selling price per unit. Chung-Tao- Chang extended the Goyal's model under the assumption of constant demand with cash discount and the difference between unit price and unit purchase cost of the product.

In the present paper we developed an EOQ model with exponential demand rate together with an optimal ordering policy under cash discount and permissible delay in payment to the customers. Sensitivity analysis is given to study the effect of variation of parameters in the optimal solution.

"2. Assumptions and Notations"

We consider the following assumptions

(1) The demand rate is $R(t)=e^{at}$, where $0\le a\le 1$

(2) h is the unit holding cost per unit time excluding interest charges.

(3) p is the selling price per unit of the product.

(4) c is the unit purchase cost.

(5) I_C is the interest charged per \$ per unit time in stock by the supplier.

(6) I_e is the interest earned per \$ per unit time.

(7) S is the ordering cost per order.

(8) Q is the order quantity or order size.

(9) r is the cash discount rate.

(10) M is the period of cash discount.

(11) N is the period of permissible delay in setting account with N > M.

(12) T is the inventory cycle length.

(13) Shortages are not allowed.
(14) The replenishment rate is infinite.
(15) Time horizon is infinite.
(16) The account is not settled during the time, generated sales revenue is deposited in an interest bearing account.
At the end of the period the customer pays off all units sold, keep profit and starts paying for the interested charges on the items in stocks
(17) I(t) be the inventory level at any time t.
(18) Z (T) is the total variable cost per unit time.

"3. Mathematical Formulations"

The instantaneous inventory level at any time t is given by the differential equation

$$\frac{dI(t)}{dt} = -e^{at}, \qquad 0 \le t \le T \quad \ldots\ldots\ldots\ldots (1)$$

With boundary conditions

$$I(0) = I_0 \quad and \quad I(T) = 0$$

The solution of (1) is

$$I(t) = \frac{1}{a}\left(e^{at} - e^{aT}\right) \quad \ldots\ldots.. (2)$$

And the order quantity is $Q = \frac{1}{a}\left(1 - e^{aT}\right)$ ……… (3)

The ordering cost is $O_C = \frac{S}{T}$ ……… (4)

The holding cost is $H_C = \frac{h}{T}\int_0^T I(t)\,dt$

$$= \frac{h}{T\,a^2}\left[e^{aT}\left(1 - aT\right) - 1\right] \quad \ldots\ldots\ldots (5)$$

Due to cash discount, interest charged and interest earned we consider the following four cases

Case 1 The payment is paid at M to get a cash discount together with $T \ge M$

Case 2 The customer pays in full at M to get a cash discount together with T<M

Case 3 The payment is paid at N to get the permissible delay together with $T \ge N$

Case 4 The customer pays in full at N together with T<N

Now we find the total variable cost in each of the cases

Case 1 Since in this case payment is paid at M and $T \ge M$

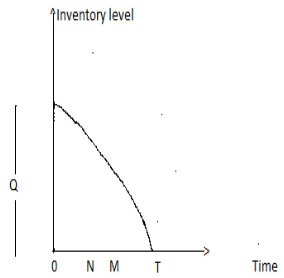

Figure 1

Then the cash discount per unit time is k

$$= \frac{r\,c\,Q}{T}$$

$$k = \frac{rc}{Ta}\left(1 - Exp[aT]\right) \ldots\ldots(6)$$

The interest payable per unit time is

$$I_{C_1} = \frac{cI_C}{T}\int_M^T I(t)dt$$

$$= \frac{cI_C}{T\,a^2}\left[\left\{1 - a\left(T - M\right)\right\}e^{aT} - e^{aM}\right] \quad \ldots\ldots\ldots (7)$$

The interest earned per unit time is

$$I_{E_1} = \frac{pI_e}{T}\int_0^T te^{at}dt$$

$$= \frac{p\,I_e}{T\,a^2}\left[1 + (a\,M - 1)\,e^{aM}\right] \quad \ldots\ldots.. (8)$$

So the total variable cost per unit time is

$$Z_1(T) = \frac{S}{T} + \frac{h}{Ta^2}\left[e^{aT}\left(1 - aT\right) - 1\right] + \frac{cI_C}{Ta^2}\left[\left\{1 - a\left(T - M\right)\right\}e^{aT} - e^{aM}\right] - \frac{p\,I_e}{Ta^2}\left[1 + (aM - 1)e^{aM}\right] - \frac{rc}{Ta}\left(1 - e^{aT}\right) \quad \ldots\ldots\ldots.. (9)$$

For the optimum value of

$$Z_1(T),\ \frac{\partial Z_1}{\partial T} = 0 \, gives\ T = T_1\ \ for\ which\ \frac{\partial^2 Z_1}{\partial T^2} \succ 0, \forall\ T =$$

Case 2 Since in this case the customer pays in full at M and T<M

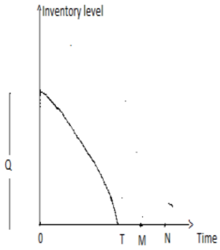

Figure 2

So there is zero interest charge and the cash discount is same as that in case 1, then the interest earned per unit time is

$$I_{E_2} = \frac{pI_e}{T}\left[\int_0^T te^{at}dt + \int_M^T Te^{at}dt\right]$$

$$= \frac{pI_e}{T}\left[(2e^{aT} - e^{aM})\frac{T}{M} - \frac{e^{aT}}{a^2} + \frac{1}{a^2}\right]$$

………. (10)

Therefore the total variable cost per unit time is

$$Z_2(T) = \frac{S}{T} + \frac{h}{Ta^2}\left[e^{aT}(1 - aT) - 1\right]$$

$$-\frac{pI_e}{T}\left[\frac{1}{a^2} - \frac{e^{aT}}{a^2} + \frac{T}{a}(2e^{aT} - e^{aM})\right] - \frac{rc}{Ta}(1 - e^{aT})$$

………(11)

For the optimum value of

$$Z_2(T),\ \frac{\partial Z_2}{\partial T} = 0 \, gives\ T = T_2\ \ for\ which$$

$$\frac{\partial^2 Z_2}{\partial T^2} \succ 0, \forall\ T = T_2$$

Case 3 Since in this case payment is paid at N and $T \geq N$,

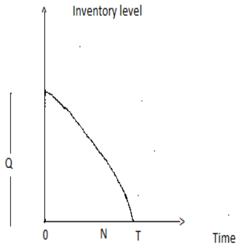

Figure 3

Then there is zero cash discount and the interest charged per unit time is

$$I_{C_3} = \frac{cI_C}{T}\int_N^T I(t)\,dt$$

$$= \frac{cI_C}{Ta^2}\left[\left\{1 - a(T - N)e^{aT}\right\} - e^{aN}\right]$$

………(12)

The interest earned per unit time is

$$I_{E_3} = \frac{pI_e}{T}\int_0^N t\,e^{at}dt$$

$$= \frac{pI_e}{Ta^2}\left[1 + (aN - 1)e^{aN}\right]$$

$$I_{E_2} = \frac{pI_e}{T}\left[\int_0^T te^{at}\right.$$

$$= \frac{pI_e}{T}\left[(2e^{a}\right.$$

…………(13)

Therefore total variable cost per unit time is

$$Z_3(T) = \frac{S}{T} + \frac{h}{Ta^2}\left[e^{aT}(1 - aT) - 1\right]$$

$$+ \frac{cI_C}{Ta^2}\left[\left\{1 - a(T - N)e^{aT}\right\} - e^{aN}\right]$$

$$-\frac{pI_e}{Ta^2}\left[1 + (aN - 1)e^{aN}\right]$$

……….. (14)

For the optimum value of

$$Z_3(T),\frac{\partial Z_3}{\partial T} = 0\ gives\ T = T_3\ \ for\ which$$

$$\frac{\partial^2 Z_3}{\partial T^2} \succ 0,\quad \forall\ T = T_3$$

Case 4 Since in this case the customer pays in full at N and T<N

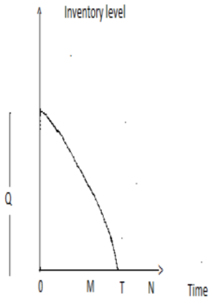

Figure 4

Then there is no interest charged and the interest earned per unit time is

$$I_{E_4} = \frac{pI_e}{T}\left[\int_0^T te^{at}dt + \int_N^T Te^{at}dt\right]$$

$$I_{E_4} = \frac{pI_e}{Ta^2}\left[(2aT-1)e^{aT} + (1-aTe^{aN})\right]$$

………. (15)

Therefore the total variable cost per unit time is

$$Z_4(T) = \frac{S}{T} + \frac{h}{Ta^2}\left[e^{aT}(1-aT)-1\right]$$

$$-\frac{pI_e}{Ta^2}\left[(1-aTe^{aN}) + (2aT-1)e^{aT}\right]$$

…….. (16)

For the optimum value of

$$Z_4(T),\ \frac{\partial Z_4}{\partial T} = 0 \ gives\ T = T_4 \ \ for\ which$$

$$\frac{\partial^2 Z}{\partial T^2} \succ 0,\ \ \forall\ T = T_4$$

"4.Theoretical Results"

For a I st order approximation of $e^{aT} = 1 + aT$ we have

$$Z_1(T) = \frac{S}{T} - hT + cI_C[M-T] - \frac{pI_eM^2}{T} + rc$$

………. (17)

$$\frac{\partial Z_1}{\partial T} = -\frac{S}{T^2} - h - cI_C + \frac{pI_eM^2}{T^2}$$

$$\frac{\partial Z_1}{\partial T} = 0,\ gives\ T_1^2(cI_C + h) + (S - pI_eM^2) = 0$$

………….. (18)

$$\frac{\partial^2 Z_1}{\partial T^2} = \frac{2}{T^3}(S - pI_eM^2) \succ 0 \quad \forall \quad T = T_1$$

And the condition $T_1 \succ M \ \ gives$

$$[pI_e - (h + cI_C)]M^2 \succ S$$ ………… (19)

$$Z_2(T) = \frac{S}{T} - hT - pI_e[2T - M] + rc$$

………… (20)

$$\frac{\partial Z_2}{\partial T} = -\frac{S}{T^2} - h - 2pI_e$$

$$\frac{\partial Z_2}{\partial T} = 0, gives\ T_2^2(h + 2pI_e) + S = 0$$

……… (21)

$$\frac{\partial^2 Z_2}{\partial T^2} = \frac{2S}{T_2^3} \succ 0$$

And the condition

$$T_2 \prec M\ gives\ \ [S + M(h + 2pI_e)] \succ 0$$

……….. (22)

$$Z_3(T) = \frac{S}{T} - hT + \frac{cI_Ca^2(N-T)}{M^2} - \frac{pI_eN^2}{T}$$

………. (23)

$$\frac{\partial Z_3}{\partial T} = -\frac{S}{T^2} - h - \frac{cI_Ca^2}{M^2} + \frac{pI_eN^2}{T^2}$$

$$\frac{\partial Z_3}{\partial T} = 0,\ gives\ T_3^2(hM^2 + cI_Ca^2) + (S - pI_eN^2)M^2 = 0$$

………… (24)

$$\frac{\partial^2 Z_3}{\partial T^2} = \frac{2}{T_3^3}(S - pI_eN^2) \succ 0$$

And the condition

$$T_3 \succ N\ gives\ SM^2 \prec \left\{(pI_eM^2 - cI_Ca^2) - h^2M^2\right\}N^2$$

……… (25)

$$Z_4(T) = \frac{S}{T} - hT - pI_e(2T - N)$$ ……. (26)

$$\frac{\partial Z_4}{\partial T} = -\frac{S}{T^2} - h - 2pI_e$$

$$\frac{\partial Z_4}{\partial T} = 0, \; gives \; T_4{}^2(h + 2pI_e) + S = 0$$

………. (27)

$$\frac{\partial^2 Z_4}{\partial T^2} = \frac{2S}{T_4^3} \succ 0$$

And the condition $T_4 \prec N \; gives \; [S + N(h + 2pI_e)] \succ 0$

………… (28)

"5. Algorithms"

If $S \prec (pI_e - h - cI_C)M^2 \;\; then \;\; T^* = T_1$

If $S = (pI_e - h - cI_C)M^2 \;\; then \;\; T^* = M$

If $SM^2 \prec \left[pI_e M^2 - cI_C a^2 - h^2 M^2\right] N^2 then \; T^* = T_3$

If $SM^2 = \left[pI_e M^2 - cI_C a^2 - h^2 M^2\right] N^2 then \; T^* = T_3$

Since T_2 & T_4 comes out to be imaginary so these two do not give the optimum value of T, If

$Z_1(T_1) \prec Z_3(T_3) \; then \;\; either \; T^* = T_1$

$or \, T^* = T_3$

"6. Numerical Results"

We consider the parametric values of the parameters in appropriate units as

[S, c, h, I_C, I_e, r, p, M, N, a] =[10, 5, 3, 0.1, 0.3, 0.1, 4, 20, 15, 0.1]

Table 1

Effect of ordering cost on optimal solution

S	T	Z	Q(T*)
10	11.5882 9.3095*	-70.6172 -55.8569*	-21.8617 -15.3692
20	11.4642 9.1287*	-69.7496 -54.7722*	-21.4691 -14.9146
50	11.0841 8.5635*	-67.0887 -51.3808*	-20.2954 -13.5455

As ordering cost increases, the optimal cycle length and total variable cost decreases

Table 2

Effect of holding cost on optimal solution

h	T	Z	Q(T*)
3	11.5882 9.3095*	-70.6172 -55.8569*	-21.8617 -15.3692
5	9.2442 7.2111*	-72.6974 -72.1109*	-15.2041 -10.5671
10	6.6904* 5.0990	-83.1663* -101.9803	-9.5236 -6.6513

As holding cost increases, the optimal cycle length decreases and total variable cost increases

Table 3

Effect of interest earned on optimal solution

I_e	T	Z	Q(T*)
0.3	11.5882 9.3095*	-70.6172 -55.8569*	-21.8617 -15.3692
0.5	15.0238 12.1106*	-94.6665 -72.6636*	-34.9237 -23.5704
0.9	20.2131 16.3299*	-130.9920 -97.9796*	-65.4821 -41.1916

As interest earned increases, the optimal cycle length increases and total variable cost increases

Table 4

Effect of interest charged on optimal solution

I_C	T	Z	Q(T*)
0.1	11.5882 9.3095*	-70.6172 -55.8569*	-21.8617 -15.3692
0.5	9.2442* 9.3094	-51.1858* -55.8566	-15.2041 -15.3689
0.9	7.9162* 9.3093	-28.2434* -55.8563	-12.0697 -15.3687

As interest charged increases, the optimal cycle length decreases and total variable cost decreases (Negative sign comes because purchase cost is not taken into account)

"7.Conclusion"

In this paper we developed an economic order quantity model with exponential demand rate under the condition of cash discount and permissible delay in payment. We have seen that the increase in the ordering cost parameter and the interest charged parameter decreases the total cost also the increase in the holding cost parameter and interest earned parameter increases the total cost so the parameters S and I_C have the significant impact on total cost for profit maximization.

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