International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 7, July - 2013

An EOQ Model For Multi-Item Inventory With Stochastic Demand

Kizito Paul Mubiru
Makerere University

Abstract

Traditional approaches towards determining the economic order quantity (EOQ) in inventory management assume deterministic demand of a single item, often at a constant rate. In this paper, an optimization model is developed for determining the EOQ that minimizes inventory costs of multiple items under a periodic review inventory system with stochastic demand. Adopting a Markov decision process approach, the states of a Markov chain represent possible states of demand for items. The decision of whether or not to order additional units is made using dynamic programming. The approach demonstrates the existence of an optimal state-dependent EOQ and produces optimal ordering policies, as well as the corresponding total inventory costs for items.

1 Introduction

Business enterprises continually face the challenge of optimizing inventory levels and decisions of items in a stochastic demand environment. Two major problems are usually encountered: (i) determining the most desirable period during which to order additional units of the item in question and (ii) determining the economic order quantity given a periodic review inventory system when demand is uncertain. In this paper, an inventory system is considered whose goal is to optimize the economic order quantity and total costs associated with ordering and holding inventory items. At the beginning of each period, a major decision has to be made, namely whether to order additional units of the items in stock or postpone ordering and utilize the available units in inventory. The paper is organized as follows. After reviewing the previous work done, a mathematical model is proposed where initial consideration is given to the process of estimating the model parameters. The model is solved thereafter and applied to a special case study. Some final remarks lastly follow.

2 Literature Reviews

Tadashi and Takeshi[1] formulated the stochastic EOQ type models with discounting using Gaussian processes in the context of the classical EOQ model. Numerical properties of the order quantities that minimize expected costs for various model parameters were examined. However, the model is restricted to a single item. The stochastic EOQ-type models to establish inventory policies were examined by Berman and Perry [2].Output can be interpreted by a random demand and the input by a deterministic production plus random returns. In this study, the authors examine a single item and the decision of how much to order at least cost is not explicit. Research results also explore the structure of optimal ordering policies for stochastic inventory system with minimum order quantity by Yao & Kateshaki [3]. In this article, ordering quantity is either zero or at least a minimum order size. The impact of such inventory policies on the cost structures of the stocked item is however not explicit. In a similar context, Broekmeullen [4] proposed a replenishment policy for a perishable inventory system based on estimated aging and retrieval behavior. The model takes into account the age of inventories and which requires only very simple calculations. The model has profound insights especially in terms of the randomness of demand. However, the model is restricted to perishable products. According to Roychowdhury [5], an optimal policy for a stochastic inventory model for deteriorating items with time-dependent selling price is feasible. The rate of deterioration of the items is assumed to be constant over time. The demand and lead time both are random. A profit-maximization model is formulated and solved for optimum order quantity. The model provides some intriguing insights to the problem. However, model extensions to handle multiple items must be embedded in the formulation to provide optimal results. The current literature presented examines modeling in the context of a single product. In this paper, an inventory system is considered whose goal is to optimize the EOQ, ordering policy and the total costs associated with multiple items in inventory. At the beginning of each period, a major decision has to be made, namely whether to order units of the stocked items or to postpone ordering and utilize the available units in stock.

The paper is organized as follows. After describing the mathematical model in §3, consideration is given to the process of estimating the model parameters. The model is solved in §4 and applied to a special case study in §5.Some final remarks lastly follow in §6.

3 Model Formulation

We consider a designated number of items in inventory whose the demand during each time period over a fixed planning horizon is classified as either *favorable* (denoted by state $\mathcal{F}$) or *unfavorable* (denoted by state $\mathcal{U}$) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to order additional stock units (a decision denoted by Z=1) or not to order additional units (a decision denoted by Z=0) during each time period over the planning horizon, where Z is a binary decision variable. Optimality is defined such that the lowest expected total inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a three item (m =3) and two-period (N=2) planning horizon is considered.

3.1 Assumptions and notation

Varying demand is modeled by means of a Markov chain with *state transition matrix* $Q^Z(m)$ where the entry $Q^Z_{ij}(m)$ in row i and column j of the transition matrix denotes the probability of a transition in demand from state iϵ**{**$\mathcal{F}$, $\mathcal{U}$**}** to state jϵ**{**$\mathcal{U}$, $\mathcal{F}$**}** for item m ϵ{1,2,3}under a given ordering policy Zϵ{0,1}. The number of customers observed in the system and the number of units demanded during such a transition is captured by the *customer matrix* $N^Z(m)$ and *demand matrix* $D^Z(m)$ respectively. Furthermore, denote the number of units in inventory and the total (ordering, holding and shortage) cost during such a transition by the *inventory matrix* $I^Z(m)$ and the *cost matrix* $C^Z(m)$ respectively. Also, denote the *expected future cost* , the already *accumulated total cost* at the end of period *n* when the demand is in state iϵ{ $\mathcal{F}$, $\mathcal{U}$ }for a given ordering policy Zϵ{0,1} by respectively $e^Z_i(m)$ and $a^Z_i(m, n)$ and let $e^Z(m) = [e^Z_{\mathcal{F}}(m), e^Z_{\mathcal{U}}(m)]^T$ and $a^Z(m,n) = [a^Z_F(m,n), a^Z_U(m,n)]^T$ where "T" denotes matrix transposition.

3.2 Finite period dynamic programming formulation

Recalling that the demand can either be in state $\mathcal{F}$ or in state $\mathcal{U}$, the problem of finding an optimal EOQ may be expressed as a finite period dynamic programming model.

Let $C_n(i, m)$ denote the optimal expected total inventory costs of item m accumulated during the periods *n,n+1,……,N* given that the state of the system at the beginning of period *n* is iϵ{ $\mathcal{F}$,$\mathcal{U}$ }.The recursive equation relating C_n and C_{n+1} is

$$C_n(i, m) = \min_Z \{Q^Z_{i\mathcal{F}}(m)(C^Z_{i\mathcal{F}}(m) + C_{n+1}(\mathcal{F}, m), \; C^Z_{i\mathcal{U}}(m) + C_{n+1}(\mathcal{U}, m))\}$$

$$i \epsilon \{\mathcal{F},\mathcal{U}\} \quad n=1,2\ldots\ldots N \quad m = 1, 2, 3 \qquad (1)$$

together with the final conditions

$C_{N+1}(\mathcal{F}, m) = C_{N+1}(\mathcal{U}, m) = 0$

.

This recursive relationship may be justified by noting that the cumulative total inventory costs $C^Z_{ij}(m) + C_{n+1}(j,m)$ resulting from reaching state j ϵ{ $\mathcal{F}$,$\mathcal{U}$ } at the start of period n+1 from state i ϵ{ $\mathcal{F}$,$\mathcal{U}$ } at the start of period *n* occurs with probability $Q^Z_{ij}(m)$.

The dynamic programming recursive equations become:

$$C_n(i,m) = \min_Z\{e^Z_i(m) + Q^Z_{i\mathcal{F}}(m)\, C_{n+1}(\mathcal{F},m) + Q^Z_{i\mathcal{U}}(m)\, C_{n+1}(\mathcal{U},m)\}$$

$$i\epsilon\{\mathcal{F}, \mathcal{U}\} \quad n= 1, 2\ldots\ldots N \qquad (2)$$

$$m = \{1, 2, 3\}$$

$$Z \epsilon \{0,1\}$$

$$C_N(i, m) = \min_Z\{e^Z_i(m)\} \qquad (3)$$

result where (3) represents the Markov chain stable state.

3.2.1 Computing $Q^Z(m)$, $C^Z(m)$ and $O^Z(m)$

The demand transition probability from state i$\epsilon\{\mathcal{F}, \mathcal{U}\}$ to state j $\epsilon\{\mathcal{F}, \mathcal{U}\}$,given ordering policy Z $\epsilon\{0,1\}$ may be taken as the number of customers for item m observed with demand initially in state i and later with demand changing to state j, divided by the sum of customers over all states. That is,

$$Q^Z_{ij}(m) = \frac{N^Z_{ij}(m)}{[N^Z_{i_{\mathcal{F}}}(m) + N^Z_{i_{\mathcal{U}}}(m)]}$$

$$i\epsilon\{\mathcal{F}, \mathcal{U}\}, \quad m = \{1, 2, 3\}, \quad Z \epsilon \{0,1\} \qquad (4)$$

When demand outweighs on-hand inventory, the inventory cost matrix $C^Z(m)$ may be computed by means of the relation

$$C^Z(m) = [c_0(m) + c_h(m) + c_s(m)][D^Z(m) - I^Z(m)]$$

where $c_0(m)$ denotes the unit ordering cost, $c_h(m)$ denotes the unit holding cost and $c_s(m)$ denotes the unit shortage cost.

Therefore,

$$C^Z_{ij}(m) = \begin{cases} [c_0(m)+c_h(m)+c_s(m)][D^Z_{ij}(m) - I^Z_{ij}(m)] \\ \text{if} \quad D^Z_{ij}(m) > I^Z_{ij}(m) \\ 0 \quad \text{if} \quad D^Z_{ij}(m) \leq I^Z_{ij}(m) \end{cases} \qquad (5)$$

for all i,j$\epsilon\{\mathcal{F}, \mathcal{U}\}$, Z$\epsilon\{0,1\}$ and m $\epsilon\{1,2,3\}$

A justification for expression (5) is that $D^Z_{ij}(m) - I^Z_{ij}(m)$ units must be ordered in order to meet the excess demand. Otherwise ordering is cancelled when demand is less than or equal to the on-hand inventory.

The following conditions must however, hold.

1. Z=1 when $c_0(m) > 0$ and Z = 0 when $c_0(m) = 0$
2. $c_s(m) > 0$ when shortages are allowed, and $c_s(m) = 0$ when shortages are not allowed.

4 Optimization

The optimal EOQ and ordering policy are found in this section for each time period separately.

4.1 Optimization during period 1

When demand is Favorable (ie. In state $\mathcal{F}$), the optimal ordering policy during period 1 is

$$Z = \begin{cases} 1 & \text{if} \quad e^1_{\mathcal{F}}(m) < e^0_{\mathcal{F}}(m) \\ 0 & \text{if} \quad e^1_{\mathcal{F}}(m) \geq e^0_{\mathcal{F}}(m) \end{cases}$$

The associated total inventory costs and EOQ are then

$$C_1(\mathcal{F}, m) = \begin{cases} e^1_{\mathcal{F}}(m) & \text{if } Z = 1 \\ e^0_{\mathcal{F}}(m) & \text{if } Z = 0 \end{cases}$$

and

$$O^Z_{\mathcal{F}}(m) = \begin{cases} [D^1_{\mathcal{F}\mathcal{F}}(m) - I^1_{\mathcal{F}\mathcal{F}}(m)] + [D^1_{\mathcal{F}\mathcal{U}}(m) \\ - I^1_{\mathcal{F}\mathcal{U}}(m)] & \text{if } Z = 1 \\ 0 & \text{if } Z = 0 \end{cases}$$

respectively. Similarly, when demand is Unfavorable (ie. in state $\mathcal{U}$), the optimal ordering policy during period 1 is

$$Z = \begin{cases} 1 & \text{if} \quad e^1_{\mathcal{U}}(m) < e^0_{\mathcal{U}}(m) \\ 0 & \text{if} \quad e^1_{\mathcal{U}}(m) \geq e^0_{\mathcal{U}}(m) \end{cases}$$

In this case, the associated total inventory costs and EOQ are

$$C_1(\mathcal{U}, m) = \begin{cases} e^1_{\mathcal{U}}(m) & \text{if } Z = 1 \\ e^0_{\mathcal{U}}(m) & \text{if } Z = 0 \end{cases}$$

and

$$O^Z_{\mathcal{U}}(m) = \begin{cases} [D^1_{\mathcal{U}\mathcal{F}}(m) - I^1_{\mathcal{U}\mathcal{F}}(m)] + [D^1_{\mathcal{U}\mathcal{U}}(m) \\ - I^1_{\mathcal{U}\mathcal{U}}(m)] & \text{if } Z = 1 \\ 0 & \text{if } Z = 0 \end{cases}$$

Using (2),(3) and recalling that $a^Z_i(m,2)$ denotes the already accumulated total inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$a^Z_i(m,2) = e^Z_i(m) + Q^Z_{i_{\mathcal{F}}}(m) \min\{e^1_{\mathcal{F}}(m), e^0_{\mathcal{F}}(m)\} + Q^Z_{i_{\mathcal{U}}}(m) \min\{e^1_{\mathcal{U}}(m), e^0_{\mathcal{U}}(m)\}$$
$$= e^Z_i(m) + Q^Z_{i_{\mathcal{F}}}(m)\, C_1(\mathcal{F}, m) + Q^Z_{i_{\mathcal{U}}}(m)\, C_1(\mathcal{U}, m)$$

4.2 Optimization during period 2

Using the dynamic programming recursive equation(1), and recalling that $a^Z_i(m)$ denotes the already accumulated total cost of item m at the end of period 1 as a result of decisions made during that period, when demand is favorable (ie. in state $\mathcal{F}$),the optimal ordering policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a^1_{\mathcal{F}}(m) < a^0_{\mathcal{F}}(m) \\ 0 & \text{if } a^1_{\mathcal{F}}(m) \geq a^0_{\mathcal{F}}(m) \end{cases}$$

While the associated total inventory costs and EOQ are

$$C_2(\mathcal{F}, m) = \begin{cases} a^1_{\mathcal{F}}(m) & \text{if } Z = 1 \\ a^0_{\mathcal{F}}(m) & \text{if } Z = 0 \end{cases}$$

and

$$O^Z_{\mathcal{F}}(m) = \begin{cases} [D^1_{\mathcal{FF}}(m) - I^1_{\mathcal{FF}}(m)] + [D^1_{\mathcal{FU}}(m) - I^1_{\mathcal{FU}}(m)] & \text{if } Z = 1 \\ 0 & \text{if } Z = 0 \end{cases}$$

respectively.

Similarly,when demand is unfavorable(ie. in state $\mathcal{U}$),the optimal ordering policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a^1_{\mathcal{U}}(m) < a^0_{\mathcal{U}}(m) \\ 0 & \text{if } a^1_{\mathcal{U}}(m) \geq a^0_{\mathcal{U}}(m) \end{cases}$$

In this case, the associated total inventory costs and EOQ are

$$C_2(\mathcal{U}, m) = \begin{cases} a^1_{\mathcal{U}}(m) & \text{if } Z = 1 \\ a^0_{\mathcal{U}}(m) & \text{if } Z = 0 \end{cases}$$

and

$$O^Z_{\mathcal{U}}(m) = \begin{cases} [D^1_{\mathcal{UF}}(m) - I^1_{\mathcal{UF}}(m)] + [D^1_{\mathcal{UU}}(m) - I^1_{\mathcal{UU}}(m)] & \text{if } Z = 1 \\ 0 & \text{if } Z = 0 \end{cases}$$

respectively.

5 Implementation

5.1 Case Description

In order to demonstrate use of the model in §3-4, a real case application from *Shoprite supermarket* in Uganda is presented in this section. The category of items examined included 400gms Full cream Milk powder, 250gms Black coffee and 250gms Cadbury Cocoa; whose demand fluctuate every week. The supermarket wants to avoid excess inventory when demand is Unfavorable (state $\mathcal{U}$) or running out of stock when demand is Favorable (state $\mathcal{F}$) and hence seeks decision support in terms of an optimal inventory policy, the associated inventory costs and specifically, a recommendation as to the EOQ of Full cream milk powder, Black coffee and Cadbury Cocoa over the next two-week period is required.

5.2 Data collection

Samples of customers were taken for each item. Past data revealed the following demand pattern and inventory levels of the three items over the first week of the month when demand was Favorable ($\mathcal{F}$) or Unfavorable ($\mathcal{U}$).

Considering Full cream milk powder (m =1), when additional units were ordered (Z=1),

$$N^1(1) = \begin{bmatrix} N^1_{FF}(1) & N^1_{FU}(1) \\ N^1_{UF}(1) & N^1_{UU}(1) \end{bmatrix}$$

$$N^1(1) = \begin{bmatrix} 91 & 71 \\ 64 & 13 \end{bmatrix}$$

$$D^1(1) = \begin{bmatrix} D^1_{FF}(1) & D^1_{FU}(1) \\ D^1_{UF}(1) & D^1_{UU}(1) \end{bmatrix}$$

$$D^1(1) = \begin{bmatrix} 156 & 115 \\ 107 & 11 \end{bmatrix}$$

$$I^1(1) = \begin{bmatrix} I^1_{FF}(1) & I^1_{FU}(1) \\ I^1_{UF}(1) & I^1_{UU}(1) \end{bmatrix}$$

$$I^1(1) = \begin{bmatrix} 95 & 93 \\ 93 & 94 \end{bmatrix}$$

When additional units were not ordered (Z=0),

$$N^0(1) = \begin{bmatrix} N^0_{FF}(1) & N^0_{FU}(1) \\ N^0_{UF}(1) & N^0_{UU}(1) \end{bmatrix}$$

$$N^0(1) = \begin{bmatrix} 82 & 50 \\ 56 & 25 \end{bmatrix}$$

$$D^0(1) = \begin{bmatrix} D^0_{FF}(1) & D^0_{FU}(1) \\ D^0_{UF}(1) & D^0_{UU}(1) \end{bmatrix}$$

$$D^0(1) = \begin{bmatrix} 123 & 78 \\ 78 & 15 \end{bmatrix}$$

$$I^0(1) = \begin{bmatrix} I^0_{FF}(1) & I^0_{FU}(1) \\ I^0_{UFF}(1) & I^0_{UU}(1) \end{bmatrix}$$

$$I^0(1) = \begin{bmatrix} 43.5 & 45 \\ 46.5 & 45.5 \end{bmatrix}$$

When additional units were ordered (Z=1) for Black coffee (m=2),

$$N^1(2) = \begin{bmatrix} N^1_{FF}(2) & N^1_{FU}(2) \\ N^1_{UF}(2) & N^1_{UU}(2) \end{bmatrix}$$

$$N^1(2) = \begin{bmatrix} 48 & 55 \\ 59 & 13 \end{bmatrix}$$

$$D^1(2) = \begin{bmatrix} D^1_{FF}(2) & D^1_{FU}(2) \\ D^1_{UF}(2) & D^1_{UU}(2) \end{bmatrix}$$

$$D^1(2) = \begin{bmatrix} 93 & 60 \\ 59 & 11 \end{bmatrix}$$

$$I^1(2) = \begin{bmatrix} I^1_{FF}(2) & I^1_{FU}(2) \\ I^1_{UF}(2) & I^1_{UU}(2) \end{bmatrix}$$

$$I^1(2) = \begin{bmatrix} 145 & 145 \\ 78.5 & 79.5 \end{bmatrix}$$

while these matrices were

$$N^0(2) = \begin{bmatrix} N^0_{FF}(2) & N^0_{FU}(2) \\ N^0_{UF}(2) & N^0_{UU}(2) \end{bmatrix}$$

$$N^0(2) = \begin{bmatrix} 54 & 46 \\ 45 & 11 \end{bmatrix}$$

$$D^0(2) = \begin{bmatrix} D^0_{FF}(2) & D^0_{FU}(2) \\ D^0_{UF}(2) & D^0_{UU}(2) \end{bmatrix}$$

$$D^0(2) = \begin{bmatrix} 72 & 77 \\ 75 & 11 \end{bmatrix}$$

$$I^0(2) = \begin{bmatrix} I^0_{FF}(2) & I^0_{FU}(2) \\ I^0_{UF}(2) & I^0_{UU}(2) \end{bmatrix}$$

$$I^0(2) = \begin{bmatrix} 81 & 78.5 \\ 79.5 & 78.5 \end{bmatrix}$$

for the case when additional units were not ordered.

When additional units were ordered (Z=1) for Cadbury cocoa (m=3),

$$N^1(3) = \begin{bmatrix} N^1_{FF}(3) & N^1_{FU}(3) \\ N^1_{UF}(3) & N^1_{UU}(3) \end{bmatrix}$$

$$N^1(3) = \begin{bmatrix} 57 & 62 \\ 62 & 9 \end{bmatrix}$$

$$D^1(3) = \begin{bmatrix} D^1_{FF}(3) & D1_{FU}(3) \\ D^1_{UF}(3) & D1_{UU}(3) \end{bmatrix}$$

$$D^1(3) = \begin{bmatrix} 82 & 93 \\ 84 & 9 \end{bmatrix}$$

$$I^1(3) = \begin{bmatrix} I^1_{FF}(3) & I^1_{FU}(3) \\ I^1_{UF}(3) & I^1_{UU}(3) \end{bmatrix}$$

$$I^1(3) = \begin{bmatrix} 67.5 & 68.5 \\ 72 & 68.5 \end{bmatrix}$$

while these matrices were

$$N^0(3) = \begin{bmatrix} N^0_{FF}(3) & N^0_{FU}(3) \\ N^0_{UF}(3) & N^0_{UU}(3) \end{bmatrix}$$

$$N^0(3) = \begin{bmatrix} 36 & 53 \\ 56 & 9 \end{bmatrix}$$

$$D^0(3) = \begin{bmatrix} D^0_{FF}(3) & D^0_{FU}(3) \\ D^0_{UF}(3) & D^0_{UU}(3) \end{bmatrix}$$

$$D^0(3) = \begin{bmatrix} 51 & 70 \\ 72 & 10 \end{bmatrix}$$

$$I^0(3) = \begin{bmatrix} I^0_{FF}(3) & I^0_{FU}(3) \\ I^0_{UF}(3) & I^0_{UU}(3) \end{bmatrix}$$

$$I^0(3) = \begin{bmatrix} 105 & 100 \\ 100 & 50 \end{bmatrix}$$

for the case when additional units were not ordered.

The following unit ordering, holding and shortage costs (in UGX) were captured for each individual product at the supermarket:

400 gms Full Cream Milk powder (m=1)
$c_0(1) = 4500$, $c_h(1) = 1200$, $c_s(1) = 300$

250 gms Black Coffee (m=2)
$c_0(2) = 4800$, $c_h(2) = 900$, $c_s(2) = 300$

250 gms Cadbury Cocoa (m=3)
$c_0(3) = 5100$, $c_h(3) = 600$, $c_s(3) = 300$

$$Q^0(1) = \begin{bmatrix} 0.6212 & 0.3722 \\ 0.6914 & 0.3086 \end{bmatrix}$$

$$Q^0(2) = \begin{bmatrix} 0.5400 & 0.4600 \\ 0.8036 & 0.1964 \end{bmatrix}$$

$$Q^0(3) = \begin{bmatrix} 0.404 & 0.596 \\ 0.862 & 0.138 \end{bmatrix}$$

$$C^0(1) = \begin{bmatrix} 0.477 & 0.198 \\ 0.189 & 0.037 \end{bmatrix}$$

$$C^0(2) = \begin{bmatrix} 0.008 & 0.001 \\ 0.004 & 0.061 \end{bmatrix}$$

$$C^0(3) = \begin{bmatrix} 0.049 & 0.027 \\ 0.025 & 0.036 \end{bmatrix}$$

for the case when additional units are *not* ordered(Z=0).

5.3 Solution procedure for Milk powder, Black Coffee and Cadbury Cocoa

Using (4) and (5), the state transition matrices and Inventory cost matrices for each respective item in week 1 are

$$Q^1(1) = \begin{bmatrix} 0.5697 & 0.4303 \\ 0.8312 & 0.1688 \end{bmatrix}$$

$$Q^1(2) = \begin{bmatrix} 0.4660 & 0.5340 \\ 0.8429 & 0.1571 \end{bmatrix}$$

$$Q^1(3) = \begin{bmatrix} 0.4790 & 0.5210 \\ 0.8732 & 0.1268 \end{bmatrix}$$

$$C^1(1) = \begin{bmatrix} 0.366 & 0.132 \\ 0.084 & 0.025 \end{bmatrix}$$

$$C^1(2) = \begin{bmatrix} 0.047 & 0.077 \\ 0.018 & 0.062 \end{bmatrix}$$

$$C^1(3) = \begin{bmatrix} 0.087 & 0.147 \\ 0.072 & 0.054 \end{bmatrix}$$

When additional units are ordered (Z = 1), the matrices $Q^1(1)$, $C^1(1)$, $Q^1(2)$, $C^1(2)$, $Q^1(3)$ and $C^1(3)$ yield the costs(in million UGX)

$$e^1_F(1) = (0.562)(0.366) + (0.438)(0.132) = 0.2634$$
$$e^1_U(1) = (0.831)(0.084) + (0.169)(0.025) = 0.074$$
$$e^1_F(2) = (0.4660)(0.047) + (0.5340)(0.077) = 0.0627$$
$$e^1_U(2) = (0.8194)(0.0176) + (0.1804)(0.0617) = 0.0256$$
$$e^1_F(3) = (0.479)(0.087) + (0.521)(0.147) = 0.118$$
$$e^1_U(3) = (0.8732)(0.072) + (0.1268)(0.0536) = 0.0697$$

However, when additional units are *not* ordered (Z= 0), the matrices Q^0 (1), C^0(1) , Q^0 (2) , C^0(2) , Q^0 (3) and C^0(3) yield the costs (in million UGX)

$$e_F^0(1) = (0.6212)(0.477) + (0.3788)(0.198) = 0.3713$$
$$e_U^0(1) = (0.6914)(0.189) + (0.3086)(0.037) = 0.1421$$
$$e_F^0(2) = (0.540)(0.008) + (0.460)(0.0014) = 0.005$$
$$e_U^0(2) = (0.804)(0.004) + (0.196)(0.0608) = 0.0152$$
$$e_F^0(3) = (0.4045)(0.0486) + (0.5955)(0.027) = 0.0357$$
$$e_U^0(3) = (0.8615)(0.017) + (0.1385)(0.036) = 0.0196$$

The results are summarized in Tables 1 and 2 below:

Table 1:
Values of Z , $e^Z_i(m)$ and $O^Z_i(m,n)$ for items during week 1(n=1)

Full Cream Milk Powder (m=1)	**Z = 1**	**Z = 0**
$e^Z_f(1)$	0.265	0.371
$e^Z_u(1)$	0.087	0.142
$O^Z_f(1,1)$	83	0
$O^Z_u(1,1)$	14	0
$e^Z_f(1)$	0.265	0.371
Black Coffee (m=2)	**Z = 1**	**Z = 0**
$e^Z_f(2)$	0.063	0.005
$e^Z_u(2)$	0.026	0.015
$O^Z_f(2,1)$	0	0
$O^Z_u(2,1)$	0	0
Cadbury Cocoa (m=3)	**Z = 1**	**Z = 0**
$e^Z_f(3)$	0.118	0.024
$e^Z_u(3)$	0.067	0.018
$O^Z_f(3,1)$	0	0
$O^Z_u(3,1)$	0	0

The cumulative total costs $a^Z_i(m,n)$ are computed using (1) for week 2 and results are summarized in Table 2 below:

Table 2:
Values of Z , $a^Z_i(m,n)$ and $O^Z_i(m,n)$ during week 2(n=2)

Full Cream Milk Powder (m=1)	**Z = 1**	**Z = 0**
$a^Z_f(1,2)$	0.265	0.371
$a^Z_u(1,2)$	0.087	0.142
$O^Z_f(1,2)$	83	0
$O^Z_u(1,2)$	14	0
Black Coffee (m=2)	**Z = 1**	**Z = 0**
$a^Z_f(2,2)$	0.073	0.015
$a^Z_u(2,2)$	0.032	0.022
$O^Z_f(2,2)$	0	0
$O^Z_u(2,2)$	0	0
Cadbury Cocoa (m=3)	**Z = 1**	**Z = 0**
$a^Z_f(3,2)$	0.139	0.044
$a^Z_u(3,2)$	0.090	0.041
$O^Z_f(3,2)$	0	0
$O^Z_u(3,2)$	0	0

5.4 The Optimal ordering policy and EOQ

Week1

Full Cream Milk powder

Since 0.265 < 0.371, it follows that Z=1 is an optimal ordering policy for week 1 with associated total inventory costs of 0.265 million UGX and an EOQ of (156- 95) + 115-93) = 83 units when demand is favorable. Since 0.087< 0.142, it follows that Z=1 is an optimal ordering policy for week 1 with associated total inventory costs of 0.087 million UGX and an EOQ of (107 – 93) = 14 units if demand is unfavorable.

Black Coffee

Since 0.005 < 0.063, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.005 million UGX when demand is favorable. Since 0.015 < 0.026, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.015 million UGX if demand is unfavorable.
EOQ = 0 units regardless of the state of demand.

Cadbury Cocoa

Since 0.024 < 0.118, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.024 million UGX when demand is favorable. Since 0.018 < 0.067, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.018 million UGX if demand is unfavorable.

Week 2

Full Cream Milk Powder

Since 0.449 < 0.568, it follows that Z=1 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.449 million UGX and an EOQ of (156- 95) + 115-93) = 83 units when demand is favorable. Since 0.320 < 0.531, it follows that Z=1 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.320 million UGX and an EOQ of (107 – 93) = 14 units if demand is unfavorable.

Black coffee

Since 0.015 < 0.073, it follows that Z=0 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.015 million UGX when demand is favorable. Since 0.022 < 0.032, it follows that Z=0 is an optimal ordering policy for week 2 with associated total inventory costs of 0.022 million UGX if demand is unfavorable. In this case, EOQ = 0 regardless of the state of demand.

Cadbury Cocoa

Since 0.044 < 0.139, it follows that Z=0 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.044 million UGX when demand is favorable. Since 0.041 < 0.090, it follows that Z=0 is an optimal ordering policy for week 1 with associated accumulated inventory costs of 0.041 million UGX if demand is unfavorable; and EOQ = 0 regardless of the state of demand.

6 Conclusion

An inventory model with stochastic demand was presented in this paper. The model determines an optimal ordering policy, inventory costs and the EOQ of a multi-item inventory problem with stochastic demand. The decision of whether or not to order additional stock units is modeled as a multi-period decision problem using dynamic programming over a finite planning horizon. The working of the model was demonstrated by means of a real case study. It would however be worthwhile to extend the research and examine the behavior of EOQ for items under non stationary demand conditions. In the same spirit, our model raises a number of salient issues to consider: Lead time of items during replenishment and customer response to abrupt changes in price of items. Finally, special interest is thought in further extending our model by considering EOQ determination in the context of Continuous Time Markov Chains (CTMC).

Acknowledgments

The authors wish to thank Makerere-Sida Bilateral Research program for the financial support and the staff of Shoprite supermarket during the data collection exercise.

REFERENCES

[1] Tadashi D,Takeshi S etal A comparative study of the structure of EOQ models with discounting, *IMA Journal of Management Mathematics*,5(2),171-179, 1993.
[2] Berman O & Perry D. Two control policies for stochastic EOQ- type models, *Probability in the Engineering and Information sciences*, 20(2), 151-163, 2006.
[3] Yao Z & Kateshakis M. On the structure of optimal ordering policies for stochastic inventory systems with minimum order quantity, *Probability in the Engineering and Information sciences*, 20(2), 151-163, 2006.
[4] Broekmeullen & Van Donselarr. Inventory control of perishables in supermarkets. *International Journal of Production Economics, 104(2),* 462-472, 2006
[5] Rowchowdhury, S.. An optimal policy for a stochastic inventory model for deteriorating items with time-dependent selling price, *Advanced modeling and optimization,* 11(3).182-191, 2009.