An EOQ Inventory Model For Two Parameter Weibull Deterioration With Time Dependent Demand And Shortages

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Abstract

In this paper, we developed an inventory model with time dependent demand rate and Weibull deterioration. The demand rate is a linear function of time. Deterioration rate is taken as two parameter Weibull distribution deterioration. In this study, shortage are allowed and completely backlogged. The objective of this model is to maximize the total profit. The numerical example of the model is provided to illustrate the problem.

Key Words: Inventory system**,** Deterioration, Weibull Distribution of two parameters, Time Dependent Demand, Shortages.

Introduction

Deterioration has a significant role in many inventory systems. Deterioration is defined as damage, declension, worsening, corrosion and loss of the utility of the product. However certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. For example, the commonly used goods like fruits, vegetables, meat, foodstuffs, gasoline, radioactive substances, electronic components etc. where deterioration is usually observed during the normal storage period. The loss due to deterioration can not be ignored.

Covert and Phillip **[1]** developed an inventory model for items with Weibull distribution deterioration. An inventory model with inventory-level-dependent demand rate, shortages and decrease in demand was developed by Jain and Kumar **[6].** RoyChaudhuri **[8]** proposes a production-inventory model under stock-dependent demand, Weibull distribution deterioration and shortage. Tripathy and Pradhan **[4]** developed an integrated Partial Backlogging Inventory Model having weibull Demand and Variable Deterioration rate with the effect of Trade credit. Chu and Chen **[5]** considered an inventory model for deteriorating items and time-varying demand. Wee [**2]** developed a deterministic inventory model for deteriorating with shortages and a declining market. Goyal and Giri **[7]** developed recent trends in modelling of deteriorating inventory. Hung **[10]** proposes an inventory model with generalized type demand deterioration and backorder rates.

In this model demand is taken as time dependent & deterioration is taken as weibull distribution with two parameter. Shortage are allowed & completely lost in this paper. A numerical example is also shown in this paper.

Assumptions and Notations

The proposed mathematical model of inventory replenishment policy is developed under the following notations and assumptions:

Assumptions:

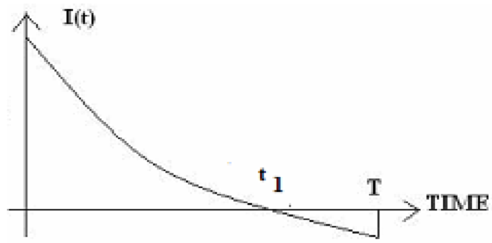
- Lead time is zero
- The inventory system involves only one time.
- The demand rate is time dependent.
- Shortage are allowed & completely lost.
- The length of the cycle is T.
- $\theta(t) = \alpha\beta t^{(\beta-1)}$ is the weibull two parameter deterioration rate, where $0 < \alpha < 1$ and $\beta > 0$ .are called scale and shape parameter respectively.

Notations:

- A: Setup cost
- C_1 : Shortage cost / Unit/ Unit time
- C_2 : Deterioration cost / Unit/ Unit time
- h : Holding cost/Unit/Unit time
- T: Duration of a cycle
- t_1 : The time at which the inventory level becomes zero.
- S: Selling Price
- α : Scale parameter
- β : Shape parameter
- θ : Deterioration rate
- D(t): a+bt, where a>b

Mathematical Formulation of the model:

The length of the cycle is T. At the time t_1 the inventory level becomes zero and shortages occurring in the period (t_1, T) which is completely backlogged. Let I (t) be the inventory level at time t ($0 \le t \le t_1$). The differential equations for the instatantaneous state over (0, T) are given by



$$\frac{dI(t)}{dt}+\alpha\beta t^{(\beta-1)}I(t)=-(a+bt), 0\le t\le t_1 \ldots\ldots (1)$$

$$\frac{dI(t)}{dt}=-(a+bt), t_1\le t\le T \ldots\ldots (2)$$

With the boundary condition $I(t_1)=0$ at $t=t_1$.

Solving equation (1) & (2) we get

$$I(t)=(1-\alpha t^{\beta})[a(t_1-t)+\frac{b}{2}(t_1^{2}-t^{2})+\frac{a\alpha}{\beta+1}(t_1^{(\beta+1)}-t^{(\beta+1)})+\frac{b\alpha}{\beta+2}(t_1^{(\beta+2)}-t^{(\beta+2)})], 0\le t\le t_1 \ldots\ldots (3)$$

$$I(t)=[a(t_1-t)+\frac{b}{2}(t_1^{2}-t^{2})], t_1\le t\le T \ldots\ldots (4)$$

The total holding cost during the time period 0 to t_1

HC= $h\int_0^{t_1} I(t)dt$

HC= $h\int_0^{t_1}(1-\alpha t^{\beta})[a(t_1-t)+\frac{b}{2}(t_1^{2}-t^{2})+\frac{a\alpha}{\beta+1}(t_1^{(\beta+1)}-t^{(\beta+1)})+\frac{b\alpha}{\beta+2}(t_1^{(\beta+2)}-t^{(\beta+2)})]dt$

$$\text{HC}=h[\frac{at_1^{2}}{2}+\frac{bt_1^{3}}{3}+\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{(\beta+2)}+\frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{(\beta+3)}+\frac{a\alpha}{2(\beta+1)^2}t_1^{(2\beta+2)}+\frac{b\alpha}{(\beta+1)(2\beta+3)}t_1^{(2\beta+3)}] \ldots\ldots (5)$$

The total Shortage cost during the time period t_1 to T is given by

SC= $-c_1\int_{t_1}^{T} I(t)dt$

SC= $-c_1\int_{t_1}^{T}[a(t_1-t)+\frac{b}{2}(t_1^{2}-t^{2})]dt$

$$\text{SC}=[\frac{c_1a}{2}(t_1-T)^2+\frac{c_1b}{6}(2t_1^3+T^3-3Tt_1^2)] \ldots\ldots (6)$$

The total Deterioration cost during the time period 0 to t_1 is given by

DC= $c_2\int_0^{t_1} \theta(t).I(t)dt$

DC=

$$c_2\int_0^{t_1} \alpha\beta t^{(\beta-1)}(1-\alpha t^{\beta})[a(t_1-t)+\frac{b}{2}(t_1{}^2-t^2)+\frac{a\alpha}{\beta+1}(t_1{}^{(\beta+1)}-t^{(\beta+1)})+\frac{b\alpha}{\beta+2}(t_1{}^{(\beta+2)}-t^{(\beta+2)})]dt$$

$$\text{DC}=\frac{at_1}{\beta+1}t_1^{\beta+1}+\frac{3a\alpha}{2\beta(2\beta+1)}t_1^{2\beta+1}+\frac{b}{\beta(\beta+2)}t_1^{\beta+2}-\frac{b\alpha}{\beta(2\beta+2)}t_1^{2\beta+2}-\frac{a\alpha^2}{2\beta(3\beta+1)}t_1^{3\beta+1}$$
$$-\frac{b\alpha^2}{2\beta(3\beta+2)}t_1^{3\beta+2} \quad \ldots\ldots (7)$$

$$\text{Sales Revenue} = \text{s}\int_0^{\text{T}}(\text{a+bt})\text{dt} = s(aT+b\frac{T^2}{2}) \quad \ldots\ldots (8)$$

From equation (5), (6), (7) & (8) the total profit per unit time is

$$P(T,t_1)=\frac{1}{T}[s.(aT+b\frac{T^2}{2})]-\frac{1}{T}[A+HC+SC+DC]$$

$$P(T,t_1)=s(a+b\frac{T}{2})-\frac{1}{T}[A+h\{\frac{at_1{}^2}{2}+\frac{bt_1{}^3}{3}+\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1{}^{(\beta+2)}+\frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1{}^{(\beta+3)}$$
$$+\frac{a\alpha}{2(\beta+1)^2}t_1{}^{(2\beta+2)}+\frac{b\alpha}{(\beta+1)(2\beta+3)}t_1{}^{(2\beta+3)}\}+\{\frac{c_1a}{2}(t_1-T)^2+\frac{c_1b}{6}(2t_1^3+T^3-3Tt_1^2)\}$$
$$+\{\frac{at_1}{\beta+1}t_1^{\beta+1}+\frac{3a\alpha}{2\beta(2\beta+1)}t_1^{2\beta+1}+\frac{b}{\beta(\beta+2)}t_1^{\beta+2}-\frac{b\alpha}{\beta(2\beta+2)}t_1^{2\beta+2}-\frac{a\alpha^2}{2\beta(3\beta+1)}t_1^{3\beta+1}$$
$$-\frac{b\alpha^2}{2\beta(3\beta+2)}t_1^{3\beta+2}\}] \quad \ldots\ldots (9)$$

Our objective is to maximize the total profit. The necessary condition for maximize the profit are

$$\frac{\partial p(T,t_1)}{\partial T}=0 \text{ And } \frac{\partial p(T,t_1)}{\partial t_1}=0 \text{ then}$$

$$\frac{bs}{2}-\frac{-ac_1(-T+t_1)+\frac{1}{6}bc_1(3T^2-3t_1^2)}{T}+\frac{1}{T^2}(A+\frac{at_1^{2+\beta}}{1+\beta}+\frac{bt_1^{2+\beta}}{\beta(2+\beta)}+\frac{3a\alpha t_1^{1+2\beta}}{2\beta(1+2\beta)}-\frac{b\alpha t_1^{2+2\beta}}{\beta(2+2\beta)}$$
$$-\frac{a\alpha^2t_1^{1+3\beta}}{2\beta(1+3\beta)}+\frac{b\alpha^2t_1^{2+3\beta}}{2\beta(2+3\beta)}+\frac{1}{2}ac_1(-T+t_1)^2+\frac{1}{6}bc_1(T^3-3Tt_1^2+2t_1^3)+h(\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{a\alpha\beta t_1^{2+\beta}}{(1+\beta)(2+\beta)}$$

$$+\frac{b\alpha\beta t_1^{3+\beta}}{(1+\beta)(3+\beta)}+\frac{a\alpha t_1^{2+2\beta}}{2(1+\beta)^2}+\frac{b\alpha t_1^{3+2\beta}}{(1+\beta)(3+2\beta)}))=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(10)}$$

And

$$-\frac{1}{T}(\frac{3\alpha a t_1^{2\beta}}{2\beta}-\frac{\alpha^2 a t_1^{3\beta}}{2\beta}+\frac{b t_1^{1+\beta}}{\beta}+\frac{a(2+\beta)t_1^{1+\beta}}{(1+\beta)}-\frac{b\alpha t_1^{1+2\beta}}{\beta}+\frac{b\alpha^2 t_1^{1+3\beta}}{2\beta}+ac_1(-T+t_1)+\frac{1}{6}bc_1$$
$$(-6Tt_1+6t_1^2)+h(at_1+bt_1^2+\frac{a\alpha\beta t_1^{1+\beta}}{1+\beta}+\frac{a\alpha(2+2\beta)t_1^{1+2\beta}}{2(1+\beta)^2}+\frac{b\alpha t_1^{2+2\beta}}{1+\beta}))=0 \quad \ldots. \text{(11)}$$

Provided

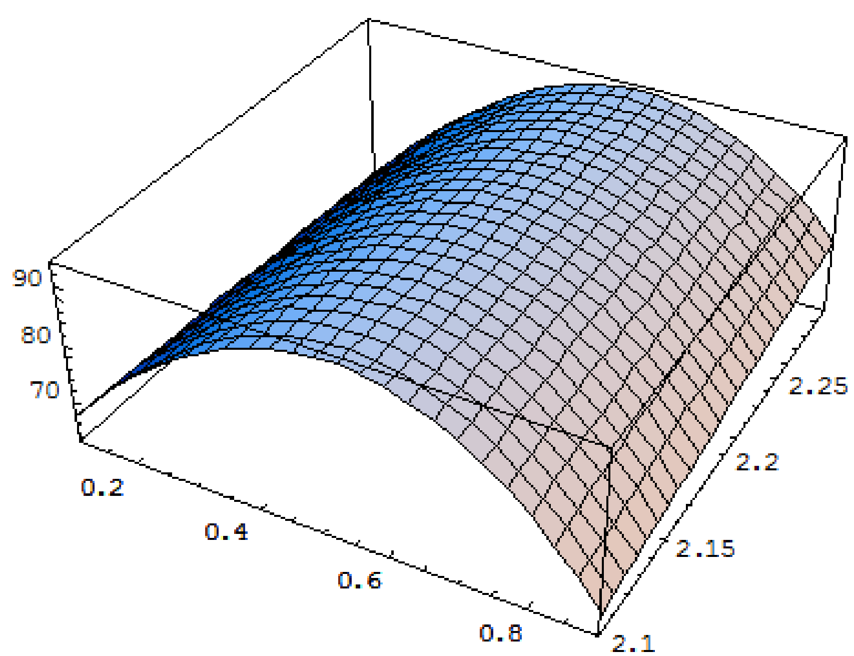
$$(\frac{\partial^2 P}{\partial T^2})<0,(\frac{\partial^2 P}{\partial t_1^2})<0 \text{ And } (\frac{\partial^2 P}{\partial T^2}).(\frac{\partial^2 P}{\partial t_1^2})-(\frac{\partial^2 P}{\partial T\partial t_1})>0$$

Using the software mathematica-5.2, we can find the optimal values of T^* and t_1^* by equation no. (10) & (11) simultaneously and also find the optimal value of $P^*(T,t_1)$ by equation no. (9).

Numerical Example & Sensitivity Analysis

We consider $[A,a,b,s,c_1,c_2,h,\alpha,\beta]$= [200, 100, 76, 2, 1.3, 1.1, 1.0, 0.1, 0.3] in proper units, where h, c_1 & c_2 are holding cost, shortage cost and deterioration cost respectively.

The optimal value of T^*= 2.21261, t_1^* = 0.550549 & $P^*(T,t_1)$ = 93.3771



This is the concavity of the total cost w.r.t. to T^* and t_1^*.

The variation in the parameter is as follows

Table- (1) Variation in parameter a

a	t_1	T	$P(T,t_1)$
100	0.550549	2.21	93.3771
120	0.526357	2.08115	113.514
140	0.503424	1.96326	134.837
160	0.482039	1.85775	157.217

Table- (2) Variation in parameter b

b	t_1	T	$P(T,t_1)$
76	0.550549	2.21261	93.3771
78	0.549563	2.21476	95.6483
80	0.548588	2.21685	97.9206
82	0.547624	2.2189	100.194

Table- (3) Variation in scale parameter α

α	t_1	T	$P(T,t_1)$
0.1	0.550549	2.21261	93.3771
0.2	0.504696	2.19272	88.9163
0.3	0.462959	2.17396	85.1296
0.4	0.425271	2.15656	81.916

Table- (4) Variation in shape parameter β

β	t_1	T	$P(T,t_1)$
0.3	0.550549	2.21261	93.3771
0.4	0.639018	2.25639	101.374
0.5	0.701465	2.28739	107.353
0.6	0.747829	2.3102	112.02

Table- (5) Variation in selling price s

s	t_1	T	$P(T,t_1)$
2	0.550549	2.21261	93.3771
2.5	0.634548	2.52823	188.313
3.0	0.738288	2.90943	289.867
3.5	0.862504	3.35414	399.276

Observations:

- From the table (1), we observed that total profit increases if we increase the parameter a.
- From the table (2), we observed that total profit increases if we increase the parameter b.
- From the table (3), we observed that total profit decrease if we increase the scale parameter α .
- From the table (4), we observed that total profit increases if we increase the shape parameter β .
- From the table (5), we observed that total profit increases if we increase the selling price s.

Concluding Remarks

In this paper, we developed an inventory model for deteriorating items with time dependent demand and shortages. The rate of deterioration follows the Weibull distribution with two parameters. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. The shortage cost, holding cost and, deterioration cost are considered in this model. A numerical example and sensitivity analysis are presented to illustrate the proposed model.

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