An Efficient Linear Precoding Scheme Based on Block Diagonalization for Multiuser MIMO Downlink System

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Abstract

Block diagonalization (BD) based on singular value decomposition (SVD) is a well-known precoding method that eliminates inter user interference in multiuser multi-input multi-output (MIMO) broadcast channels, but it is computationally inefficient. In this paper we examine the use of generalized singular value decomposition (GSVD) for coordinated beam-forming in MIMO systems. GSVD facilitates joint decomposition of a class of matrices from source to destination MIMO broadcast scenarios. GSVD allows two channels of suitable dimensionality to be jointly diagonalized, through the use of jointly determined transmit precoding and receiver reconstruction matrices. Simulation results demonstrate that the proposed GSVD technique achieves considerable gains in terms of bit error rate (BER) over SVD technique with traditional interference aware block diagonalization (BD) scheme. Analysis and simulation results show that our proposal has lower complexity than the conventional BD method, with a significant increase there in sum throughput.

1. Introduction

Multiple-input multiple-output communication systems have attracted attention in recent years. Such multiple antennas potentially allow higher throughput, increased diversity, and reduced interference as they communicate with multiple wireless users [1]. In the multiuser multiple-input multiple-output (MIMO) downlink system, a base station with multiple antennas transmits data streams to multiple mobile users, each with one or more receive antennas. Due to the fact that the co-channel interference is hard to be managed by the receiver operation, and more importantly, capacity of interference channel can be made equivalent to capacity without interference, pre-cancellation of the interference via the precoding at the transmitter has received much attention in recent years [2].

In [3][4], it was shown that the maximum sum rate in the multiuser MIMO broadcast channels (BC) can be achieved by dirty paper coding (DPC). However, the DPC is difficult to implement in practical systems due to its high computational complexity. As a suboptimal strategy of the DPC [5], the Tomlinson-Harashima precoding, is still not practical due to its complexity, since this algorithm is based on nonlinear modulo operations.

In linear processing systems, several practical precoding techniques have been proposed, typically as the channel inversion method [6], the block diagonalization (BD) method [7]. A zero-forcing channel inversion (ZF) scheme [6] can suppress CCI completely for the case where all users employ a single antenna. However its performance is rather poor due to a transmit power boost issue. Although a minimum mean-squared error channel inversion (MMSE) method [6] overcomes the drawback of the ZF, this is still confined to a single receive antenna case. For the case where the users in the network have multiple antennas, the block diagonalization (BD) is well-known precoding algorithm [7]. As a generalization of the ZF scheme, the BD can eliminate the multiuser interference (MUI) completely. Owing to the singular value decomposition (SVD) in the algorithm, the BD is not computationally efficient.

In this paper, we present a new generalized singular value decomposition based on block diagonalization interference suppression precoding (GSVD) scheme for downlink multi-stream MU-MIMO channel. As a new approach of the conventional BD-SVD scheme, BD-GSVD shows a significant advantage in computational complexity. From the sum rate analysis as well as the bit error rate simulations in multiuser MIMO downlink, we show that the proposed BD-GSVD brings substantial performance gain over existing multiuser MIMO algorithms scheme.

The rest of this paper is organized as follows. In Section II the system model and the summary of the conventional BD-

SVD technique is described. In Section III, we present the proposed GSVD-BD method. The complexity analysis is given in section IV. We present the simulation results in Section V and conclude in Section VI.

2. Multiuser MIMO Downlink

In this section, we present the system model for single cell multiuser MIMO downlink and then review the BD technique.

2.1 System Model

In the multiuser multiple-input multiple-output (MIMO) downlink system, a base station with multiple antennas transmits data streams to multiple mobile users, each with one or more receive antennas [8]. Consider a single-cell multiuser MIMO downlink system with independent users and a base station. A base station is equipped with N_T antennas transmits information to K mobile users. Each mobile user has $N_{R,k}$ receive antennas and thus the total number of receive antennas is $N_R = \sum_{k=1}^{K} N_{R,k}$. In the sequel, the notation $(N_{R,1}, \ldots, N_{R,K}) \times N_T$ to describe the multiuser system MIMO downlink system. The downlink channel between the base station and user is described by the matrix $H = [H_1^T\ H_2^T\ \ldots\ H_K^T]^T$ is perfectly known at the base station. The downlink channel between the base station and user is described by the matrix $H_k \in \mathbb{C}^{N_k \times N_T}$, In the MIMO BC, the signal received at user k can be written as
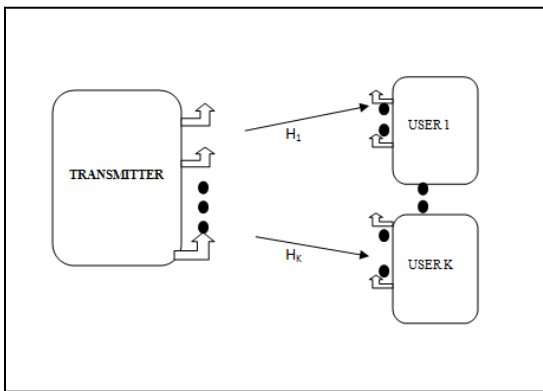

Figure1. Multiuser MIMO Downlink Chanel

$$y_k = H_k W_k x_k + \sum_{j=1}^{K} H_k W_j x_j + n_k \qquad (1)$$

Where x_k represents information data of user k, W_k is an $N_T \times 1$ precoding vector associated with user k and denotes the complex-valued additive white Gaussian noise at user k, which has a variance of σ^2. Note that the precoding vectors are normalized to be unity, i.e., $\|W_k\|^2 = 1$ for k=1,…, K. Furthermore, the power allocated to user k is given by $P_k = E(x\ x^H)$ and a sum power constraint is applied at the BS, corresponding to $\sum_{k=1}^{K} P_k \leq P$, the second term on the right hand side represents the co-channel interference (CCI), which is desirable to be eliminated by designing appropriate precoding algorithms.

2.2 Review of conventional BD-SVD method

The BD algorithm is an extension of zero-forcing method for multiuser MIMO systems where each user has multiple antennas. Each user's linear precoder and receive filter can be obtained by twice SVD operations. In the first SVD operation, we get precoding matrix which can eliminate the other user's interference. In the second SVD operation, each user's block channel is decomposed into parallel subchannels in order to optimize the power allocation for each user N_k The key idea of the BD algorithm is to employ the precoding matrix W to suppress the MUI completely. To eliminate all MUI, the following constraint is improved.

$$H_k W_k = 0 \qquad (2)$$

Let the SVD of H_k be

$$H_k = U_k \Sigma_k [\ V_{k,1}\ V_{k,0}\ ]^H \qquad (3)$$

Where U_k and Σ_k are matrices of the left singular vectors and the ordered singular values of H_k, respectively, and $V_{k,1}$ & $V_{k,0}$ are right singular matrices corresponding to non zero singular values and zero singular values, respectively. We choose $V_{k,0}$ as an orthonormal basis for null space of H_k. $H_k V_{k,0}$ is the equivalent channel for user k after elimination MUI, these equivalent channel has the same properties as a conventional SU-MIMO channel. In order to maximize the achieve sum rate of the BD, the water filling algorithm can be additionally incorporated. Define the SVD of $H_k V_{k,0}$ is

$$H'\ V_{k,0} = U_k'\ \Sigma_k'\ [\ V_{k,1}'\ V_{k,0}'\ ]^H \qquad (4)$$

Thus we define the precoding matrix as

$$W^{BD\text{-}SVD} = [V_{1,0}\ V_{1,1}'\ \ V_{2,0}\ V_{2,1}'\ \ldots\ V_{k,0}\ V_{k,1}'\ ]\ \wedge^{1/2} \qquad (5)$$

where $\wedge$ is a diagonal matrix whose element λ_i scale the power transmitted into each of columns of $W^{BD\text{-}SVD}$. With $W^{BD\text{-}SVD}$ chosen in (5), the capacity of the BD-SVD is [9].

$$C_{BD\text{-}SVD} = \max \log_2 |\ I + \sum^2 \frac{\wedge}{\sigma^2}\ | \qquad (6)$$

Where $\Sigma = \text{diag}(\Sigma_1', \ldots, \Sigma_k')$ the optimal power loading coefficient in $\wedge$ are determined by using water-filling on the diagonal elements of Σ under the power constraint.

3. Proposed BD-GSVD Scheme

Generalized singular value decomposition is a joint matrix decomposition technique. Consider two matrices, $H_k \in \mathbb{C}^{m \times n}$

and $H_{k+1} \in \mathbb{C}^{p \times n}$ for m>n. The GSVD of the matrices is given by pair of factorizations.

$$H_k = U_k \Sigma_1 V^T \qquad H_{k+1} = U_{k+1} \Sigma_2 V^T \qquad (7)$$

The matrices in this factorization have following properties: U_k is an (m × m) orthogonal matrix, U_{k+1} is a (p × p) orthogonal matrix, and V is an (n × n) non singular matrix. The columns of V are called the right singular vectors. The columns of U are called the left singular vectors. Σ_1 is an (m × r) and Σ_2 is a (p × r) real non-negative and diagonal matrix.

$$\Sigma_1 = \text{diag}[\beta_1 , \ldots \beta_r] \qquad (8)$$

$$\Sigma_2 = \text{diag}[\lambda_1 , \ldots \lambda_r] \qquad (9)$$

The ratios $\mu_r = \beta_r / \lambda_r$ are called the generalized singular values of the matrix pair H_k, H_{k+1}[10]. In our GSVD calculation we are only using V vector for both the matrix i.e. common orthogonal basis for null spaces for both the matrix and the reason for using this matrix is that only V satisfy the condition with H_k ,H_{k+1} and all other arithmetic operations. The V_r (the columns of V) are referred to as 'generalized singular vectors'. Thus we define the precoding matrix as

$$W^{BD\text{-}GSVD} = \alpha \, V^{-1} \qquad (10)$$

where α is chosen to satisfy the power constraint. The capacity for BD-GSVD is given by

$$C_{BD\text{-}GSVD} = \max \log_2 | I + \frac{\Phi}{\sigma}_2 | \qquad (11)$$

where Φ is a diagonal matrix corresponding to the singular values μ_r. As an example, suppose that H_k and H_{k+1} above represents the channel matrices of MIMO system having two users. Assume perfect channel-state-information (CSI) for H_k and H_{k+1}. With a transmit precoding matrix V^{-1}, and receiver reconstruction matrices U_k, U_{k+1}. The invertible factor V in facilitates joint precoding for the MIMO subsystems. Diagonal elements of Σ, Φ represents the gains of these virtual channels. Since V is non-unitary, precoding would cause the instantaneous transmit power to fluctuate. This is a drawback not present in SVD-based beamforming. Transmit signal should be normalized to maintain the average total transmit power at the desired level. This is the essence of 'GSVD-based beamforming' for a single source and two destinations.

4. Complexity Analysis

The mathematical analysis for computational complexity reduction of proposed BD-GSVD method over BD-SVD method is shown in this section. For the sake of the simplicity, all users are assumed to have the same number of the receive antennas N_R and $KN_R \leq N_T$. The complexities of the methods are usually compared by the number of floating point operations (flops) [11]. A flop is defined as real floating operations, e.g. a real addition, multiplication or division. One complex addition and multiplication involve 2 and 6 flops respectively

The complexity of some utilized operations in these for an m×n complex-valued matrix A with m≤n, its multiplication with another n×p complex-valued matrix B requires 8mnp flops. But when B has a special form the complexity reduces. For instance, when B= A^H, the complexity reduces to 4nm(m+1) flops.

For the conventional SVD-BD method, obtaining the orthogonal complementary basis $V_{k,0}$ requires K times SVD operations in step 1.a. In step 1.b calculating all $H_k V_{k,0}$ requires K matrix multiplications while in step 1.c obtaining the singular vectors $V'_{k,1}$ and the singular values $\lambda_{k,1}$ requires another K SVD operations. In step 2, one water filling is needed to find P_k. In step 3, the square root of the real-valued diagonal matrix $P_k^{1/2}$ needs to be first calculated and then multiplied by $V_{k,1}$ and $V_{k,0}$,respectively. Those operations repeat K times as well. By treating all operations as multiplications, the SVD operation on A takes about $24m n^2 + 48m^2 n + 54m^3$.

For the conventional GSVD-BD method, obtaining the orthogonal Complementary basis V requires K times GSVD operations in step 1.a. If we only want to know V it is not computationally efficient. In step 1.bcalculating all $H_k V$ for precoding matrix W requires K matrix multiplications while in step 1.c obtaining the singular vectors and the singular values requires another K GSVD operations. Here no water filling is needed to find P_k because all diagonal elements are same. In step 2, the square root of the real-valued diagonal matrix $P_k^{1/2}$ needs to be first calculated and then multiplied. Those operations repeat K times as well. By treating all operation as multiplication, the GSVD operation of A & B takes $6(5m+5mp^2+9m^2p+9m^3)$.

5. Simulation Results

Computer simulations are performed to evaluate the performance of GSVD-ISP approach. In all simulations, each user's data are modulated by quadrature phase shift keying (QPSK) signaling. We assume the channel matrix and the interference channel matrix are perfectly known both at the transmitter and the receiver.

These results compare the sum-rate of proposed BD-GSVD scheme and BD-SVD scheme for multiuser MIMO downlink system having various antenna configurations both at transmitter and receiver side. Parameters and values for single cell multiuser MIMO downlink channel are considered with number of frames per packet as 10 with

number of packets as 100, number of bits per QPSK symbol = 2, number of bits per packet = 40 and total number of bits transmitted = 4000.

Fig.1.1, Fig.1.2 compares the sum rates of proposed BD-GSVD scheme against SNR with the perfect channel information in the cases of {2,2}×4,{3,3,3}×10. It is observed from the figures that BD-GSVD has more sum-rate compared to BD-SVD[12][13] and outperforms the BD-SVD scheme.
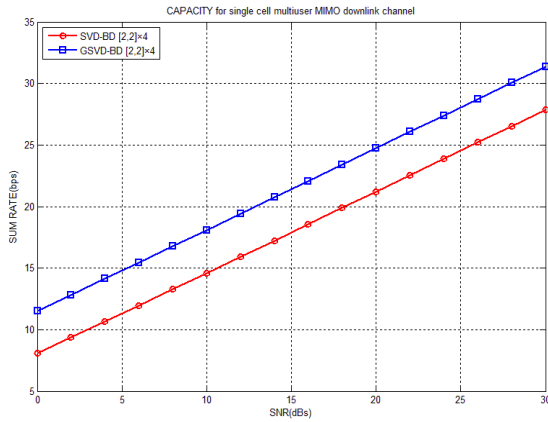

Figure 1.1 Sum rate capacity with NT = 4, NR = 2 and K = 2

Fig.2 compares the simulated bit error rate (BER) or single cell multiuser MIMO downlink system. The bit error rate analysis at different signal to noise ratio (SNR) is done by considering number of transmitting antennas (N_T = 4) and number of receiving antennas (N_R = 2) for K=2 users. From the figure 2 shown below it can be seen that the BER decreases as SNR increases by using the proposed BD-GSVD method as compared to conventional BD-SVD method. From the BER curves versus the transmit SNR, it can be seen that BD-GSVD achieves less BER.

Figure 1.2 Sum rate capacity with NT = 10, NR = 3 and K = 3

Fig. 3.1(a), we consider the case that $N = KN_R$. We set $N_R = 2$ and express the computation cost as a function of N_T (or equivalently K). Later we consider the case that $KN_k < N_T$.
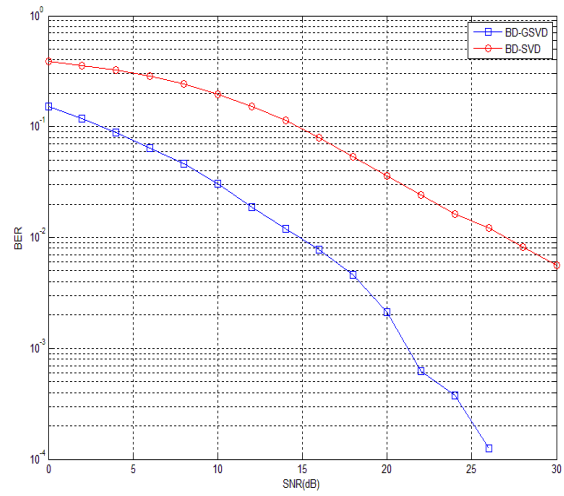

Figure 2. BER versus SNR for NR = 4, NT = 2 for K = 2

We fix $N_T = 64$ and $N_R = 2$ while express the computation cost as a function of K. The result is shown in Fig. 3.1(b). Obviously, BD-GSVD method shows a clear advantage in both comparisons. The larger N_T or N_R, the more it shows the significant difference.

6. Conclusion

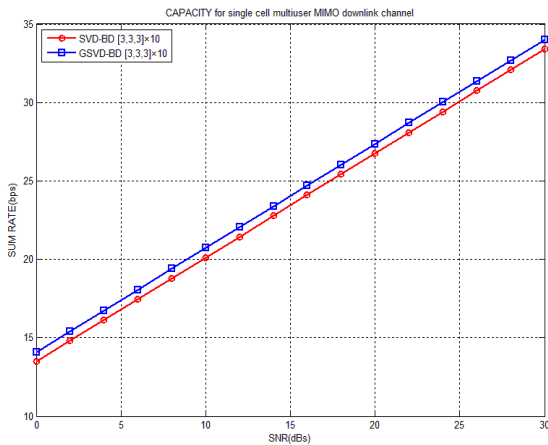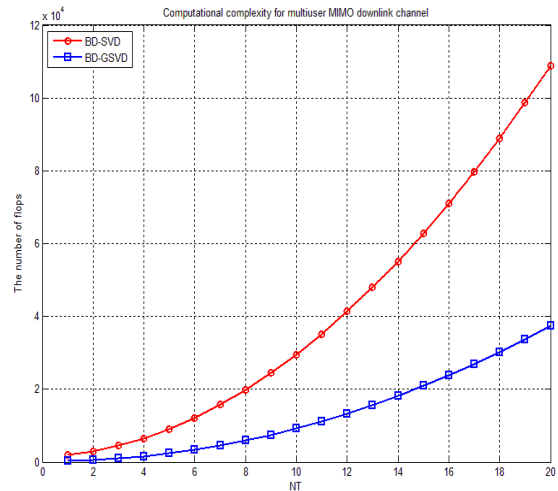In this paper, we proposed BD-GSVD method to obtain the

Figure 3.1(a) Required flops versus the number of transmitting antennas, NT

precoding matrix for multiuser MIMO downlink systems where each user has more than one antenna. We have proved that BD-GSVD scheme has more sum-capacity than the BD-SVD scheme. Through complexity analysis, we show that the new method has lower complexity than the conventional BD method and efficiency improvement becomes significant

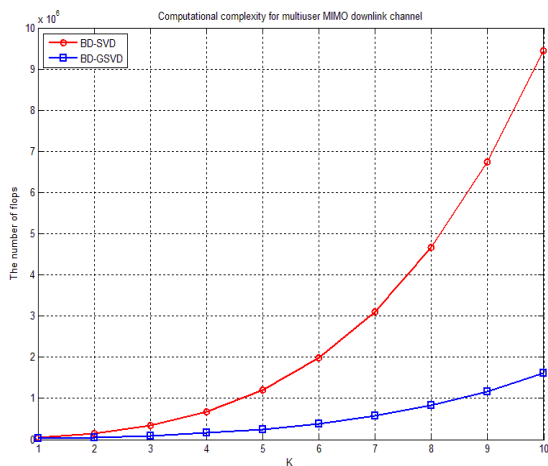when there have a large number of transmit antennas and users.

Figure 3.1(b) Required flops versus the number of users, K

7. References

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