International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 8, October - 2012

An Efficient Implementation of Generalized DFT Filter Banks for subband decomposition

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Abstract — **In this paper Generalized DFT filter bank is developed for subband coding. The analysis starts from the signal definition in the analog domain. Simple observations of sampling period changes and matrix decompositions played key role in the development of efficient implementation of the proposed filter bank. We simulated and verified the proposed algorithm using MATLAB software.**

Index Terms — multirate filter bank, square root raised cosine filter

I. INTRODUCTION

Multirate signal processing is an active research having applications in diverse fields like transmultiplexers, signal and image compression, low complexity adaptive filtering [1, 2, 3] etc.

Among these signal and image compression gained a lot of attention by researchers [1,2,5]. The idea behind this compression is dividing the given input signal into different subbands and coding each subband separately and transmitting. Number of bits is varying in each subband depending upon the information [1,4,8] in it. At the receiver end these subband signals are combined to generate the signal of interest. Subband decomposition is performed by analysis filters at the transmitter side, where as the synthesis filters at the receiver side reconstruct the signal.

Main interest is to reconstruct the signal as close as possible to the original signal. Studies have been performed regarding the design of the analysis/synthesis filters for perfect reconstruction. Perfect reconstruction is possible by removing two distortions that are usually arising. One is amplitude distortion, other is aliasing distortion. To remove aliasing completely conditions are derived in [1,3,6] as pseudo circulant property.

Another important issue of interest is reducing the complexity of implementation. Low complexity solutions are obtained from polyphase decomposition. DFT filter bank is one which implementing subband decomposition with sinc filters. Using ploy phase decomposition, it can be implemented with the complexity of a single filter and efficient DFT algorithms [1,7,8]. Cosine modulated filters also developed from a simple prototype filters [3,8].

In this paper, we present a generalized DFT filter with reduced implementation complexity. We extended the theory developed into filter banks for subband composition. We consider an analog signal as a sum of subband signals. The sampling rate requirement for subband signals is less compared with the original signal, since frequency content is limited. Number of subbands, analysis filters, and filter length can be varied as designer's freedom. By proper matrix formulation of the problem and appropriate decompositions we achieved simplified implementations.

This paper is organized as follows. Section II presents the analog domain problem formulation and its interpretation in frequency domain. Section III presents the digital domain due to sampling. Section IV implementation of multicarrier filter banks. Simulations are illustrated in section V. Conclusions remarks are given in section VI.

II. ANALOG DOMAIN REPRESENTATION

The signal $x_a(t)$ has to be decomposed into subband signals, $s_{a,m}(t)$ where m=0, 1……M-1.We can express $x_a(t)$ as sum of modulated versions of subband signals.

$$x_a(t) = \sum_{m=0}^{M-1} s_{a,m}(t) e^{j\Omega_m t} \qquad (1)$$

Ω_m denote centre frequency of subband signal $s_{a,m}(t)$.
Since $s_{a,m}(t)$ is a band limited signal it can be sampled and reconstructed. Let T_s be the sampling period of $s_{a,m}(t)$ and $g_{a,m}(t)$ be the reconstructed filter. Then

$$s_{a,m}(t) = \{s_{a,m}(t)\sum_{l=-\infty}^{\infty} \delta(t - lT_s)\} * g_{a,m}(t) \qquad (2)$$

$$s_{a,m}(t) = \sum_{l=-\infty}^{\infty} s_{a,m}(lT_s) g(t - lT_s) \qquad (3)$$

ISSN: 2278-0181
Vol. 1 Issue 8, October - 2012

Here ideal impulse sampling is performed and reconstruction is done convolving with $g_{a,m}(t)$.

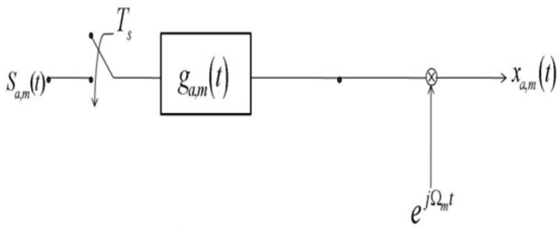

Fig. 1. Reconstruction of original signal analog representation

From the original signal $x_a(t)$, we can obtain the subband signals with proper frequency shifting and filtering .To obtain $s_{a,m}(t)$, we need to shift frequency Ω_m to 0 , then low pass filtering will do. Thus

$$s_{a,m}(t) = (x_a(t)e^{-j\Omega_m t}) * h_{a,m}(t) \qquad (4)$$

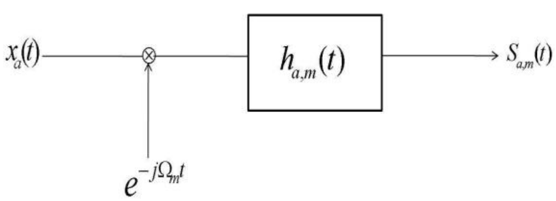

Fig.2.Decomposition of original signal

Equation (4) can be expressed as

$$s_{a,m}(t) = \int_{\tau=-\infty}^{\tau=\infty} x_a(\tau)e^{-j\Omega_m \tau} h_{a,m}(t-\tau)d\tau$$
$$s_{a,m}(t) = e^{-j\Omega_m t} \int_{\tau=-\infty}^{\tau=\infty} x_a(\tau)h_{a,m}(t-\tau)e^{j\Omega_m (t-\tau)}d\tau \qquad (5)$$

Now Figure 2 can be modified as Figure 3

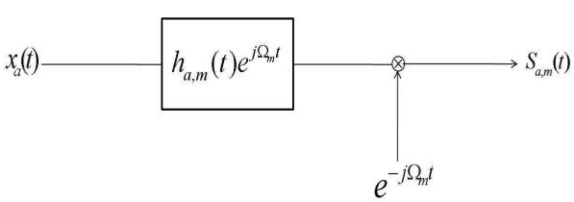

Fig.3.Modified version of figure 2

Now looking at the frequency domain Figure 2 implies

$$X_a(\Omega) \Rightarrow X_a(\Omega+\Omega_m) \Rightarrow X_a(\Omega+\Omega_m)H_{a,m}(\Omega) \quad (6)$$

Figure 3 implies

$$X_a(\Omega) \Rightarrow X_a(\Omega)H_{a,m}(\Omega-\Omega_m) \Rightarrow X_a(\Omega+\Omega_m)H_{a,m}(\Omega) \quad (7)$$

Similarly if we are interested to look at the frequency domain of Figure 1

$$S_{a,m}(\Omega) \Rightarrow \Omega_s \sum_{k=-\infty}^{\infty} S_{a,m}(\Omega - k\Omega_s)$$
$$\Rightarrow \Omega_s \sum_{k=-\infty}^{\infty} S_{a,m}(\Omega - k\Omega_s)G_{a,m}(\Omega) \qquad (8)$$

If $s_{a,m}(t)$ is modulated by $\exp(jt\Omega_m)$ we obtain $x_{a,m}(t)$.Thus

$$X_{a,m}(\Omega) = S_{a,m}(\Omega-\Omega_m)$$
$$= \Omega_s \sum_{k=-\infty}^{\infty} S_{a,m}(\Omega-\Omega_m - k\Omega_s)G_{a,m}(\Omega-\Omega_m) \qquad (9)$$

Using the equivalent developed between Figure 2 and Figure 3,we can modulated part of Figure 1 towards input side and obtain Figure 4

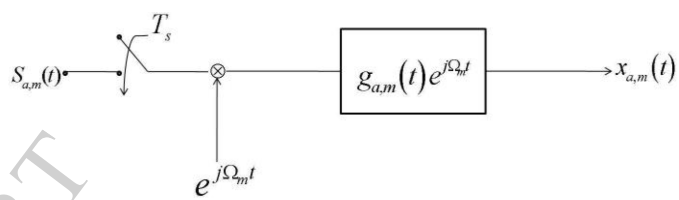

Fig.4. Modulated version of Fig.1.

The frequency domain relation from Figure 4 is given as follows

$$S_{a,m}(\Omega) \Rightarrow \Omega_s \sum_{k=-\infty}^{\infty} S_{a,m}(\Omega - k\Omega_s)$$
$$\Rightarrow \Omega_s \sum_{k=-\infty}^{\infty} S_{a,m}(\Omega - \Omega_m - k\Omega_s) \qquad (10)$$
$$\Rightarrow \Omega_s \sum_{k=-\infty}^{\infty} S_{a,m}(\Omega - \Omega_m - k\Omega_s)G_{a,m}(\Omega-\Omega_m)$$

Clearly (9), (10) give same final result.

III. Digital Domain Representation

In the previous discussion is based on analog implementation. With reference with the original signal sampling period, we develop the digital equivalent analysis. We choose T, as the sampling rate of $x_a(t)$ for proper reconstruction based on Nyquist criteria. Subband sampling period T_s is larger compared to T. The relationship between T and T_s is given by $T= T_s/N$, for some positive integer N. If we sample $x_a(t)$, it result $x_a(nT)=x(n)$.
For subband signals

$$s_{a,m}(nT_s) = s_{a,m}(nNT) = s_m(nN) \qquad (11)$$

The same relation can be concluded from Figure 5.

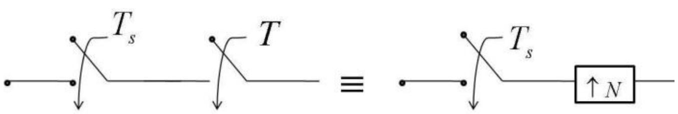

Fig.5 (a). Upsampling process

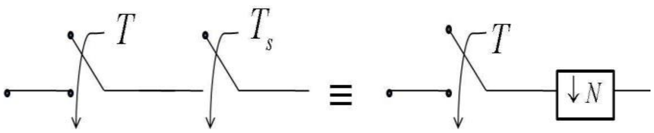

Fig.5 (b). Downsampling process

Equation (1) after samples t=nT

$$x_a \ \ nT \ = \sum_{m=0}^{M-1} s_{a,m}(nT)e^{j\Omega_m nT} \qquad (12)$$

Using (12) its equivalent to

$$\sum_{m=0}^{M-1}\{\sum_{l=-\infty}^{\infty} s_{a,m}(lT_s)g_{a,m}(nT-lT_s)\}e^{j(\Omega_m T)n} \qquad (13)$$

Using T_s=NT, $\Omega_m T=\omega_m$

$$x(n) = \sum_{m=0}^{M-1}\sum_{l=-\infty}^{\infty} s_m(Nl)g_m(n-Nl)e^{j\omega_m n} \qquad (14)$$

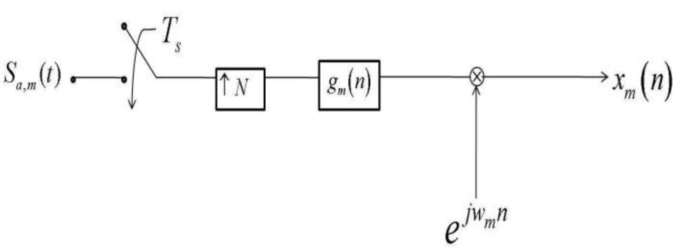

Fig.6 (a). Reconstruction of the signal by upsampling

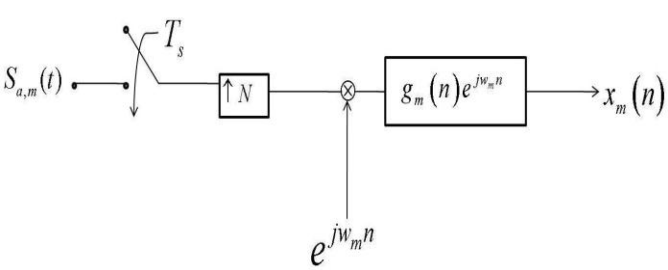

Fig.6 (b) Modulated version Fig.6 (a).

Using (14) Figure 6 can be developed. It is easy to note that using Figure 1 and Figure 5(a) one can develop Figure 6(a). Simply by applying result in Figure 5(a) on Figure 4 we obtain Figure 6(b). Similarly by applying 5(b) on Figure 2, 3 we obtain Figures 7(a), 7(b).

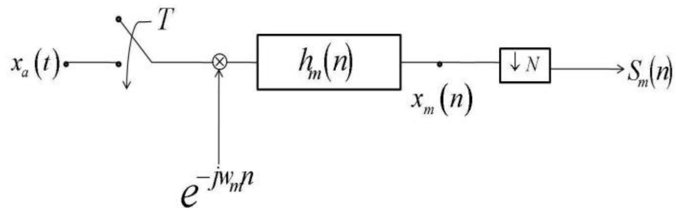

Fig.7 (a). Decomposing of the signal by downsampling

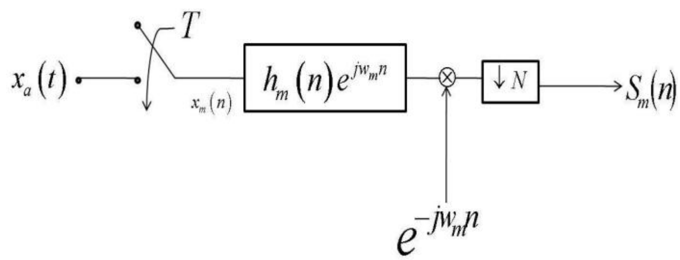

Fig.7 (b). Modulated version of the Fig.7 (a).

Now we look at mathematical analysis of Figures .From Figure 6(b)

$$((s_m(n)_{\uparrow N})e^{j\omega_m n}) * (g_m(n)e^{j\omega_m n})$$
$$= (s_m(n)e^{j\omega_m nN})_{\uparrow N} * (g_m(n)e^{j\omega_m n})$$

Let $g_m(n)e^{j\omega_m n} = b_m(n)$, $s_m(n)e^{j\omega_m Nn} = y_m(n)$

$$x_m(n) = \sum_{k=-\infty}^{\infty} y_m(k)b_m(n-kN) \qquad (15)$$

From Figure 7(b) $s_m(n) = ((x(n) * h_m(n)e^{j\omega_m n})e^{-j\omega_m n})_{\downarrow N}$

Let $h_m(n)e^{j\omega_m n} = a_m(n)$ then

$$s_m(n) = e^{j\omega_m nN}\sum_{k=-\infty}^{\infty} x(k)a_m(nN-k) \qquad (16)$$

Equation (16) given decomposition of x (n) into subbands $s_m(n)$. Where (15) with x(n) =$x_0(n)$ +$x_1(n)$ +………$x_{M-1}(n)$, gives synthesis of original signal from subband signals. It is easy to combine Figure7 (b) followed by 6(b)

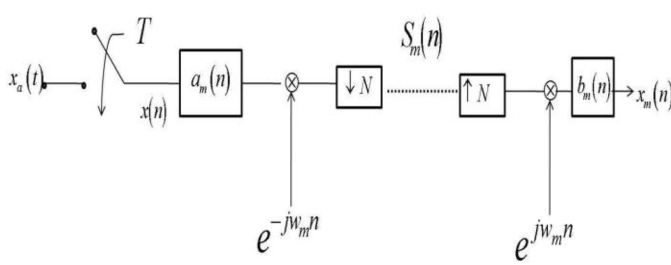

Fig.8. The reformed digital implementation of generalized multicarrier filter banks

IV. IMPLEMENTATION OF MULTICARRIER FILTER BANKS

To simplify the implementation task, all the filters can be developed from a prototype filter g(n). We choose it be the length L=2kN+1 for group delay k. We know that

Analysis filter $a_m \; n \; = h_m \; n \; e^{j\omega_m n}$

Synthesis filter $b_m \; n \; = g_m \; n \; e^{j\omega_m n}$

To make all filters casual a common shift of (L-1)/2 samples is introduced.

$$\tilde{a}_m \; n \; = g \; L-1-n \; e^{j2\pi/M(m-(M-1/2))(n-(L-1/2))}$$
$$\tilde{b}_m \; L-1-n \; = \; \tilde{a}_m(n) \; ^* \qquad (17)$$

Implementation of synthesis/analysis filter bank algorithm as follows.

A. Synthesis Filter Bank:

Now we will discuss development of synthesis filter bank. The transmitted signal can be expressed as

$$x_m \; n \; = \sum_{k=-\infty}^{\infty} y_m \; k \; b_m \; n-Nk$$
$$y_m \; n \; = s_m \; n \; e^{j\omega_m nN} \qquad (18)$$

This can be depicts as follows

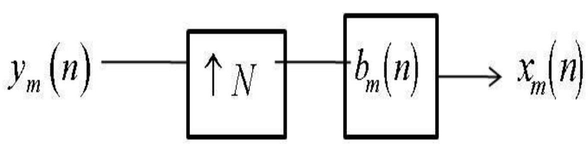

Fig.9. Synthesis filter bank representation

$$f_k \; n \; = \sum_{m=0}^{M-1} y_m \; k \; b_m \; n$$
$$f_k \; n-kN \; = \sum_{m=0}^{M-1} y_m \; k \; b_m \; n-kN \qquad (19)$$

Where $f_k \; n \; = [b_0 \; n \quad b_1 \; n \quad ...... \quad b_{M-1} \; n] \begin{bmatrix} y_0 \; k \\ y_1 \; k \\ : \\ y_{M-1} \; k \end{bmatrix}$

n=0, 1….L-1 k=0, 1 ….∞

$$\begin{bmatrix} f_k \; 0 \\ f_k \; 1 \\ : \\ f_k \; L-1 \end{bmatrix}_{L\times 1} = \begin{bmatrix} b_0 \; 0 & b_1 \; 0 & .... & b_{M-1} \; 0 \\ b_0 \; 1 & b_1 \; 1 & .... & b_{M-1} \; 1 \\ : & : & .... & : \\ b_0 \; L-1 & b_1 \; L-1 & .... & b_{M-1} \; L-1 \end{bmatrix}_{L\times M} \begin{bmatrix} y_0 \; k \\ y_1 \; k \\ : \\ y_{M-1} \; k \end{bmatrix}_{M\times 1} \qquad (20)$$

$$B_{n,m} = b_m \; n \; = g \; n \; e^{j2\pi/M(m-(M-1/2))(n-(L-1/2))} \qquad (21)$$

$$B^T = T\left[I_m \quad -1^{\;M-1} I_m \right]\times$$
$$\left[I_{2M} \quad I_{2M} \quad ..... \quad I_{2M} \quad I_{2M,\tilde{L}} \right]\times \qquad (22)$$
$$diag \;\; g \;\; 0 \qquad g \;\; 1 \qquad ..... \qquad g \;\; L-1$$

Where $T = \Lambda_1 W_M{}^H \Lambda_2$, W_m= normalized DFT matrix

Λ_1, Λ_2 diagonal matrix with equations

$$\Lambda_1 \;\; _{k,k} = e^{-j\pi(k-(M-1/2))(L-1)/M}$$
$$\Lambda_2 \;\; _{k,k} = e^{-j\pi k(M-1)/M}$$

The relationship is $f_k = By_k$

1) The Synthesis filter bank is implemented by below algorithm

Step 1) Set the initial value of m and k: k=0, n=0. Set an L-point sequence: d (l) =0, l=0, 1…L-1.

Step 2) Perform M-point IFFT on M subband inputs $y_m(k)$, with the transform matrix $T = \Lambda_1 W_M{}^H \Lambda_2$.Gives the result $\tilde{f}_k \;\; m$, m=0, 1, 2….M-1.

Step3) Expand M-point sequence to 2M-point sequence

$$\breve{f}_k \;\; m \; = \tilde{f}_k \;\; m \qquad 0 \le m \le M-1$$
$$= \; -1^{\;M-1} \; \tilde{f}_k \;\; m \qquad M \le m \le 2M-1$$

And then expand L-point $\bar{f}_k \;\; l \; = \breve{f}_k \;\; l \;\; _{2M}$ sequence, l=0, 1….L-1.

Step 4) Calculate $f_k \;\; l \; = g \;\; l \;\; \bar{f}_k \;\; l$ l=0,1,….L-1.

Step 5)Calculate $d(l) = d(l) + f_k(l)$ l=0,1,….L-1

Step 6) Output the first N samples of the L point sequence d(l) : $f(n+l) = d(l)$ l=0,1,….N-1. Update the L-point sequence d(l) as

$$d(l) = d(l+N) \quad 0 \le l \le L-N-1$$
$$= 0 \qquad L-N \le l \le L-1$$

Step 7) Set $k = k+1, n = n+N$ and go to step 2.

B. Analysis Filter Bank:

Now we will discuss development of Analysis filter bank.

$$y_m \;\; n = \sum_{k=0}^{L-1} \tilde{a}_m \;\; k \;\; x \;\; nN-k$$
$$= \sum_{k=0}^{L-1} \tilde{a}_m \;\; L-1-k \;\; x \;\; nN-L+1+k$$
$$= \sum_{k=0}^{L-1} (\tilde{b}_m \;\; k \;)^* x \;\; nN-L+1+k \qquad (23)$$

This can be depicts as follows

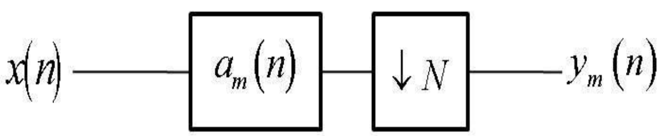

Fig. 10. Analysis filter bank representation

Let $x_{n=} \begin{bmatrix} x \;\; nN-L+1 \\ x \;\; Nn-L+2 \\ \vdots \\ x \;\; nN \end{bmatrix} \quad y_{n=} \begin{bmatrix} y_0 \;\; n \\ y_1 \;\; n \\ \vdots \\ y_{M-1} \;\; n \end{bmatrix}$

$$y_n = B^H x_n \qquad (24)$$

This expression decomposes input signal x_n into subbands y_n.Thus analysis filter bank can be implemented as follows.

2) The Analysis filter bank is implemented by below algorithm

Step 1) set the initial value of n: n= (L-1)/N

Step2) Take an L-point sequence from input signal: $x_n(k) = x(Nn-L+1+k)$, k=0, 1,…L-1.

Step 3) calculate $\overline{x}_n(l) = g(l)x_n(l)$, k=0,1,…L-1

Step 4) calculate $\breve{x}_n(m) = \sum_{q=0}^{Q_m} \overline{x}_n(2Mq+m)$,m=0, 1...2M-1

and Qm is an integer no more than (L-1-m)/M .calculate $\tilde{x}_n(m) = \breve{x}_{\boldsymbol{n}}(m) + (-1)^{M-1}\breve{x}_n(m+M)$ m=0,1..2M-1.

Step 5) perform FFT on M-point sequence $\tilde{x}_n(m)$ with transform matrix $T^* = \Lambda_1{}^* W_M \Lambda_2{}^*$ to get reconstructed $y_m(n)$ m=0,1..2M-1.

Step 6) set n=n+1 and go to step 2.

V. Simulation Results

Based on the analysis done so far, simulations are performed using MATLAB to justify the effectiveness of the algorithm. The prototype filter is chooses to be a square root raised cosine filter of length L=433.The impulse response, frequency response are plotted in below

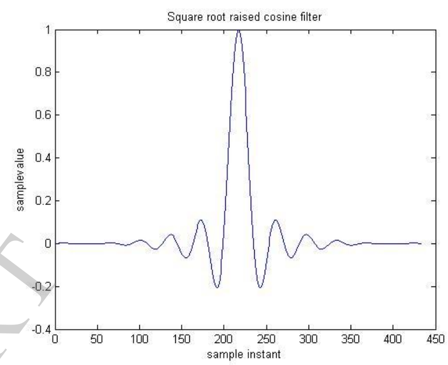

Fig.11. Square root raised cosine filter

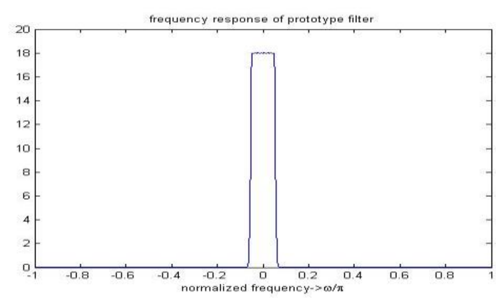

Fig.12.FFT filter response

We are using the prototype filter and various modulations on it and from the original signal different bands are extracted the original signal spectrum and the extracted subband spectrum are illustrated in below figures.

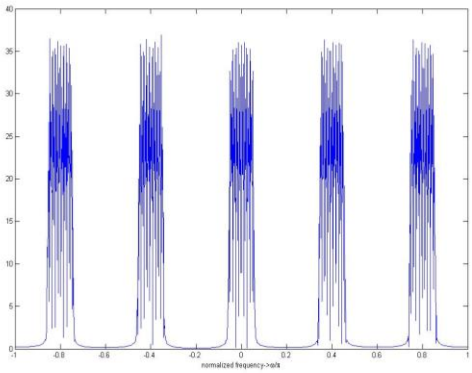

Fig.13 (a). Spectrum consists of five bands

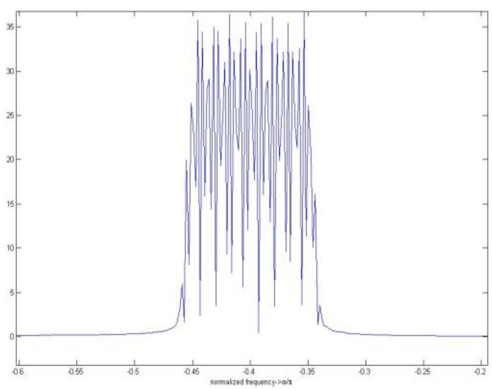

Fig.13 (b) Decomposition of band 1

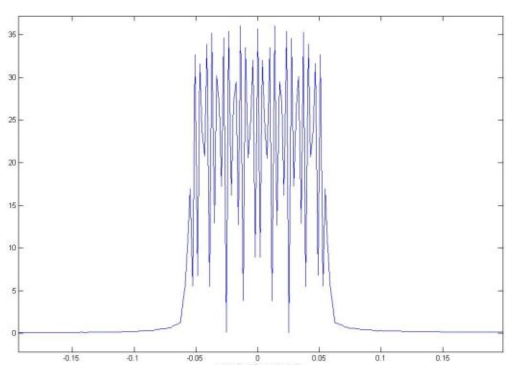

Fig.13 (c) Decomposition of band 2

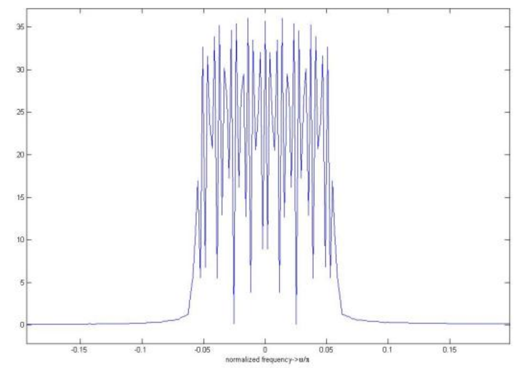

Fig.13 (d) Decomposition of band 3

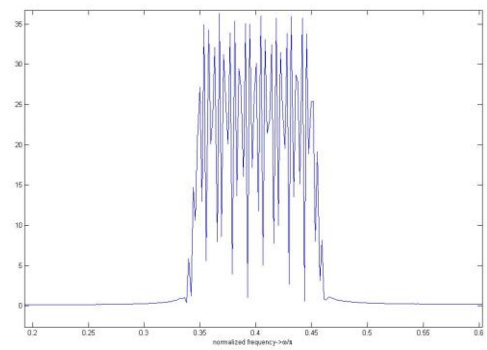

Fig.13 (e) Decomposition of band 4

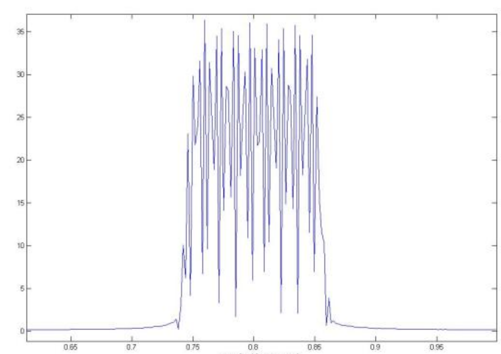

Fig.13 (f) Decomposition of band 5

VI. CONCLUSION

In this paper we presented algorithms to implement generalized DFT filter bank. From a single prototype filter various subband filters are developed using the generalized DFT matrix operation. Factorization of generalized DFT matrix played the major role in developing efficient implementation algorithm.

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