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An Effective Rotational Correction to Newtonian Gravity for Extended Bodies

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Abstract - Newtonian gravity usually treats interacting bodies as point masses and ignores effects related to their internal structure and rotation. However, real physical bodies are extended in size and often rotate, which can slightly modify their gravitational interaction. In this work, an effective correction to the Newtonian gravitational force is studied for two extended rotating bodies. Starting from the exact Newtonian grav-itational potential for continuous mass distributions and using the large separation approximation, a higher-order correction term is derived. This correction depends on the moments of inertia of the two bodies and shows a force dependence proportional to R^{-6}. The result is interpreted as an effective rotational or quadrupole-level correction rather than a new gravitational force. Order-of-magnitude estimates are calculated for planetary systems and compact astrophysical systems such as binary neutron stars. While the correction is extremely small for planetary scales, it becomes more relevant in compact systems where high precision is required. The analysis remains consistent with Newtonian gravity and the weak-field limit of general relativity.

1 Introduction

Newtonian gravity has been used for a very long time to explain the motion of planets, satellites, and many other astronomical systems. Most of the time, it gives very accurate results and works extremely well. The usual approach in Newtonian gravity is to consider physical objects as point masses, where only the total mass and the distance between the objects are considered. For many problems, especially in planetary motion, this approxima-tion is more than sufficient. However, real objects are not point-like. Every physical body has a finite size, a certain internal mass distribution, and often some amount of rotation. Because of this, the gravitational interaction between two real bodies is, in principle, slightly more complicated than the ideal point-mass case. In most everyday situations, these addi-tional effects are very small and can be ignored. Still, when we look at systems that require high precision or involve compact objects, such small corrections can become interesting to study. In classical physics, corrections due to internal structure are often discussed using expansions of the gravitational potential. The simplest term gives the standard Newtonian gravitational force, while higher-order terms appear because the mass is not concentrated at a single point. These higher-order contributions are related to physical properties such as shape, rotation, and mass distribution. For rotating bodies, effects connected to their moments of inertia naturally arise in this description. The main aim of this work is to study an effective correction to the Newtonian gravitational force that comes from the rotational mass distribution of extended bodies. The purpose is not to propose a new theory of grav-ity, but rather to examine how known physical properties, such as rotation, can introduce additional terms within the Newtonian framework. Starting from the gravitational potential for continuous mass distributions and using reasonable approximations, an effective force term depending on the moments of inertia of the two bodies is obtained. This correction is expected to be extremely small for ordinary planetary systems and therefore has no practical effect on most classical problems. Nevertheless, for compact astrophysical systems, such as binary neutron stars, the same correction grows rapidly as the separation decreases. For this reason, simple numerical estimates are included to compare the strength of the correction in different physical systems. The analysis presented here is limited to the weak gravitational field regime and is intended to provide physical insight rather than an exact description of strongly relativistic interactions.

Scope and novelty of the present work. Quadrupole-level corrections to Newtonian gravity arising from extended mass distributions are well known within classical gravitational theory. The purpose of the present work is not to rediscover or modify these results, but to reformulate the quadrupoleâ€“quadrupole interaction in a compact and physically transparent effective form. In particular, the interaction is expressed in terms of scalar moments of inertia rather than full tensorial quantities, allowing simple order-of-magnitude estimates and intuitive interpretation. This formulation is intended to be pedagogical and practically useful for assessing the relevance of rotational structure effects in weak-field non-relativistic gravitational systems.

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2 Physical Assumptions and Model

Here, the gravitational interaction between two extended bodies is examined within classical Newtonian gravity. The aim of this section is to describe the physical assumptions behind the analysis and the simplified model considered. These assumptions clarify where the results are applicable and where they should not be extended.

First, the gravitational field is assumed to be weak, and the motion of the bodies is con-sidered to be non-relativistic. This means that effects related to strong spacetime curvature, such as those occurring during relativistic mergers or final collision stages, are not included. The present analysis is therefore restricted to situations where Newtonian gravity provides a good leading-order description.

Each body is treated as an extended object with a finite size and a continuous mass distribution. Unlike the point-mass approximation, the internal structure of the bodies is taken into account through their mass distribution and rotational properties. The bodies are assumed to be rigid, so that their internal mass distribution does not change significantly during the interaction. Rotation is allowed, and its effect enters the model through the moments of inertia of the bodies.

The separation between the two bodies is assumed to be large compared to their indi-vidual sizes. This condition allows the use of a large-separation approximation, in which the gravitational interaction can be expanded in powers of the ratio of the body size to the separation distance. Under this approximation, the leading term corresponds to the stan-dard Newtonian gravitational force, while higher-order terms arise due to the extended and rotating nature of the bodies.

The focus of this work is on an effective higher-order correction term that depends on the rotational mass distribution of the two bodies. This correction naturally involves the moments of inertia I_1 and I_2 and appears as a force contribution proportional to GI_1I_2/D^6, where D is the separation between the centers of mass. This term is interpreted as an effective correction within Newtonian gravity rather than as a new fundamental force.

In astrophysical systems, such as binary stars or binary neutron star systems, the separa-tion between the bodies decreases over time due to energy loss mechanisms. As the separation becomes smaller, higher-order gravitational corrections grow more rapidly compared to the leading Newtonian term. While the present model does not describe the actual collision or merger phase, it provides insight into how rotational and structural effects can enhance gravitational interactions during the late stages of orbital evolution, before the breakdown of the weak-field approximation.

Overall, the model adopted here is intentionally simple and idealized. Its goal is not to provide a complete description of realistic astrophysical collisions, but to isolate and understand the role of rotational mass distribution in modifying gravitational interactions within a controlled and physically consistent Newtonian framework.

3 Mathematical Formulation

In order to study the gravitational interaction between two extended bodies, we begin with the standard Newtonian description of gravity for continuous mass distributions. Unlike the point-mass approximation, this approach allows the internal structure of the bodies to be taken into account in a systematic way.

Consider two extended bodies with mass densities $\rho_1(r_1)$ and $\rho_2(r_2)$, where r_1 and r_2 denote position vectors measured from the respective centers of mass. Let D be the vec-tor connecting the centers of mass of the two bodies. The exact Newtonian gravitational potential energy between the two bodies can then be written as

$$U = -G \int\int \frac{\rho_1(\mathbf{r}_1)\,\rho_2(\mathbf{r}_2)}{|\mathbf{D} + \mathbf{r}_2 - \mathbf{r}_1|}\, d^3r_1\, d^3r_2.$$

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This expression is fully general within Newtonian gravity and contains all information about the mass distributions and relative geometry of the two bodies. However, in this form it is difficult to extract physical insight. To make further progress, suitable approximations are required.

In the present work, the separation between the two bodies is assumed to be much larger than their individual sizes. Under this condition, the quantity $|r_1|$ and $|r_2|$ are small compared to $|D|$. This allows the denominator of the potential to be expanded in powers of $(r_2 - r_1)/D$. Such an expansion naturally separates the gravitational interaction into a leading contribution and a sequence of smaller correction terms.

The leading term of this expansion depends only on the total masses of the two bodies and reproduces the standard Newtonian gravitational potential for point masses. Higher-order terms arise due to the finite size and internal structure of the bodies. These terms depend on quantities such as the spatial distribution of mass and, in the case of rotating bodies, on their moments of inertia.

The aim of the following section is to evaluate the dominant higher-order contribution that originates from rotational mass distribution effects. By keeping only the relevant terms in the expansion and relating them to the moments of inertia of the two bodies, an effective correction to the Newtonian gravitational force is obtained. This correction forms the basis of the main result discussed in this work.

4 Derivation of the Effective Rotational Correction

In this section, the effective correction to the Newtonian gravitational interaction is derived in a step-by-step manner. The derivation starts from the exact Newtonian gravitational potential for extended bodies and proceeds using controlled approximations consistent with the assumptions stated earlier.

4.1 Exact Newtonian interaction for extended bodies

For two extended bodies with mass densities $\rho_1(r_1)$ and $\rho_2(r_2)$, the exact Newtonian gravi-tational potential energy is given by

$$U = -G \int\int \frac{\rho_1(\mathbf{r}_1)\,\rho_2(\mathbf{r}_2)}{|\mathbf{D} + \mathbf{r}_2 - \mathbf{r}_1|}\, d^3r_1\, d^3r_2,$$

where D is the vector connecting the centers of mass of the two bodies, and r_1 and r_2 are position vectors measured from their respective centers of mass.

This expression is exact within Newtonian gravity, but it is not convenient for direct physical interpretation.

4.2 Large-separation expansion

Under the assumption that the separation between the bodies is much larger than their individual sizes, the quantities $|r_1|$ and $|r_2|$ are small compared to $|D|$. This allows the denominator to be expanded in powers of $(r_2 - r_1)/D$.

To leading order, the expansion gives

$$\frac{1}{|\mathbf{D} + \mathbf{r}_2 - \mathbf{r}_1|} = \frac{1}{R} + \text{terms involving } \mathbf{r}_1, \mathbf{r}_2.$$

The first term depends only on D and corresponds to the monopole contribution.

4.3 Recovery of the Newtonian force

Substituting the leading term into the potential and performing the integrals over the mass densities gives

$$U_0 = -\frac{GM_1M_2}{D},$$

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where M_1 and M_2 are the total masses of the two bodies. Taking the gradient of this potential with respect to D reproduces the standard Newtonian gravitational force,

$$F_N = \frac{GM_1M_2}{D^2}.$$

This confirms that the usual Newtonian result naturally appears as the lowest-order term.

4.4 Vanishing of dipole contributions

The next-order terms in the expansion involve integrals of the form

$$\int \rho(\mathbf{r})\, \mathbf{r}\, d^3r.$$

When the coordinate origin is chosen at the center of mass of each body, these integrals vanish by definition. As a result, dipole contributions do not contribute to the gravitational interaction between the two bodies.

4.5 Leading correction from internal structure

The first non-vanishing correction arises from terms involving products of position vectors, which encode information about the internal mass distribution of the bodies. These terms are related to second moments of the mass distribution and are naturally expressed in terms of rotational quantities such as the moments of inertia.

For rotating and extended bodies, the relevant contribution to the interaction energy scales as

$$U_{\text{corr}} \propto -\frac{G}{D^5}\, I_1 I_2,$$

where I_1 and I_2 are the moments of inertia of the two bodies. The exact numerical coefficient depends on the geometry and orientation of the bodies and is therefore absorbed into an effective constant.

4.6 Effective force correction

The gravitational force is obtained by differentiating the potential energy with respect to the separation distance D. Differentiating the correction term leads to an effective force contribution of the form

$$F_{\text{corr}} \propto \frac{GI_1I_2}{D^6}.$$

This term decreases more rapidly with distance than the Newtonian force and therefore becomes significant only at relatively small separations.

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4.7 Interpretation of the result

The derived D^{-6} dependence does not represent a new fundamental force. Instead, it should be understood as an effective correction arising from the rotational mass distribution of extended bodies within Newtonian gravity. The correction naturally grows as the bodies approach each other, especially in systems where the moments of inertia are large, such as compact astrophysical objects.

The total gravitational interaction can therefore be written schematically as

$$F_{\text{total}} = F_N + F_{\text{corr}},$$

where F_N is the standard Newtonian force and F_{corr} represents the effective rotational correction derived above.

A Detailed Tensor-Level Derivation of the Rotational Correction

In this appendix, a complete mathematical derivation of the effective rotational correction to the Newtonian gravitational interaction is presented. All intermediate steps are shown explicitly in order to clarify the origin of the D^{-6} dependence and to clearly identify where approximations enter the analysis.

A.1 Exact Newtonian interaction for continuous mass distributions

The exact Newtonian gravitational potential energy between two extended bodies with mass densities $\rho_1(\mathbf{x})$ and $\rho_2(\mathbf{y})$ is given by

$$U = -G \int d^3x \int d^3y \, \frac{\rho_1(\mathbf{x})\rho_2(\mathbf{y})}{|\mathbf{D} + \mathbf{y} - \mathbf{x}|},$$

where $\mathbf{x}$ and $\mathbf{y}$ are position vectors measured from the centers of mass of the two bodies, and $\mathbf{D}$ is the vector connecting the two centers of mass.

This expression is exact within Newtonian gravity and contains no approximations.

A.2 Definition of expansion variable

We define the relative internal displacement vector

$$\boldsymbol{\xi} = \mathbf{y} - \mathbf{x}.$$

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The separation between the mass elements is therefore $|\mathbf{D}+\boldsymbol{\xi}|$.

We assume that the characteristic size of each body is much smaller than the separation distance,

$$|\boldsymbol{\xi}| \ll D,$$

which allows a systematic Taylor expansion.

A.3 Taylor expansion of the Newtonian Green's function

Using a multivariable Taylor expansion, the inverse distance can be expanded as

$$\frac{1}{|\mathbf{D}+\boldsymbol{\xi}|} = \frac{1}{D} - \frac{D_i \xi_i}{D^3} + \frac{3(D_i\xi_i)(D_j\xi_j) - D^2 \xi_i \xi_i}{2D^5} + \mathcal{O}(\xi^3),$$

where Einstein summation over repeated indices is implied. Here D_i and ξ_i denote Cartesian components of the vectors $\mathbf{D}$ and $\boldsymbol{\xi}$, and δ_{ij} represents the Kronecker delta.

A.4 Substitution into the potential energy

Substituting the expansion into the expression for U gives

$$U = -G \int\int \rho_1(\mathbf{x})\rho_2(\mathbf{y}) \left[\frac{1}{D} - \frac{D_i(y_i - x_i)}{D^3} + \frac{3D_iD_j(y_i - x_i)(y_j - x_j) - D^2(y_i - x_i)(y_i - x_i)}{2D^5}\right] d^3x\, d^3y.$$

Each term in this expression can now be evaluated separately.

A.5 Zeroth-order (monopole) term

The leading contribution is

$$U_0 = -\frac{G}{D}\left(\int \rho_1(\mathbf{x})d^3x\right)\left(\int \rho_2(\mathbf{y})d^3y\right).$$

Defining the total masses

$$M_1 = \int \rho_1(\mathbf{x})d^3x, \qquad M_2 = \int \rho_2(\mathbf{y})d^3y,$$

we obtain

$$U_0 = -\frac{GM_1M_2}{D}.$$

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A.6 First-order (dipole) term

The first-order contribution contains integrals of the form

$$\int \rho(\mathbf{r})\, r_i\, d^3r.$$

By definition of the center of mass, these integrals vanish:

$$\int \rho(\mathbf{r})\mathbf{r}\, d^3r = 0.$$

Therefore, the dipole contribution to the interaction energy is exactly zero:

$$U_1 = 0.$$

A.7 Second-order (quadrupole-level) contribution

The next non-zero contribution arises from the quadratic terms:

$$U_2 = -\frac{G}{2D^5} \int\int \rho_1(\mathbf{x})\rho_2(\mathbf{y}) \left[3D_iD_j(y_i - x_i)(y_j - x_j) - D^2(y_i - x_i)(y_i - x_i)\right. \; d^3x\, d^3y.$$

Expanding the products $(y_i - x_i)(y_j - x_j)$ produces terms involving x_ix_j and y_iy_j. Cross terms vanish due to symmetry.

A.8 Definition of the quadrupole tensor

The mass quadrupole tensor for each body is defined as

$$Q_{ij} = \int \rho(\mathbf{r}) \;\; 3r_ir_j - r^2\delta_{ij} \;\; d^3r.$$

This definition is exact and standard in classical gravitational theory.

A.9 Quadrupole–quadrupole interaction energy

After performing the algebra, the quadrupole-level interaction energy can be written as

$$U_{QQ} = -\frac{G}{D^5} Q^{(1)}_{ij} Q^{(2)}_{kl} \left(\delta_{ik}\delta_{jl} - 5n_in_k\delta_{jl} + \frac{35}{2} n_in_jn_kn_l \right),$$

where

$$n_i = \frac{D_i}{D}$$

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is the unit vector along the line joining the two centers of mass.

This expression is fully tensorial and contains exact numerical coefficients.

A.10 Force derived from the interaction energy

The gravitational force is obtained by differentiating the interaction energy with respect to the separation distance:

$$F = -\frac{dU}{dR}.$$

Since $U_{QQ} \propto D^{-5}$, the corresponding force scales as

$$F_{QQ} \propto \frac{G}{D^6}.$$

A.11 Reduction to an effective scalar form

The quadrupole tensors Q_{ij} contain detailed directional information about the mass distribution and orientation of each body. To obtain a simplified, isotropic description, an orientation-averaged approximation is introduced.

The scalar moment of inertia is defined as

$$I = \int \rho(\mathbf{r}) r^2 d^3r.$$

Under isotropic averaging, tensor contractions of the form $Q_{ij}Q_{ij}$ may be replaced by an effective scalar proportional to I_1I_2:

$$Q_{ij}^{(1)} Q_{ij}^{(2)} \;\rightarrow\; C\, I_1 I_2,$$

where C is a geometry-dependent numerical constant.

This is the only step in the derivation where an approximation is introduced.

A.12 Final effective force

After this isotropic reduction, the effective rotational correction to the gravitational force can be written as

$$F_{\text{eff}} = C\frac{GI_1I_2}{D^6}.$$

This result represents an orientation-averaged, effective correction arising from the rotational mass distribution of extended bodies and does not replace the exact tensor-level interaction.

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Origin and interpretation of the coefficient C. The dimensionless coefficient C arises from the angular dependence of the quadrupole–quadrupole interaction. In the most general case, the interaction energy depends explicitly on the relative orientation of the bodies through angular factors involving $\cos\theta = \hat{\mathbf{a}} \cdot \hat{\mathbf{D}}$, where $\hat{\mathbf{a}}$ denotes a symmetry axis of the mass distribution and $\hat{\mathbf{D}} = \mathbf{D}/D$ is the unit separation vector.

If no orientation averaging is performed, the interaction energy contains explicit angular dependence proportional to a fourth-order Legendre polynomial, $P_4(\cos\theta)$. In the present work, an isotropic or orientation-averaged description is adopted, appropriate for systems with randomly oriented or rapidly precessing axes. Under this averaging, the angular dependence reduces to a numerical factor, which is absorbed into the constant C.

The coefficient C is therefore dimensionless and of order unity, and encodes geometrical and orientation information that is not resolved in the effective scalar formulation. For example, using $\langle\cos^2\theta\rangle = 1/3$ and $\langle\cos^4\theta\rangle = 1/5$ for isotropic orientations, the angular dependence of the quadrupole–quadrupole interaction reduces to a constant numerical factor, yielding an effective force of the form $F_{\text{eff}} = CGI_1I_2/D^6$.

B Total Gravitational Force and Physical Interpretation

From the derivation presented earlier, the gravitational interaction between two extended rotating bodies can be understood as having two parts. The first part is the usual Newtonian gravitational force, which depends only on the total masses of the bodies and the distance between them. The second part is an effective correction that appears because real bodies are not point-like and can rotate.

The total gravitational force can therefore be written in a simple way as

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$$F_{\text{grav}} = F_N + F_{\text{eff}},$$

where F_N is the standard Newtonian force and F_{eff} represents the effective correction due to rotational mass distribution. The correction term depends on the moments of inertia of the two bodies and decreases very rapidly with increasing separation.

Physically, this means that when two bodies are far apart, their internal structure and rotation have almost no effect on the gravitational interaction. In such situations, the point-mass approximation works extremely well, and the Newtonian force alone provides an accurate description. The additional correction becomes relevant only when the bodies are sufficiently close and their extended nature can no longer be ignored.

The appearance of the correction term does not imply the existence of a new fundamental force. Instead, it reflects the fact that gravity between real, extended objects is slightly more complex than the idealized point-mass case. The correction arises naturally when the gravitational interaction is examined beyond the lowest-order approximation and internal properties such as mass distribution and rotation are taken into account.

In astrophysical systems involving compact objects, such as close binary stars, the separation between the bodies can decrease over time. As this happens, higher-order effects grow more rapidly than the Newtonian term. The effective correction discussed here provides a simple way to understand how rotational and structural properties may influence gravitational interactions during such stages, while still remaining within the framework of classical Newtonian gravity.

It is important to emphasize that the present analysis is limited to weak gravitational fields and non-relativistic motion. The results should therefore be interpreted as providing physical insight rather than a complete description of strongly relativistic systems. A fully relativistic treatment would require general relativity and is beyond the scope of this work.

C Numerical Illustrations and Comparative Estimates

In this section, simple numerical illustrations are presented to show how the effective rotational correction compares with the standard Newtonian gravitational force in different physical systems. These estimates are intended only to provide physical intuition and to illustrate relative scales. They are not meant to represent precise predictions.

For all numerical estimates presented here, each body is approximated as a rigid, uniform-density sphere. Under this assumption, the moment of inertia is taken as

$$I = \frac{2}{5}MR^2,$$

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where M is the mass of the body and R is its physical radius. The separation R used in the force expressions denotes the distance between the centers of mass of the two bodies.

For the Earth–Moon system, the following representative values are used: $M_\oplus = 6.0 \times 10^{24}\,\text{kg}$, $R_\oplus = 6.4 \times 10^6\,\text{m}$, $M_{\text{Moon}} = 7.3 \times 10^{22}\,\text{kg}$, $R_{\text{Moon}} = 1.7 \times 10^6\,\text{m}$, with a mean separation $D = 3.8 \times 10^8\,\text{m}$. For the Earth–Moon system, we use the parameters

$$M_\oplus = 6.0 \times 10^{24}\,\text{kg}, \qquad R_\oplus = 6.4 \times 10^6\,\text{m}, \tag{1}$$
$$M_{\text{Moon}} = 7.3 \times 10^{22}\,\text{kg}, \qquad R_{\text{Moon}} = 1.7 \times 10^6\,\text{m}, \tag{2}$$
$$D = 3.8 \times 10^8\,\text{m}. \tag{3}$$

The moments of inertia are

$$I_\oplus = \frac{2}{5} M_\oplus R_\oplus^2 = \frac{2}{5}(6.0 \times 10^{24})(6.4 \times 10^6)^2 \approx 9.8 \times 10^{37}\,\text{kg}\,\text{m}^2, \tag{4}$$
$$I_{\text{Moon}} = \frac{2}{5} M_{\text{Moon}} R_{\text{Moon}}^2 \approx 8.4 \times 10^{34}\,\text{kg}\,\text{m}^2. \tag{5}$$

The Newtonian gravitational force is

$$F_N = \frac{G M_\oplus M_{\text{Moon}}}{D^2} \approx 2.0 \times 10^{20}\,\text{N}. \tag{6}$$

The effective rotational correction is

$$F_{\text{eff}} = C \frac{G I_\oplus I_{\text{Moon}}}{D^6} \approx 10^{-15}\,\text{N}, \tag{7}$$

where $C \sim 1$ has been assumed.

The total gravitational force is therefore

$$F_{\text{total}} = F_N + F_{\text{eff}} \tag{8}$$
$$= \quad 2.0 \times 10^{20}\,\text{N} \quad + \quad 1.0 \times 10^{-15}\,\text{N} \tag{9}$$
$$= 2.0 \times 10^{20}\,\text{N} + 1.0 \times 10^{-15}\,\text{N} \tag{10}$$
$$\tag{11}$$

Table 1: Order-of-magnitude estimates for the Earth–Moon system using the rigid-sphere moment of inertia approximation.

Quantity	Value
Center-of-mass separation D	$3.8 \times 10^8\,\text{m}$
Newtonian force F_N	$\sim 10^{20}\,\text{N}$
Effective correction F_{eff}	$\sim 10^{-15}\,\text{N}$
Ratio F_{eff}/F_N	$\sim 10^{-35}$

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For the Sun–Earth system, the estimates use $M_\odot = 2.0 \times 10^{30}\,\text{kg}$, $R_\odot = 7.0 \times 10^{8}\,\text{m}$, $M_\oplus = 6.0 \times 10^{24}\,\text{kg}$, $R_\oplus = 6.4 \times 10^{6}\,\text{m}$, and a mean separation $D = 1.5 \times 10^{11}\,\text{m}$.

For the Sun–Earth system, we use

$$M_\odot = 2.0 \times 10^{30}\,\text{kg}, \qquad R_\odot = 7.0 \times 10^{8}\,\text{m}, \tag{12}$$

$$M_\oplus = 6.0 \times 10^{24}\,\text{kg}, \qquad R_\oplus = 6.4 \times 10^{6}\,\text{m}, \tag{13}$$

$$D = 1.5 \times 10^{11}\,\text{m}. \tag{14}$$

The moments of inertia are

$$I_\odot = \frac{2}{5} M_\odot R_\odot^2 \approx 3.9 \times 10^{47}\,\text{kg}\,\text{m}^2, \tag{15}$$

$$I_\oplus \approx 9.8 \times 10^{37}\,\text{kg}\,\text{m}^2. \tag{16}$$

The Newtonian force is

$$F_N = \frac{G M_\odot M_\oplus}{D^2} \approx 3.5 \times 10^{22}\,\text{N}. \tag{17}$$

The effective rotational correction is

$$F_{\text{eff}} = C \frac{G I_\odot I_\oplus}{D^6} \approx 10^{-12}\,\text{N}. \tag{18}$$

Thus, the total gravitational force is

$$F_{\text{total}} = F_N + F_{\text{eff}} \tag{19}$$

$$= \quad 3.5 \times 10^{22}\,\text{N} \quad + \quad 1.0 \times 10^{-12}\,\text{N} \tag{20}$$

$$= 3.5 \times 10^{22}\,\text{N} + 1.0 \times 10^{-12}\,\text{N} \tag{21}$$

$$\tag{22}$$

Table 2: Order-of-magnitude estimates for the Sun–Earth system assuming spherical bodies.

Quantity	Value
Center-of-mass separation D	$1.5 \times 10^{11}\,\text{m}$
Newtonian force F_N	$\sim 10^{22}\,\text{N}$
Effective correction F_{eff}	$\sim 10^{-12}\,\text{N}$
Ratio F_{eff}/F_N	$\sim 10^{-34}$

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For a compact binary neutron star system, typical parameters are

$$M_1 \approx M_2 \approx 1.4\,M_\odot \approx 2.8 \times 10^{30}\,\text{kg}, \tag{23}$$

$$R_1 \approx R_2 \approx 10^4\,\text{m}, \tag{24}$$

$$D \sim 10^6\,\text{m}. \tag{25}$$

The moments of inertia are

$$I_1 \approx I_2 = \frac{2}{5}MR^2 \approx 1.1 \times 10^{38}\,\text{kg}\,\text{m}^2. \tag{26}$$

The Newtonian gravitational force is

$$F_N = \frac{GM_1M_2}{D^2} \approx 5 \times 10^{30}\,\text{N}. \tag{27}$$

The effective rotational correction is

$$F_{\text{eff}} = C\frac{GI_1I_2}{D^6} \approx 10^{26}\,\text{N}. \tag{28}$$

The total force is therefore

$$F_{\text{total}} = F_N + F_{\text{eff}} \tag{29}$$

$$= \quad 5.0 \times 10^{30}\,\text{N} \quad + \quad 1.0 \times 10^{26}\,\text{N} \tag{30}$$

$$= 5.0 \times 10^{30}\,\text{N} + 1.0 \times 10^{26}\,\text{N} \tag{31}$$

$$. \tag{32}$$

Quantity	Value
Center-of-mass separation D	10^6 m
Newtonian force F_N	$\sim 10^{30}$ N
Effective correction F_{eff}	$\sim 10^{26}$ N
Ratio F_{eff}/F_N	$\sim 10^{-4}$

Table 3: Order-of-magnitude estimates for a compact binary neutron star system using the spherical moment of inertia approximation.

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Note: All graphical illustrations presented below are plotted on log–log scales.

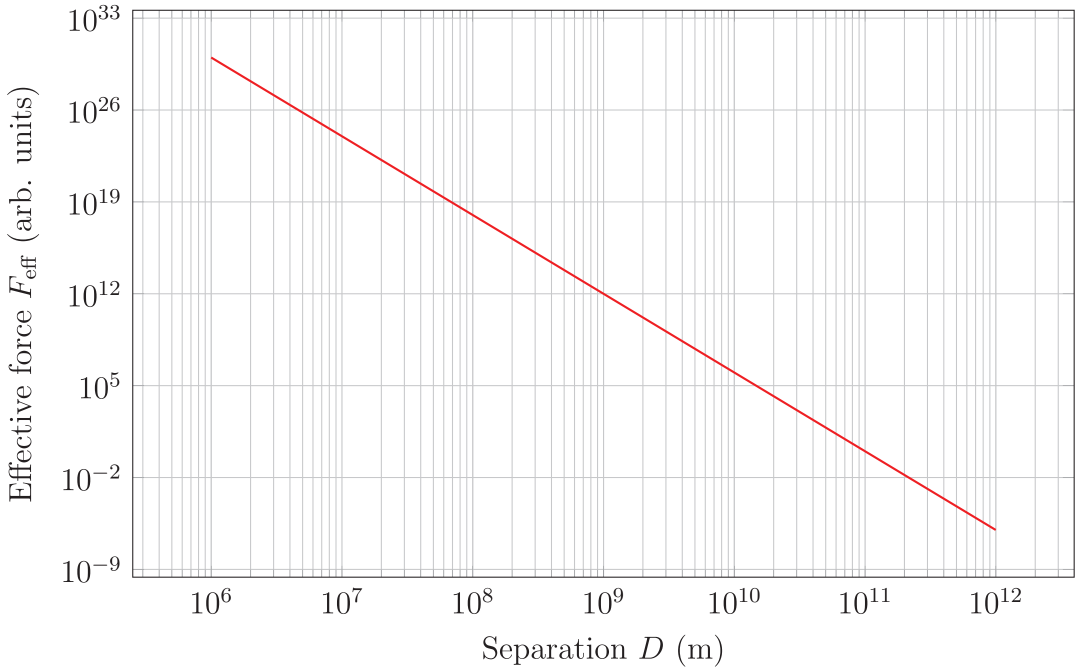

Figure 1: Log–log plot of the effective rotational correction as a function of separation distance, illustrating the inverse sixth-power dependence $F_{\text{eff}} \propto D^{-6}$.

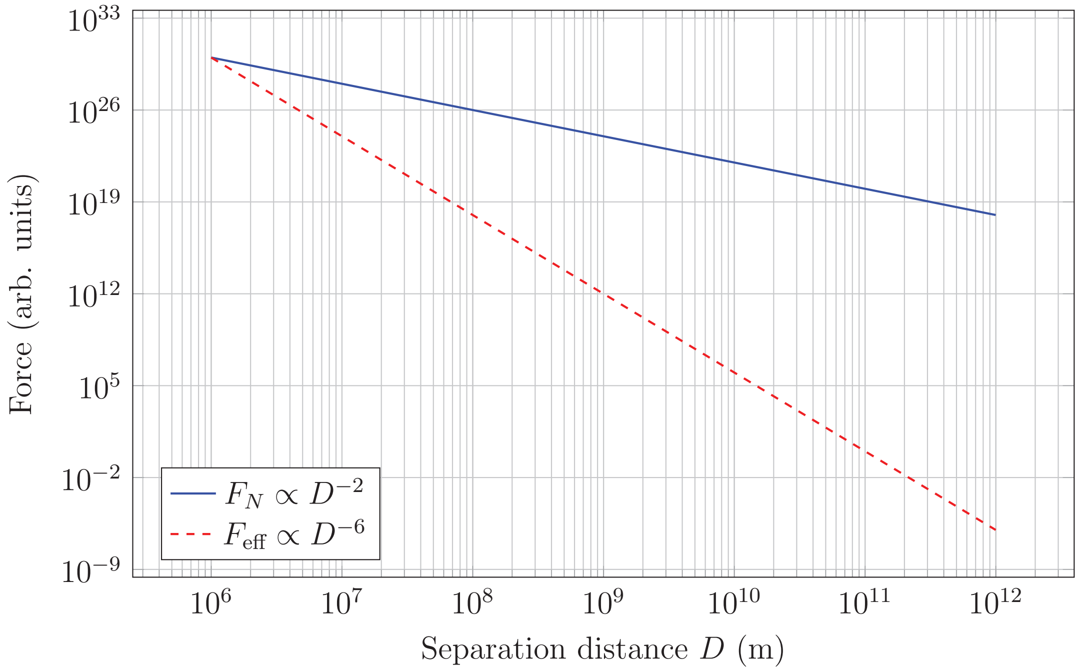

Figure 2: Log–log comparison of the Newtonian gravitational force and the effective rota-tional correction as functions of separation distance.

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The graphical results presented above are obtained directly from the analytical force rela-tions derived in this work. They illustrate physically meaningful trends and scaling behavior that may become relevant in specific regimes, particularly for compact or closely interacting systems, within the validity of the stated assumptions.

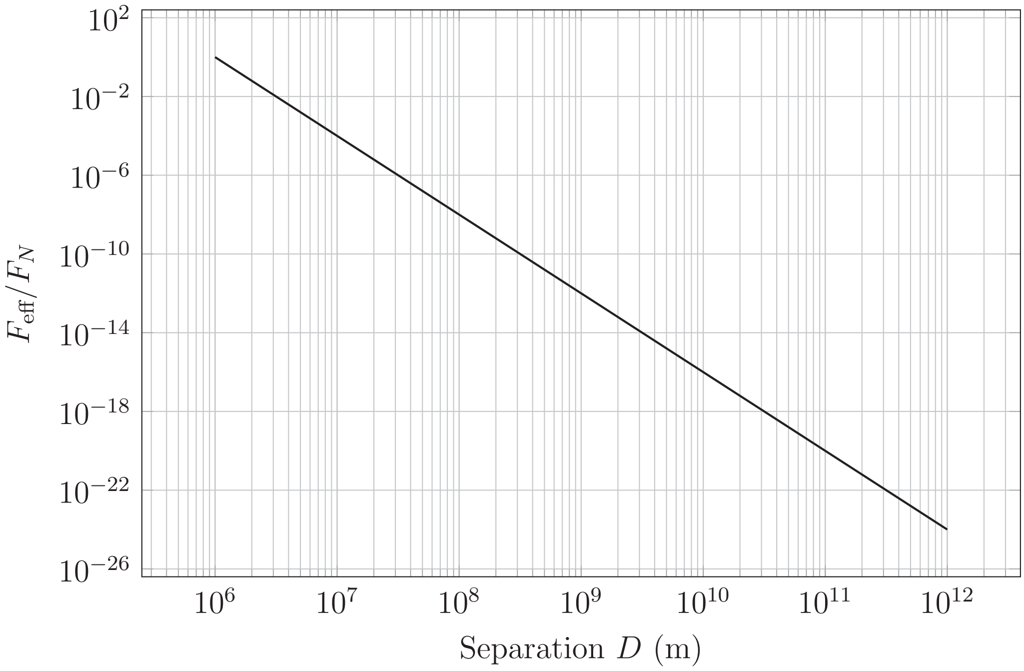

Figure 3: Ratio of the effective rotational correction to the Newtonian force as a function of separation distance, illustrating the rapid suppression at large separations.

Note: The following figures are code-generated schematic illustrations based on the analytical relations developed in this work. They are intended to visualize rotational motion, separation distance, and force dependence within the stated assumptions.
All illustrations above are generated directly from the theoretical framework developed in this work and are intended to provide physical intuition regarding rotational motion, separation dependence, and force scaling in different gravitational systems.

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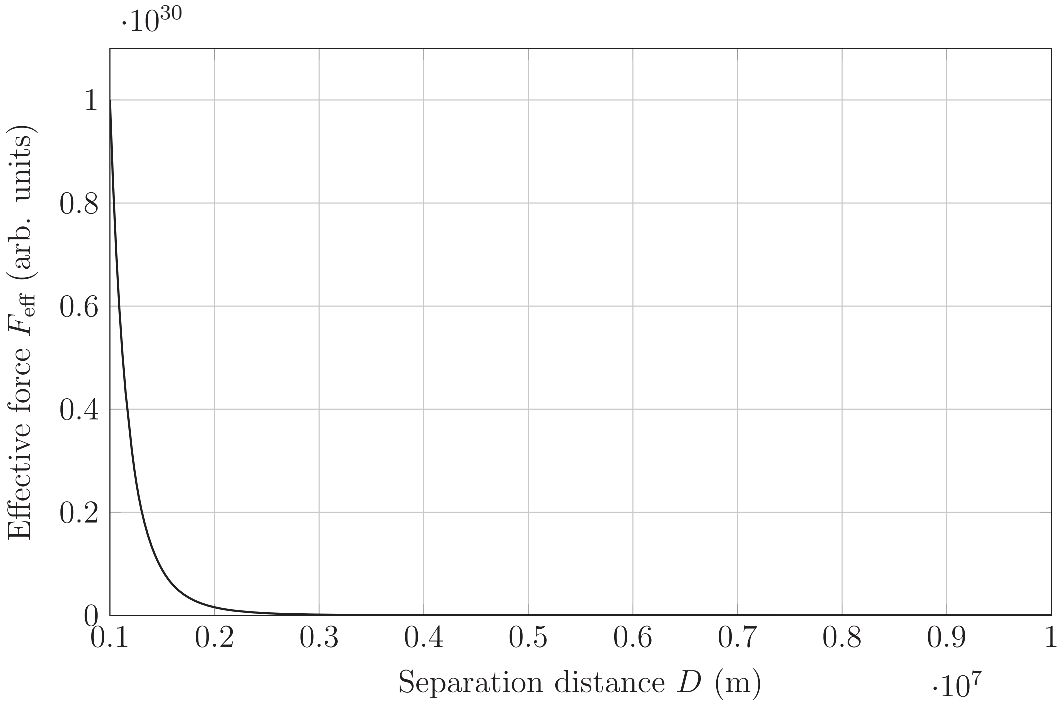

Figure 4: Effective rotational correction as a function of separation distance plotted on linear axes [no logarithmic scaling], showing a steep hyperbola-like decay consistent with $F_{\text{eff}} \propto D^{-6}$.

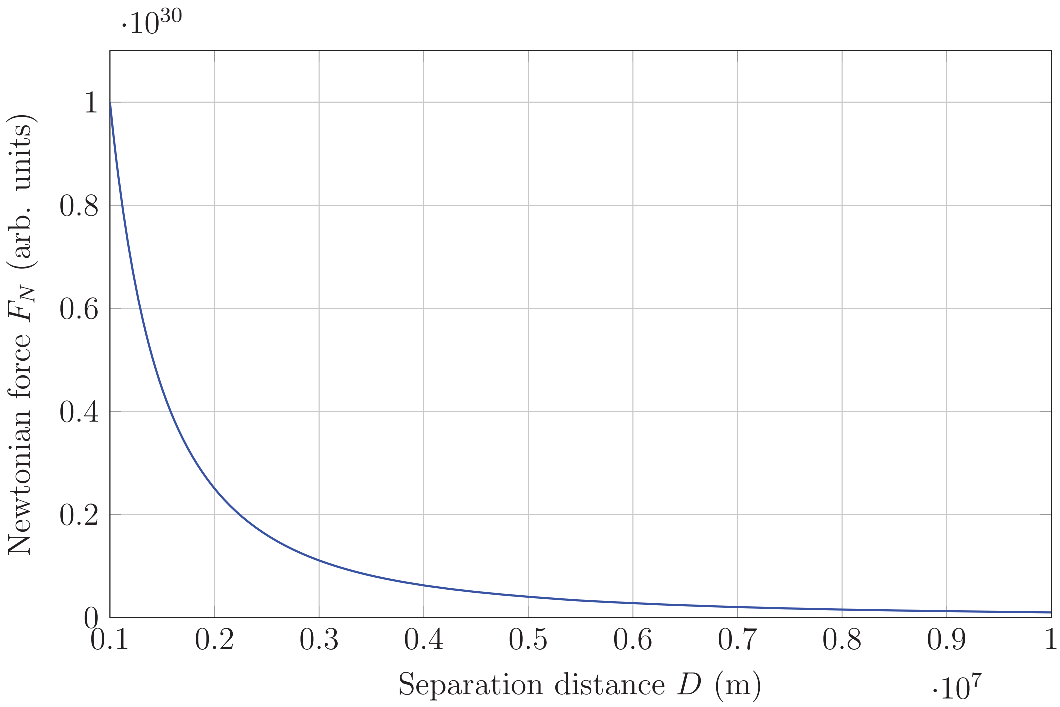

Figure 5: Newtonian gravitational force as a function of separation distance plotted on normal linear axes[no logarithmic scaling], showing the inverse-square dependence $F_N \propto D^{-2}$.

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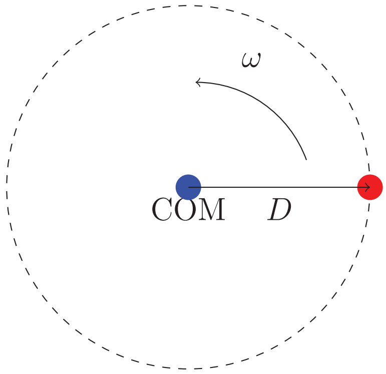

Figure 6: Schematic illustration of Newtonian circular motion with angular velocity ω and separation distance D.

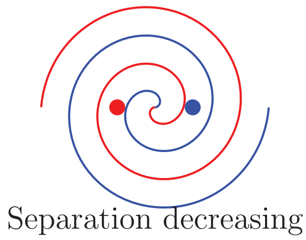

Figure 7: Code-generated schematic illustration of a binary inspiral showing decreasing separation prior to collision.

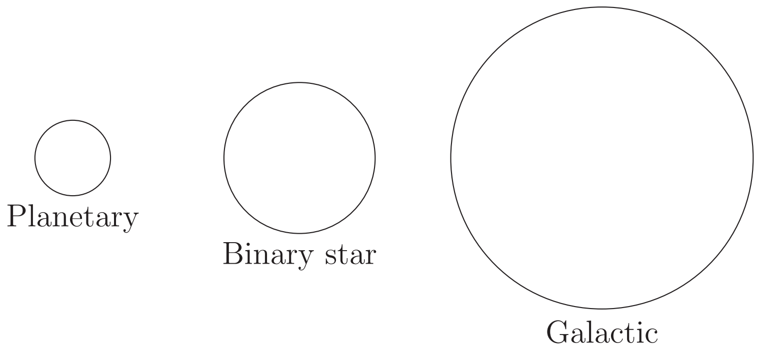

Figure 8: Qualitative comparison of characteristic separation scales across different gravitational systems.

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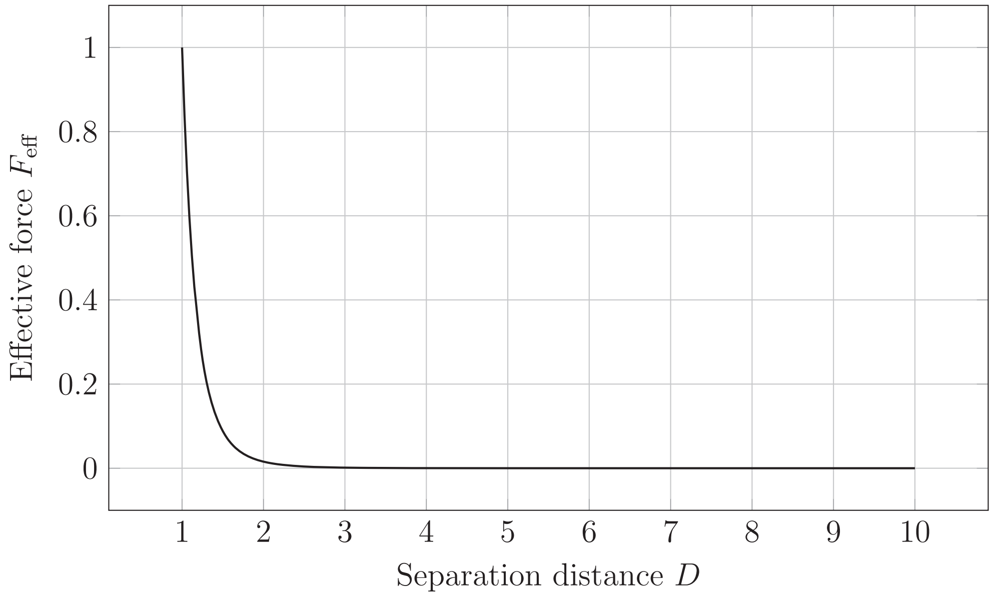

Figure 9: Linear-axis plot illustrating the inverse sixth-power dependence of the effective force on separation distance.

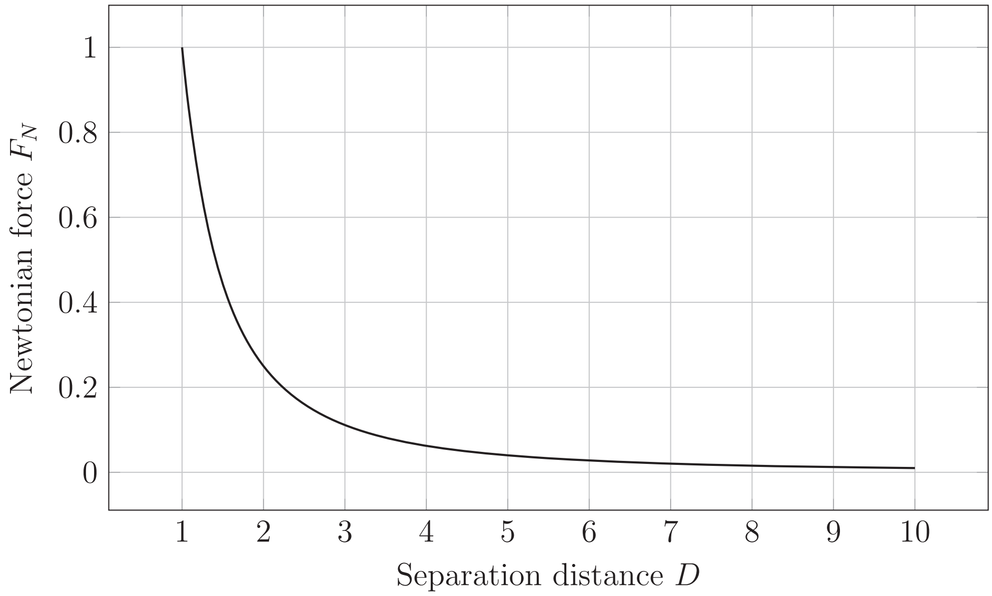

Figure 10: Linear-axis plot illustrating the inverse-square dependence of the Newtonian gravitational force on separation distance.

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D Discussion and Limitations

The results discussed in the previous section help us understand how the effective rotational correction behaves in different gravitational systems. The aim of this work is not to change or challenge Newtonian gravity, but to explore how real physical properties of bodies, such as rotation and internal mass distribution, can introduce additional terms when extended objects are considered instead of ideal point masses.

From the derived expressions and the graphs presented, it can be seen that the effective correction depends much more strongly on the separation distance than the usual Newtonian force. While the Newtonian force follows an inverse square dependence on distance, the effective term decreases with the inverse sixth power. Because of this, the correction becomes extremely small for systems with large separations, such as planetary or galactic systems. This explains why Newtonian gravity works very well in most practical situations and why the point-mass approximation is usually sufficient.

At the same time, the numerical estimates and schematic illustrations suggest that the effective correction may become more important in compact systems. When the separation between objects is small and their moments of inertia are large, the correction grows rapidly, even though it remains smaller than the Newtonian force. Examples of such systems include close binary stars or binary neutron stars during the late stages of their inspiral. In these cases, rotational and structural effects can contribute in a noticeable way while the weak-field approximation is still applicable.

The code-generated diagrams and drawings included in this work are meant to help build physical intuition. The orbital sketches, inspiral-like figures, and force–distance graphs provide a visual understanding of how rotation, separation, and force scaling are related. These figures are qualitative in nature and are not intended to represent exact dynamical evolution or realistic astrophysical simulations. Their role is simply to illustrate the behavior implied by the derived formulas.

There are several important limitations to the present analysis. First, the entire treat-ment is limited to weak gravitational fields and non-relativistic motion. Effects related to strong gravity, relativistic corrections, tidal deformation, and gravitational radiation are not included. For this reason, the results should not be applied to the final merger stages of compact objects or to systems where general relativity dominates the dynamics.

Second, the bodies are modeled as rigid objects with simplified mass distributions, often taken to be uniform spheres. In reality, astrophysical objects can have complex internal structures, differential rotation, and time-dependent deformation. These features can affect the detailed numerical form of higher-order corrections and are beyond the scope of this study.

Finally, the effective force term derived here should be understood as an additional con-tribution within the Newtonian framework, not as a new fundamental force. Its purpose is to show how known physical properties, such as rotation and mass distribution, can naturally lead to higher-order corrections when extended bodies are treated more realistically.

Overall, this work presents a simple and physically consistent approach to examining rotational effects in gravitational interactions. Although limited in scope, it provides useful insight and can serve as a starting point for further studies, including more realistic modeling or extensions into relativistic regimes

E Conclusion

In this work, an effective rotational correction to the Newtonian gravitational interaction has been studied for extended and rotating bodies. The main idea of this work was not to change Newtonian gravity, but to understand how real physical properties such as rotation and internal mass distribution can affect gravitational interaction when objects are not treated as ideal point masses.

By starting from the classical Newtonian framework and using reasonable physical as-sumptions, an effective force term proportional to the product of the moments of inertia and inversely proportional to the sixth power of the separation distance was obtained. This term appears naturally as a higher-order correction and does not replace the usual Newtonian force. Instead, it adds a small contribution that represents rotational and structural effects that are normally ignored in simple models.

Analytical results, numerical estimates, and code-generated illustrations were used to examine how this effective correction behaves in different gravitational systems. The results show that the correction is extremely small for systems with large separation distances, such as planetary or galactic systems. This explains why Newtonian gravity works very well in most everyday and astronomical situations. However, as the separation between objects decreases, the correction increases rapidly, suggesting that it may become more important in compact systems such as close binary stars or binary neutron stars, as long as the weak-field approximation remains valid.

The graphs and schematic illustrations included in this work help in building physical understanding. Force–distance plots and simple drawings make it easier to see how the effective term depends on distance and how it compares with the Newtonian force. These figures are not meant to describe real astrophysical evolution, but they clearly show the behavior predicted by the derived formulas.

There are several limitations to this study. The analysis is limited to weak gravitational ï¬ elds, non-relativistic motion, and simpliï¬ ed models of extended bodies, which are often assumed to be rigid and uniform. Important eï¬€ects such as strong gravity, relativistic motion, tidal deformation, and gravitational radiation are not included. Because of this, the results should not be applied to strongly relativistic systems or to the ï¬ nal stages of compact object mergers.

Despite these limitations, this work shows that rotational and structural properties can be included in classical gravity in a clear and consistent way. The approach presented here provides a simple starting point for further study and can be extended in the future to include more realistic body structures or relativistic eï¬€ects. Overall, this study aims to improve understanding of how classical gravitational theory can be reï¬ ned when real physical properties of extended bodies are taken into account.

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References

[1] Walter Lewin, For the Love of Physics: From the End of the Rainbow to the Edge of Time, Free Press, New York, 2011.

[2] Richard P. Feynman, Robert B. Leighton, and Matthew Sands, The Feynman Lectures on Physics, Vol. I, Addison-Wesley, 1964.

[3] Standard online educational materials and lecture notes were consulted for learning vector and tensor calculus, index notation, and related mathematical methods required for this work.

[4] MathAddict.net, Planetary and Astronomical Data Resources, available online at: https://www.mathaddict.net

[5] E. F. Taylor and J. A. Wheeler, Spacetime Physics, W. H. Freeman (1992).