

An Economic Design of a Modified Synthetic Double Sampling Control Chart for Process Monitoring

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Abstract:- The synthetic double sampling control chart proposed in this article is an alternative to the classical double sampling (DS) \bar{X} control chart for the monitoring of small process mean shift (δ). This study constructs an economic design model of the synthetic DS control chart and DS \bar{X} control chart in order to estimate the control charts parameters using MATLAB (R2013a) program to solve the optimization problem. Random samples were used to illustrate the use of this model in detecting small process mean shift. The results from this study showed; the DS \bar{X} control chart required an additional lapse of 6 Average Run Length (ARL) and 50 lapse of Average Number of Observations to Sample (ANOS) before an out-of-control status is issued. The synthetic DS control chart reduces the out-of-control ARL and ANOS values by more than half when compared with the DS \bar{X} control chart. In terms of detection ability the synthetic DS control chart is superior to the DS \bar{X} control chart.

Keywords: Synthetic control chart; Double Sampling; Head start rule; Conforming run length; Average run length.

1. INTRODUCTION

Many manufacturing processes produce inherent or natural variability as a cumulative effect of unavoidable causes [17]. Control chart is one of the well-known statistical process control (SPC) tools used to monitor process variation, either as a result of un-assignable or assignable causes. As a result, the Shewhart \bar{X} control chart has been widely used for monitoring large process shifts. A number of alternatives control charts has been proposed to improve the sensitivity of the control charts by employing control charts with warning limits that uses the head start rule to monitor process variability [21]. Giving a head start to a control chart is important in situations where after restarting the control chart following an alarm, one is interested to know instantaneously whether the process returns under statistical control; i.e., whether the process adjustment has successfully return the mean to its target value [16]. The Average Run Length (ARL) is the usual metric use to measure the performance of a control chart when the sample size in each sampling stage is fixed [16]. The Average Run Length is the expected number of samples required by the chart to signal an out-of-control status (false alarm). When the sample size in each sampling stage is not fixed, the Average Number of Observation to

Samples (ANOS) is used to evaluate the control chart's performance.

The synthetic DS chart is a combination of the synthetic control chart proposed by Wu and Spedding [28] and DS \bar{X} chart of Daudin [12], this control chart can be represented as a runs rule chart with a head start feature. The head start rule increases the sensitivity of the synthetic DS chart to detecting an out-of-control situation at first time. This article proposes a modified synthetic DS control chart that integrates the DS \bar{X} of Daudin [12] and the Conforming Run Length chart of Bourke [19] by employing a mathematical model different from the one proposed by [16]. The proposed models were used to reduce the second sample control limit (L_2) in order to increase the sensitivity of the chart to detecting the out-of-control process. This modified chart is superior to the synthetic DS chart of Khoo and others [16], synthetic control chart of Wu and Spedding [28] and the DS \bar{X} chart of [12].

1.1. Literature of the DS \bar{X} and synthetic chart

The DS \bar{X} control chart was first introduced by [22]. In this DS \bar{X} chart, information from the first and second stage is evaluated separately. Daudin and others [11] proposed a DS \bar{X} chart that uses information from both stages at the second stage to arrive at a conclusion. Irianto and Shinozaki [8] examined both procedures and concluded that Daudin and others [11] DS \bar{X} chart was better than that of [22]. This DS \bar{X} chart uses optimization of the expected sample size as the objective function to obtain the control chart parameters. However, this was not effective in minimizing the expected sample size. Irianto and Shinozaki [8] proposed a model that increased the sensitivity of the chart by maximizing the power of the chart to detect small mean shift in the process. Double Sampling S chart was developed to monitor process standard deviation by [5]. He and others [6] solved the DS \bar{X} chart design parameters using genetic algorithm and develop a triple \bar{X} charts. He and Grigoryan [7] presented a joint statistical design of DS \bar{X} chart and S charts. Costa and Claro [2] developed a DS \bar{X} chart for a first order autoregressive process, to minimize the chance of variability due to assignable causes within a sample and to maximize the chance between samples if assignable causes are present. Torng and Lee [3] proposed a modified statistical design of DS \bar{X} chart based on the

multi-objective programming method and a genetic algorithm. Torng and Lee [4] studied the performance of DS \bar{X} chart under non-conformity. Lee and others [20] proposed an economic design of combined DS \bar{X} chart and Variable interval \bar{X} control chart. This chart uses genetic algorithm to determine the chart parameters, to successfully reduce the detection time of small process mean shift.

The synthetic control chart was first introduced by [27]. This control chart combines the usefulness of the shewhart \bar{X} chart and Conforming Run Length (CRL) chart. The proposed synthetic chart was effective in detecting small shift in the process mean; this chart outperforms its component charts. I.e. the shewhart \bar{X} chart and CRL chart. The robustness of the synthetic chart to non-normality was proposed by [14]. Davis and Woodall [21] analyzed and improved on the performance of the synthetic control chart. Scariano and Calzada [25] developed a synthetic chart for detecting decrease in the exponential mean, which combines the shewhart \bar{X} chart for individual data and the CRL chart. Chen and Huang [10] merged the sample range (R) chart and CRL chart and also combined the standard deviation (S) chart with CRL chart. Costa and Rahim [1] proposed a synthetic chart based on non-central chi-square statistics. Khoo and others [15] developed a synthetic chart to monitor the process mean of skewed populations. A synthetic scaled weighted variance chart to monitor the process mean of skewed populations was developed by [18]. Machado [13] proposed a synthetic chart based on a new statistic (VMAX), this new statistic is a function of the sample variances of two quality characteristics, used to control the covariance matrix of bi-variate processes. The synthetic T^2 chart which combines the CRL chart and a Hotelling's T^2 chart was proposed by [9]. Khilare and Shirke [24] proposed a non parametric shewhart-type synthetic chart based on the signed-rank statistic to monitor process shifts in process median. A synthetic chart that hybridizes a sign chart and a CRL chart for detecting shifts in process median was proposed by [27]. Khoo and others [16] developed a synthetic DS chart that integrates the DS \bar{X} chart and the CRL chart to monitor the process mean. Haridy and others [23] proposed a combined synthetic chart and np scheme for detecting increase in fraction of non-conforming in a process, this chart was used to monitor fraction of non-conformities in a process for attributes.

2. MATERIALS AND METHODS

A review of the DS \bar{X} control chart of Daudin [12] and synthetic DS control chart of Khoo and others [16] is discussed extensively in this section.

2.1. DS \bar{X} control chart

The double sampling (DS) procedure constructed by Daudin [12] uses information from both stages to arrive at a conclusion. The following are the unions of the intervals of the proposed control chart model;

First stage;

$$I_1 = [\mu_0 - L_1\sigma_{\bar{X}_1}; \mu_0 + L_1\sigma_{\bar{X}_1}];$$

$$I_2 = [(\mu_0 - L\sigma_{\bar{X}_1}; \mu_0 - L_1\sigma_{\bar{X}_1}) \cup (\mu_0 + L_1\sigma_{\bar{X}_1}; \mu_0 + L\sigma_{\bar{X}_1})];$$

$$I_3 = [(-\infty; \mu_0 - L\sigma_{\bar{X}_1}) \cup (\mu_0 + L\sigma_{\bar{X}_1}; +\infty)];$$

Second stage;

$$I_4 = [\mu_0 - L_2\sigma_{\bar{X}}; \mu_0 + L_2\sigma_{\bar{X}}];$$

$$I_5 = [(-\infty; \mu_0 - L_2\sigma_{\bar{X}}) \cup (\mu_0 + L_2\sigma_{\bar{X}}; +\infty)];$$

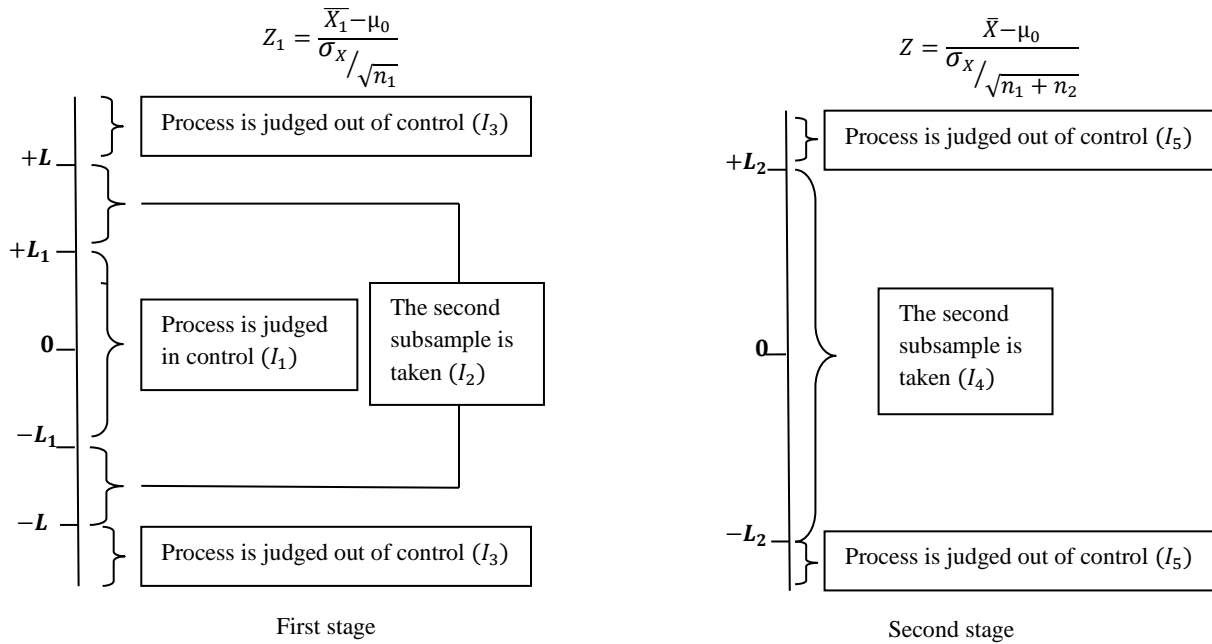


Figure 1: Graphical view of Daudin's [12] DS \bar{X} control chart

Let the probability of the random variable \bar{X}_1 in the interval of I_1 of the first stage be;

$$P_1 = \Pr[\bar{X}_1 \in I_1] = \Pr[\mu_0 - L_1 \sigma_{\bar{X}_1} < \bar{X}_1 < \mu_0 + L_1 \sigma_{\bar{X}_1}];$$

$$P_1 = \Phi(L_1 - \delta \sqrt{n_1}) - \Phi(-L_1 - \delta \sqrt{n_1}) \quad (2.1)$$

where $\delta = \frac{\mu_1 - \mu_0}{\sigma_{\bar{X}}}$ is the size of the mean shift of the process; when the process is in-control ($\delta=0$), μ_0 is the in-control mean, μ_1 is the out-of-control mean, $\sigma_{\bar{X}}$ is the in-control standard deviation (population mean), $\Phi(\cdot)$ is the cumulative distribution function of normal distribution, n_1 is the observations in the first sample and $\sigma_{\bar{X}_1}$ is the first stage standard deviation; where $\sigma_{\bar{X}_1} = \sigma_X / \sqrt{n_1}$ which can also be expressed as $\sqrt{n_1} = \frac{\sigma_X}{\sigma_{\bar{X}_1}}$.

Let the probability of the random variable \bar{X}_1 in the interval of I_2 (warning limit) of the first stage be;

$$P_2 = \Pr[\bar{X}_1 \in I_2] = \Pr[\mu_0 - L \sigma_{\bar{X}_1} < \bar{X}_1 < \mu_0 - L_1 \sigma_{\bar{X}_1}] + \Pr[\mu_0 + L_1 \sigma_{\bar{X}_1} < \bar{X}_1 < \mu_0 + L \sigma_{\bar{X}_1}];$$

$$P_2 = \Pr[\bar{X}_1 \in I_2] = \Phi(-L_1 - \delta \sqrt{n_1}) - \Phi(-L - \delta \sqrt{n_1}) + \Phi(L - \delta \sqrt{n_1}) - \Phi(L_1 - \delta \sqrt{n_1}); \quad (2.2)$$

Let the probability of the master sample mean \bar{X} in the interval of I_4 of the second stage be;

$$P_4 = \Pr[\bar{X} \in I_4] = \Pr[\mu_0 - L_2 \sigma_{\bar{X}} < \bar{X} < \mu_0 + L_2 \sigma_{\bar{X}}];$$

$$P_4 = \Phi(L_2 - \delta \sqrt{n_1 + n_2}) - \Phi(-L_2 - \delta \sqrt{n_1 + n_2}) \quad (2.3)$$

Where the master sample mean is given as;

$\bar{X} = \frac{(n_2 \bar{X}_2 + n_1 \bar{X}_1)}{(n_2 + n_1)}$. The index 1 & 2 represents the first stage and second stage. n_1 & n_2 are the observations in the first and second samples. $\sigma_{\bar{X}}$ is the master sample standard deviation which can be expressed as

$$\sigma_{\bar{X}} = \sigma_X / \sqrt{n_1 + n_2}.$$

Supposing the real process is in-control state, but the sample mean Z_1 or Z falls in I_2, I_3 & I_5 it will be concluded that the process is a Type I error (α) or false alarm (Producer's risks).

The Type I error (α) or false alarm is expressed as;

$$\alpha = P(Z_1 \in I_3) + P(Z_1 \in I_2) + P(Z \in I_5); \quad (2.4)$$

From common laws of probability the expressions below can be deduced;

From first stage;

$$P(Z_1 \in I_3) + P(Z_1 \in I_2) + P(Z_1 \in I_1) = 1;$$

$$P(Z_1 \varepsilon I_3) = 1 - P(Z_1 \varepsilon I_1) - P(Z_1 \varepsilon I_2);$$

Also from second stage;

$$P(Z \varepsilon I_4) + P(Z \varepsilon I_5) = 1;$$

$$P(Z \varepsilon I_5) = 1 - P(Z \varepsilon I_4);$$

By direct substitution into equation (2.4) the type I error becomes;

$$\alpha = 1 - P(Z_1 \varepsilon I_1) - [P(Z_1 \varepsilon I_2) \times P(Z \varepsilon I_4)];$$

$$\alpha = 1 - P1 - [P2 \times P4];$$

Substituting the parameters P1, P2 and P4.

$$\alpha = 1 - [\Phi(L_1 - \delta\sqrt{n_1}) - \Phi(-L_1 - \delta\sqrt{n_1})] - [\Phi(-L_1 - \delta\sqrt{n_1}) - \Phi(-L - \delta\sqrt{n_1}) + \Phi(L - \delta\sqrt{n_1}) - \Phi(L_1 - \delta\sqrt{n_1})] \times [\Phi(L_2 - \delta\sqrt{n_1 + n_2}) - \Phi(-L_2 - \delta\sqrt{n_1 + n_2})];$$

The size of the mean shift is ($\delta = 0$) at type I error so that;

$$\alpha = 1 - [\Phi(L_1) - \Phi(-L_1)] - [\Phi(-L_1) - \Phi(-L) + \Phi(L) - \Phi(L_1)] \times [\Phi(L_2) - \Phi(-L_2)] \quad (2.5)$$

In the case of process variation, when the mean shift to ($\mu_1 = \mu_0 + \delta\sigma_{\bar{x}}$) from the initial mean μ_0 . The process will be wrongly determined as an in-control state, that is Type II error (consumer's risks). The probability of this false determination is given by the expression below;

$$\beta_\delta = P(Z_1 \varepsilon I_1) + P(Z_1 \varepsilon I_2) \times P(Z \varepsilon I_4);$$

$$\beta_\delta = [\Phi(L_1 - \delta\sqrt{n_1}) - \Phi(-L_1 - \delta\sqrt{n_1})] + [\Phi(-L_1 - \delta\sqrt{n_1}) - \Phi(-L - \delta\sqrt{n_1}) + \Phi(L - \delta\sqrt{n_1}) - \Phi(L_1 - \delta\sqrt{n_1})] \times [\Phi(L_2 - \delta\sqrt{n_1 + n_2}) - \Phi(-L_2 - \delta\sqrt{n_1 + n_2})]; \quad (2.6)$$

The probability of the out-of-control process when the mean has shifted ($\delta > 0$) is called the Power of the control chart.

$$P_\delta = (1 - \beta_\delta) \quad (2.7)$$

The Average Run length (ARL) of the control chart which represents the expected number of points plotted on a control chart until an out-of-control signal is observed is given as;

In-control ARL ($\delta=0$);

$$ARL_o = (1/\alpha) \quad (2.8)$$

Out-of-control ARL ($\delta > 0$);

$$ARL_\delta = (1/P_\delta) \quad (2.9)$$

The average number of observation (ANOS) required in detecting a process mean shift of size (δ) before an out-of-control signal is issued for the DS \bar{X} control chart is computed as;

$$ANOS_\delta = ARL_\delta \times n$$

Where n represents the expected sample size, such that $ANOS_\delta$ becomes;

$$= ARL_\delta \times [n_1 + n_2 \times P(Z_1 \varepsilon I_2 | \delta)] \quad (2.10)$$

When the expected sample size is in-control ($\delta=0$);

$$n = n_1 + n_2 \times P(Z_1 \varepsilon I_2 | \delta = 0);$$

$$n = n_1 + n_2 \times P(Z_1 \varepsilon I_2)$$

But $P(Z_1 \varepsilon I_2) = P_2$;

$$n = n_1 + n_2 \times [\Phi(-L_1) - \Phi(-L) + \Phi(L) - \Phi(L_1)]$$

Expand the brackets by removing the (-) from the brackets;

$$n = n_1 + n_2 \times [\Phi(-L_1) - \Phi(L_1) - \Phi(-L) + \Phi(L)]$$

$$n = n_1 + n_2 \times [-2\Phi(L_1) + 2\Phi(L)]$$

$$[-2\Phi(L_1) + 2\Phi(L)] = \frac{n - n_1}{n_2}$$

L can be expressed as shown below;

$$2\Phi(L) = 2\Phi(L_1) + \frac{n - n_1}{n_2}$$

$$L = \Phi^{-1}[\Phi(L_1) + (\frac{n-n_1}{2n_2})]; \quad (2.11)$$

Irianto and Shinozaki [12] showed that, the range of L_1 can be expressed as;

$$\Phi^{-1}(1 - \frac{n - n_1}{2n_2} - \frac{\alpha}{2}) \leq L_1 \leq \Phi^{-1}(1 - \frac{n - n_1}{2n_2})$$

Where the median of L_1 is used as the range of the inequality.

$$L_1 = \frac{\Phi^{-1}(1 - \frac{n-n_1}{2n_2} - \frac{\alpha}{2}) + \Phi^{-1}(1 - \frac{n-n_1}{2n_2})}{2}$$

By simplification the above expression can be expressed as follow;

$$L_1 = \Phi^{-1}(1 - \frac{(n-n_1)}{2n_2} - (\frac{\alpha}{4})); \quad (2.12)$$

When the process is in-control the power of the control chart becomes;

$$(P_\delta | \delta = 0) = \alpha;$$

$$(P_\delta | \delta = 0) = (1 - \beta_\delta | \delta = 0)$$

$$\alpha = 1 - [\Phi(L_1) - \Phi(-L_1)] - [\Phi(-L_1) - \Phi(-L)] + \Phi(L) - \Phi(L_1)]x[\Phi(L_2) - \Phi(-L_2)]$$

The parameter L_2 can be expressed as;

$$[\Phi(L_2) - \Phi(-L_2)] = \frac{1 - \alpha - [\Phi(L_1) - \Phi(-L_1)]}{[\Phi(-L_1) - \Phi(-L) + \Phi(L) - \Phi(L_1)]}$$

From grid search and application of normal distribution rule of interval the L.H.S of the above expression becomes;

$$2\Phi(L_2) - 1 = \frac{1 - \alpha - [\Phi(L_1) - \Phi(-L_1)]}{[\Phi(-L_1) - \Phi(-L) + \Phi(L) - \Phi(L_1)]}$$

$$L_2 = \Phi^{-1}\left[\left(\frac{1 - \alpha + \Phi(-L_1) - \Phi(L_1)}{2[\Phi(L) - \Phi(L_1) + \Phi(-L_1) - \Phi(-L)]}\right) + \left(\frac{1}{2}\right)\right] \quad (2.13)$$

2.2. Estimating the DS \bar{X} chart parameters

There are five parameters required to specify the Daudin's [12] DS \bar{X} charts limits, i.e. n_1, n_2, L_1, L_2 and L . Daudin [12] suggested an optimization procedure that minimizes the expected sample size to be inspected; the goal is to reduce the inspection cost of monitoring the manufacturing process (*producer risk 'α'*). Alternatively, Irianto and Shinozaki [8] proposed an optimization procedure that minimizes the ARL_δ (when the process mean has shifted) by maximizing the capability (P_δ) of the control chart in detecting the process mean shift (δ). The goal; is to minimize the risk of not knowing the process has deviated from the in-control process mean position (*customers risk 'β_δ'*).

The proposed Irianto and Shinozaki [8] single objective optimization program is given below;

$$\max_{L, L_1, L_2} (1 - \beta_\delta)$$

Subject to:

(i) n [expected sample size $|\mu_1 = \mu_0| =$

$$n = n_1 + n_2 \times P(Z_1 \in I_2 | \delta = 0);$$

$$L = \Phi^{-1}[\Phi(L_1) + (\frac{n-n_1}{2n_2})]; \quad L_1 = \Phi^{-1}[1 - (\frac{n-n_1}{2n_2}) - \frac{\alpha}{4}];$$

(ii) $(P_\delta | \delta = 0) = \alpha;$

From the second constraint L_2 can be uniquely determined at fixed values of L_1 & L as:

$$L_2 = \Phi^{-1}\left[\left(\frac{1 - \alpha + \Phi(-L_1) - \Phi(L_1)}{2[\Phi(L) - \Phi(L_1) + \Phi(-L_1) - \Phi(-L)]}\right) + \left(\frac{1}{2}\right)\right];$$

The proposed heuristic algorithm is shown in Figure 2;

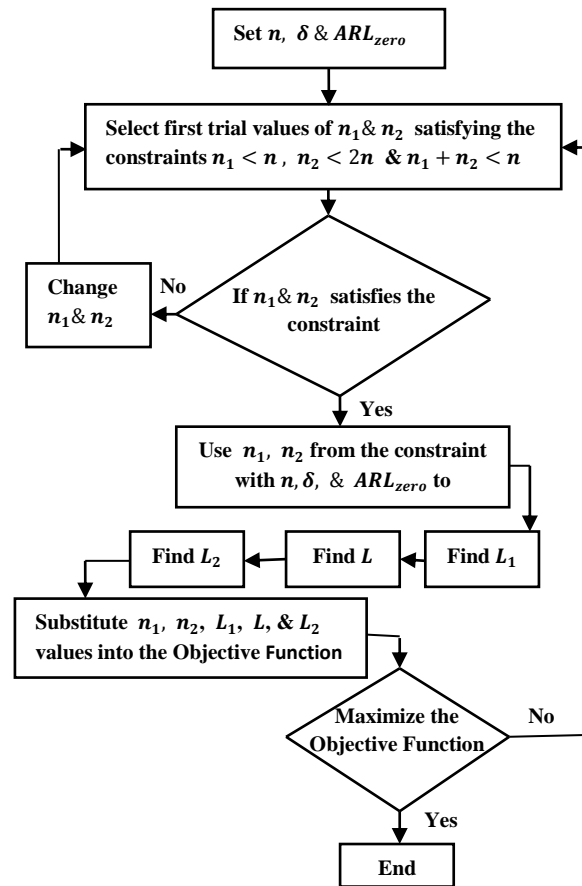


Figure 2: Heuristic algorithm for finding the DS \bar{X} control chart parameters

2.3. How to construct the DS \bar{X} control chart

- Step 1 Determine the parameters n_1, n_2, L, L_1 and L_2 .
- Step 2 Take the first samples of size n_1 at sampling stage j ; $X_{1i}, i = 1, 2, \dots, n_1$ where i is the number of sample observations from a population with a mean value of μ_0 and in-control standard deviation σ_X . Compute the sample mean $\bar{X}_{1,j} = \sum_{i=1}^{n_1} (X_{1i}/n_1)$ and normalize it as $Z_{1,j} = \frac{\bar{X}_{1,j} - \mu_0}{\sigma_X/\sqrt{n_1}}$.
- Step 3 if $Z_{1,j}$ is in I_1 , the process is in-control. The process flows back to step 2.
- Step 4 if $Z_{1,j}$ is in I_3 , the process is out of control. The process flows to step 8.
- Step 5 if $Z_{1,j}$ is in I_2 , a second sample of size n_2 , $X_{2i}, i = 1, 2, \dots, n_2$ from a population in which the first sample was taken. Then compute the second stage sample mean as $\bar{X}_{2,j} = \sum_{i=1}^{n_2} (X_{2i}/n_2)$.
- Step 6 The overall sample mean is computed as $\bar{X}_j = \frac{(n_2\bar{X}_{2,j} + n_1\bar{X}_{1,j})}{(n_2 + n_1)}$. Then normalize it as $Z_j = \frac{\bar{X}_j - \mu_0}{\sigma_X/\sqrt{n_1 + n_2}}$.

- Step 7 if Z_j is in I_4 , the process is in-control state and the control flow returns to step 2, else the process is out-of-control and flows to step 8.
- Step 8 an out-of-control signal is observed and corrective action is taken by removing the assignable causes so that the process flows back step 2.

2.4. Synthetic DS control chart

The proposed synthetic DS chart integrates the DS \bar{X} chart of Daudin [12] and Conforming Run Length (CRL) chart of Bourke [19]. This chart consists of DS/S sub-chart and the CRL/S sub-chart respectively.

2.5. Conforming run length control chart

When 100% inspection is carried out in a manufacturing process, the CRL is the number of inspected units between two consecutive non-conforming units including the end non-conforming unit. Suppose a process starts from $t=0$, such that p represents individual samples at each sampling stage. The non-conforming unit is represented with a solid circle while the conforming unit is represented with a hollow circle. Hence, the two samples of CRL are given as $CRL_1 = 3$ and $CRL_2 = 4$ (see Figure 3). The CRL increases as the fraction of non-conforming unit (p) in a process reduces and vice versa.

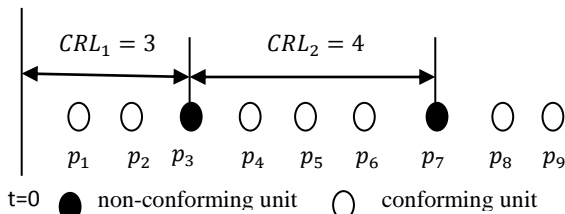


Figure 3: Conforming run length control chart [19]

The average number of the inspected sample required to signal a fraction of non-conforming shift is represented as;
 $ANS_{CRL} = \mu_{CRL} \times ARL_{CRL}$

$$= \frac{1}{p} \times \frac{1}{1-(1-p)^L} \quad (2.14)$$

Where ARL_{CRL} is the average number of CRL samples required to detect a change in p , μ_{CRL} is the mean value of the CRL and $CRL=L=1, 2, 3, \dots, m$.

After the integration of the CRL chart with the DS \bar{X} control chart to form the synthetic DS control chart. The CRL is replaced with the number of sampling stages between two consecutive non-conforming sampling stages (including the last non-conforming sampling stage). A sampling stage is regarded as the number of observations of the first samples (n_1) or the combination of both samples (n_1 and n_2). The DS/S sub chart description for in-control and out-of-control processes are replaced with conforming and non-conforming sampling stages.

2.6. Estimating the synthetic DS control chart parameters

There are six parameters required to specify the synthetic DS control charts limits, i.e. $n_1, n_2, L, L_1, L_2, \text{ and } L_3$. The first three L parameters are estimated from the optimization procedures presented in section (2.2). In addition, L_3 is used to denote the lower limit of the CRL/S sub-chart.

The equation in (2.14) was given by Wu and Spedding [28] for estimating the CRL chart parameters. Where p ; the probability of having non-conforming unit is replaced with the detecting power (P_δ) see equation (2.15). Let $ARL_{SDS}(\delta)$ represent the average number of sampling stages required to detect a mean shift of size (δ) under a zero state be computed as;

$$\delta > 0; ARL_{SDS}(\delta) = \frac{1}{P(\delta)} \times \frac{1}{1-[1-P(\delta)]^{L_3}} \quad (2.15)$$

$$\delta = 0; ARL_{SDS}(0) = \frac{1}{P(0)} \times \frac{1}{1-[1-P(0)]^{L_3}} \quad (2.16)$$

Equation (2.16) is reduced to;

$$P(0) - P(0)[1 - P(0)]^{L_3} - \alpha = 0; \quad (2.17)$$

Similarly, $ANOS_{SDS}(\delta) = ARL_{SDS}(\delta) \times n$

$$= ARL_{SDS}(\delta) \times [n_1 + n_2 \times P(Z_1 \in I_2 | \delta)] \quad (2.18)$$

2.7. How to construct synthetic DS chart

- Step 1 Determine the optimal parameters n_1, n_2, L, L_1, L_2 and L_3 using the single objective optimization procedures above.
- Step 2 Take the first sample of size n_1 at the first sampling stage j with observations $X_{1i}, i = 1, 2, \dots, n_1$, where i represents the number of observations from a population with a mean value of μ_0 and in-control standard deviation σ_X . Then compute the sample mean $\bar{X}_{1j} = \sum_{i=1}^{n_1} (X_{1i}/n_1)$ and normalize it as $Z_{1,j} = \frac{\bar{X}_{1j} - \mu_0}{\sigma_X/\sqrt{n_1}}$.
- Step 3 if $Z_{1,j}$ is in I_1 , sampling stage j is conforming. The process flows back to step 2.
- Step 4 if $Z_{1,j}$ is in I_3 , sampling stage j is non-conforming. The process flows to step 8.
- Step 5 if $Z_{1,j}$ is in I_2 , a second sample of size n_2 , $X_{2i}, i = 1, 2, \dots, n_2$ from a population in which the first sample was taken. Then compute the second sample mean as $\bar{X}_{2j} = \sum_{i=1}^{n_2} (X_{2i}/n_2)$
- Step 6 The overall sample mean is computed as $\bar{X}_j = \frac{(n_2 \bar{X}_{2j} + n_1 \bar{X}_{1j})}{(n_2 + n_1)}$. Then normalize it as $Z_j = \frac{\bar{X}_j - \mu_0}{\sigma_X/\sqrt{n_1 + n_2}}$
- Step 7 if Z_j is in I_4 , sampling stage j is conforming. The control flows back to step 2, else the process is non-conforming and flows to step 8.
- Step 8 Count the number of sampling stages between the present included in the count and the last excluded

in the count non-conforming stages and take it as the CRL value.

Step 9 If $CRL > L_3$ the process is in-control state, hence return to step 2. Otherwise an out-of-control is signalled. The process flows to step 10.

The proposed heuristic algorithm for finding the synthetic DS control chart parameters is shown in Figure 4 as follows;

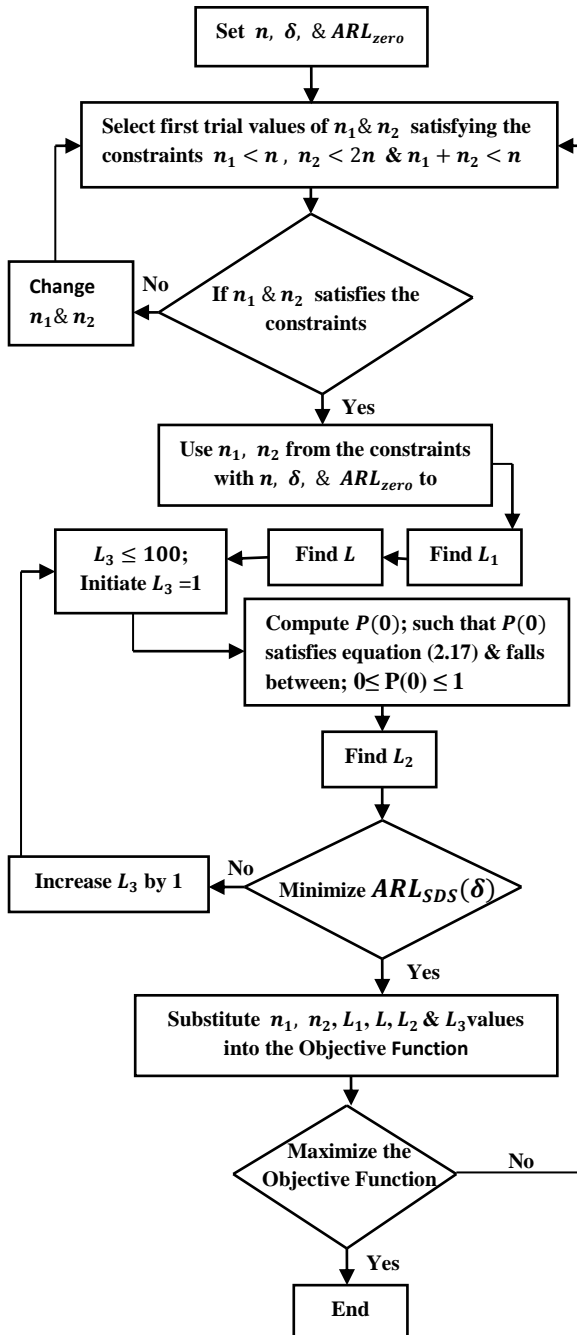


Figure 4: Heuristic algorithm for finding the synthetic DS control chart parameters

Step 10 Take the appropriate corrective action and remove the assignable causes so that the process flows back to step 2.

3. RESULTS AND DISCUSSION

This section presents the results and discussion of the DS \bar{X} control chart and synthetic DS control chart optimal parameters and their corresponding control chart plots based on random seed simulation using the optimization program presented in this work. It also shows a comparative analysis of both control charts to determine which chart is more superior in detecting the out-of-control situation.

3.1. Optimal parameters for the DS \bar{X} chart

The optimization programs were written in MATLAB (R2013a) to enable the end user determine the optimal parameters of the DS \bar{X} control chart. MATLAB (R2013a) is used since it has the capability of processing standardized statistics (Z) and it is widely available.

The optimization program computes the optimal values of n_1, n_2, L, L_1 & L_2 when the end user specifies the mean sample size (n), in-control $ARL_{DS}(0)$ and the magnitude of the desired mean shift (δ_{opt}) where a quick detection is desired. The program is based on standardized statistics, so $\mu_0 = 0$ & $\sigma = 1$.

For illustrative purpose, if the end user specifies $n = 5, ARL_{DS}(0) = 370$ & $\delta_{opt} = 0.5$. From Figure 5; the input parameters produced 30 pairs of n_1 & n_2 based on the following inequalities $n_1 < n$ & $n_2 < 2n$ satisfying the constraint $n_1 + n_2 > n$ when the mean sample size $n = 5$ at the point where $ARL_{delta} = 12.0619$ is minimum, the corresponding values gives the optimal parameters as shown in Figure 5. $n_1 = 4, n_2 = 10, L = 3.2052, L_1 = 1.6383$ & $L_2 = 2.4703$. Table 1 and Table 2 shows the various optimal parameters of the DS \bar{X} chart for combinations of $ARL_{DS}(0) \in \{250, 370\}, \delta_{opt} \{0.2, 0.4, 0.5, 0. \dots 2\}$ & $n \in \{2, 3, \dots 10\}$. Other combinations of $ARL_{DS}(0), \delta_{opt}$ & n can be computed using the optimization MATLAB (R2013a) program presented in this work. This program can be requested from the first author.

P1	P2	P4	Beta	Pdelta	n1	n2	L	L1	L2	ARLdelta
SOLUTION-MATRIX =										
0.7342	0.2521	0.7256	0.9171	0.0829	4.0000	10.0000	3.2052	1.6383	2.4703	12.0619
0.7166	0.2696	0.7596	0.9214	0.0786	4.0000	9.0000	3.2052	1.5872	2.5078	12.7298
0.6958	0.2905	0.7930	0.9261	0.0739	4.0000	8.0000	3.2052	1.5287	2.5491	13.5398
0.6704	0.3159	0.8256	0.9312	0.0688	4.0000	7.0000	3.2052	1.4603	2.5954	14.5321
0.6388	0.3474	0.8569	0.9366	0.0634	4.0000	6.0000	3.2052	1.3786	2.6479	15.7636
0.5980	0.3883	0.8866	0.9423	0.0577	4.0000	5.0000	3.2052	1.2777	2.7090	17.3179
0.5426	0.4437	0.9143	0.9483	0.0517	4.0000	4.0000	3.2052	1.1471	2.7822	19.3243
0.4612	0.5251	0.9396	0.9545	0.0455	4.0000	3.0000	3.2052	0.9647	2.8743	21.9986
0.3244	0.6619	0.9621	0.9612	0.0388	4.0000	2.0000	3.2052	0.6724	3.0000	25.7421
0.6437	0.3466	0.8176	0.9271	0.0729	3.0000	10.0000	3.2052	1.2777	2.7090	13.7138
0.6186	0.3717	0.8442	0.9324	0.0676	3.0000	9.0000	3.2052	1.2171	2.7437	14.7890
0.5886	0.4017	0.8695	0.9379	0.0621	3.0000	8.0000	3.2052	1.1471	2.7822	16.0964
0.5519	0.4384	0.8933	0.9435	0.0565	3.0000	7.0000	3.2052	1.0646	2.8252	17.7055
0.5057	0.4846	0.9153	0.9493	0.0507	3.0000	6.0000	3.2052	0.9647	2.8743	19.7146
0.4452	0.5451	0.9354	0.9551	0.0449	3.0000	5.0000	3.2052	0.8392	2.9314	22.2661
0.3612	0.6291	0.9532	0.9609	0.0391	3.0000	4.0000	3.2052	0.6724	3.0000	25.5708
0.2333	0.7570	0.9687	0.9666	0.0334	3.0000	3.0000	3.2052	0.4289	3.0866	29.9415
0.5871	0.4066	0.8662	0.9393	0.0607	2.0000	10.0000	3.2052	1.0335	2.8408	16.4792
0.5544	0.4393	0.8880	0.9445	0.0555	2.0000	9.0000	3.2052	0.9647	2.8743	18.0188
0.5147	0.4790	0.9083	0.9498	0.0502	2.0000	8.0000	3.2052	0.8846	2.9113	19.9077
0.4655	0.5282	0.9268	0.9551	0.0449	2.0000	7.0000	3.2052	0.7893	2.9527	22.2566
0.4023	0.5914	0.9436	0.9604	0.0396	2.0000	6.0000	3.2052	0.6724	3.0000	25.2231
0.3173	0.6764	0.9584	0.9656	0.0344	2.0000	5.0000	3.2052	0.5225	3.0551	29.0339
0.1953	0.7985	0.9711	0.9706	0.0294	2.0000	4.0000	3.2052	0.3169	3.1214	34.0144
0.5425	0.4540	0.8985	0.9504	0.0496	1.0000	10.0000	3.2052	0.8392	2.9314	20.1627
0.5001	0.4964	0.9166	0.9551	0.0449	1.0000	9.0000	3.2052	0.7624	2.9639	22.2723
0.4479	0.5486	0.9332	0.9598	0.0402	1.0000	8.0000	3.2052	0.6724	3.0000	24.8941
0.3818	0.6147	0.9480	0.9645	0.0355	1.0000	7.0000	3.2052	0.5640	3.0404	28.2061
0.2952	0.7013	0.9611	0.9692	0.0308	1.0000	6.0000	3.2052	0.4289	3.0866	32.4697
0.1758	0.8207	0.9723	0.9737	0.0263	1.0000	5.0000	3.2052	0.2516	3.1404	38.075
OPTIMAL PARAMETERS										
P1	P2	P4	beta	Pdelta	n1	n2	L	L1	L2	ARLdelta
0.7342	0.2521	0.7256	0.9171	0.0829	4.0000	10.0000	3.2052	1.6383	2.4703	12.0619

Figure 5: Possible pairs of n_1 & n_2 at $n = 5$, $ARL_{SDS}(0) = 370$ & $\delta_{opt} = 0.5$

Table 1: Optimal combinations of n_1, n_2, L, L_1, L_2 for DS \bar{X} control chart for $ARL_{DS}(0) = 250$

ARL _{DS} (0) = 250											
(δ) _{opt}											
n	0.2	0.4	0.5	0.6	0.8	1	1.2	1.4	1.5	2	
2	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521	1,4,3.0902, 1.1455, 2.6521
3	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,6,3.0902, 1.3765, 2.5121	2,5,3.0902, 1.2759, 2.5758	2,5,3.0902, 1.1455, 2.6521	2,5,3.0902, 1.1455, 2.6521
4	3,8,3.0902, 1.5260, 2.4089	3,8,3.0902, 1.5260, 2.4089	3,8,3.0902, 1.5260, 2.4089	3,8,3.0902, 1.5260, 2.4089	3,8,3.0902, 1.5260, 2.4089	3,8,3.0902, 1.5260, 2.4089	3,7,3.0902, 1.4579, 2.4573	3,6,3.0902, 1.3765, 2.5121	3,5,3.0902, 1.2759, 2.5758	3,4,3.0902, 1.1455, 2.6521	3,4,3.0902, 1.1455, 2.6521
5	4,10,3.0902, 1.6352, 2.3263	4,10,3.0902, 1.6352, 2.3263	4,10,3.0902, 1.6352, 2.3263	4,10,3.0902, 1.6352, 2.3263	4,10,3.0902, 1.6352, 2.3263	4,9,3.0902, 1.58844, 2.3656	4,7,3.0902, 1.4579, 2.4573	4,6,3.0902, 1.3765, 2.5121	4,5,3.0902, 1.2759, 2.5758	4,3,3.0902, 0.9634, 2.7478	4,3,3.0902, 0.9634, 2.7478
6	5,12,3.0902, 1.7205, 2.2571	5,12,3.0902, 1.7205, 2.2571	5,12,3.0902, 1.7205, 2.2571	5,12,3.0902, 1.7205, 2.2571	5,12,3.0902, 1.7205, 2.2571	5,9,3.0902, 1.5844, 2.3656	5,7,3.0902, 1.4579, 2.4573	5,5,3.0902, 1.2759, 2.5758	5,5,3.0902, 1.2759, 2.5758	5,3,3.0902, 0.9634, 2.7478	5,3,3.0902, 0.9634, 2.7478
7	6,14,3.0902, 1.7902, 2.1973	6,14,3.0902, 1.7902, 2.1973	6,14,3.0902, 1.7902, 2.1973	5,14,3.0902, 1.4579, 2.4573	5,14,3.0902, 1.4579, 2.4573	5,11,3.0902, 1.3291, 2.5427	6,6,3.0902, 1.3765, 2.5121	6,5,3.0902, 1.2759, 2.5758	6,5,3.0902, 1.2759, 2.5758	6,3,3.0902, 0.9634, 2.7478	6,3,3.0902, 0.9634, 2.7478
8	7,16,3.0902, 1.8487, 2.1444	7,16,3.0902, 1.8487, 2.1444	6,16,3.0902, 1.5260, 2.4089	6,16,3.0902, 1.5260, 2.4089	6,15,3.0902, 1.4934, 2.4324	6,11,3.0902, 1.3291, 2.5427	7,6,3.0902, 1.3765, 2.5121	7,5,3.0902, 1.2759, 2.5758	7,4,3.0902, 1.1455, 2.6521	7,3,3.0902, 1.1455, 2.6521	7,3,3.0902, 1.9634, 2.7478
9	7,18,3.0902, 1.5844, 2.3656	7,18,3.0902, 1.5844, 2.3656	7,18,3.0902, 1.5844, 2.3656	7,18,3.0902, 1.5844, 2.3656	7,15,3.0902, 1.4934, 2.4324	7,11,3.0902, 1.3291, 2.5427	8,7,3.0902, 1.3765, 2.5121	8,5,3.0902, 1.2759, 2.5758	8,4,3.0902, 1.1455, 2.6521	8,3,3.0902, 1.1455, 0.9634, 2.7478	8,3,3.0902, 1.1455, 0.9634, 2.7478
10	8,20,3.0902, 1.6352, 2.3263	8,20,3.0902, 1.6352, 2.3263	8,20,3.0902, 1.6352, 2.3263	8,20,3.0902, 1.6352, 2.3263	8,15,3.0902, 1.4934, 2.4324	8,11,3.0902, 1.3291, 2.5427	9,6,3.0902, 1.3765, 2.5121	9,4,3.0902, 1.1455, 2.6521	9,4,3.0902, 1.1455, 2.6521	9,2,3.0902, 0.6713, 2.8782	9,2,3.0902, 0.6713, 2.8782

Table 2: Optimal combinations of n_1, n_2, L, L_1, L_2 for DS \bar{X} control chart for $ARL_{DS}(0) = 370$

$ARL_{DS}(0) = 370$											
$(\delta)_{opt}$											
n	0.2	0.4	0.5	0.6	0.8	1	1.2	1.4	1.5	2	
2	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822	1,4,3.2052, 1.1471, 2.7822
3	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,6,3.2052, 1.3786, 2.6479	2,4,3.2052, 1.1471, 2.7822
4	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,8,3.2052, 1.5287, 2.5491	3,6,3.2052, 1.3787, 2.6479	3,6,3.2052, 1.3787, 2.6479	3,4,3.2052, 1.1471, 2.7822
5	4,10,3.2052, 1.6383, 2.4703	4,10,3.2052, 1.6383, 2.4703	4,10,3.2052, 1.6383, 2.4703	4,10,3.2052, 1.6383, 2.4703	4,10,3.2052, 1.6383, 2.4703	4,10,3.2052, 1.6383, 2.4703	4,10,3.2052, 1.6383, 2.4703	4,8,3.2052, 1.5287, 2.5491	4,6,3.2052, 1.3787, 2.6479	4,6,3.2052, 1.3787, 2.6479	4,4,3.2052, 1.1471, 2.7822
6	5,12,3.2052, 1.7241, 2.4044	5,12,3.2052, 1.7241, 2.4044	5,12,3.2052, 1.7241, 2.4044	5,12,3.2052, 1.7241, 2.4044	5,12,3.2052, 1.7241, 2.4044	5,10,3.2052, 1.6383, 2.4703	5,7,3.2052, 1.4603, 2.5954	5,6,3.2052, 1.3786, 2.6479	5,5,3.2052, 1.2777, 2.7090	5,5,3.2052, 1.2777, 2.7090	5,3,3.2052, 0.9647, 2.8743
7	6,14,3.2052, 1.7942, 2.3475	6,14,3.2052, 1.7942, 2.3475	6,14,3.2052, 1.7942, 2.3475	6,14,3.2052, 1.7942, 2.3475	5,14,3.2052, 1.4603, 2.5954	5,12,3.2052, 1.3786, 2.6479	6,7,3.2052, 1.4603, 2.5954	6,6,3.2052, 1.3786, 2.6479	6,5,3.2052, 1.2777, 2.7090	6,5,3.2052, 1.2777, 2.7090	6,3,3.2052, 0.9647, 2.8743
8	7,16,3.2052, 1.8532, 2.2974	7,16,3.2052, 1.8532, 2.2974	7,16,3.2052, 1.8532, 2.2974	6,16,3.2052, 1.5287, 2.5491	6,16,3.2052, 1.5287, 2.5491	6,12,3.2052, 1.5287, 2.6479	7,7,3.2052, 1.4603, 2.5954	7,6,3.2052, 1.3786, 2.6479	7,5,3.2052, 1.2777, 2.7090	7,5,3.2052, 1.2777, 2.7090	7,3,3.2052, 0.9647, 2.8743
9	8,18,3.2052, 1.9040, 2.2524	7,18,3.2052, 1.5872, 2.5075	7,18,3.2052, 1.5872, 2.5075	7,17,3.2052, 1.5590, 2.5279	7,17,3.2052, 1.5590, 2.5279	7,12,3.2052, 1.3786, 2.6479	8,7,3.2052, 1.4603, 2.5954	8,5,3.2052, 1.2777, 2.7090	8,5,3.2052, 1.2777, 2.7090	8,5,3.2052, 1.2777, 2.7090	8,3,3.2052, 0.9647, 2.8743
10	8,20,3.2052, 1.6383, 2.4703	8,20,3.2052, 1.6383, 2.4703	8,20,3.2052, 1.6383, 2.4703	8,20,3.2052, 1.6383, 2.4703	8,17,3.2052, 1.5590, 2.5279	8,12,3.2052, 1.3786, 2.6479	8,9,3.2052, 1.2171, 2.7437	9,5,3.2052, 1.2777, 2.7090	9,5,3.2052, 1.2777, 2.7090	9,5,3.2052, 1.2777, 2.7090	9,3,3.2052, 0.9647, 2.8743

3.2. Construction of the DS \bar{X} control chart

The procedures in section (2.3) are used to construct the DS \bar{X} control chart. For illustrative purpose, 300 random samples generated using MATLAB (R2013a) program were used to simulate the construction of the DS \bar{X} chart. If the desired expected sample size $n = 5$, in-control/zero state Average Run Length ($ARL_{DS}(0) = 370$) & magnitude of the mean shift desired is $(\delta)_{opt} = 0.5$; which gives the corresponding optimal parameters as shown in Table 2 where $n_1, n_2, L, L_1, & L_2$ are 4, 10, 3.2052, 1.6383 & 2.4703. From Step 2, let the first sampling stage be $j=1$ with first sample size $n_1 = 4$ with observations, $X_{1(2)} = -0.9595, X_{1(3)} = 1.6527$ & $X_{1(4)} = 0.2824$ be taken (see Table 3). The first sample mean is computed as $\bar{X}_{1(1)} = 0.6037$ and normalized as $Z_{1(1)} = 1.2073$. Next is Step 3, since $Z_{1(1)} \in I_1 = 1.2073 \in (-1.6383/1.6383)$ the process is in-control. Therefore, the process flows back to Step 2. For the second sampling stage $j=2$, the same process is observed until the fourth sampling stage $j=4$, where $n_1 = 4$ with first sample observations of $X_{1(1)} = -1.7036, X_{1(2)} = 0.7245, X_{1(3)} = -1.9864$ & $X_{1(4)} = -1.1173$. The first stage sample mean is computed as $\bar{X}_{1(4)} = -1.0207$ and normalized as $Z_{1(4)} = -2.0414$; but $Z_{1(4)} \in I_2 = -2.0414 \in (-3.2052/-1.6383)$. Next is Step 5, a second sample size of $n_2 = 10$ is observed with observations $X_{2(1)} = -1.9948, X_{2(2)} = -2.6734, X_{2(3)} = 0.9277, X_{2(4)} = 1.3004, X_{2(5)} = 1.4454,$

$X_{2(6)} = 0.3955, X_{2(7)} = 1.6042, X_{2(8)} = 1.7575, X_{2(9)} = -1.1153$ & $X_{2(10)} = 2.0693$ is taken from the same population from which the first sample is taken. The second stage sample mean is computed as $\bar{X}_{2(4)} = 0.1116$. Next is Step 6, the overall sample mean is computed as $\bar{X}_{(4)} = -0.2119$ and normalized as $Z_{(4)} = -0.7930$. Next is Step 7, since $(Z_{(4)} \in I_4) = -0.7930 \in (-2.4703/2.4703)$ the process is in-control, the process flows to Step 2. The process continues until the 12th sampling stage $j=12$, where the normalized first stage sample mean is $Z_{1(12)} = 3.4137$, since $3.4137 \in I_3$ the process is out-of-control, the process flows to Step 8. From Table 3, 30 sampling stages observed showed out-of-control samples at $Z_{1(12)} = 3.4137, Z_{1(22)} = 3.4435,$ & $Z_{1(23)} = 3.4332$ which shows three out-of-control situations at $j = 12, j = 22$ & $j = 23$ during the sampling of the data. The first Average Run length (ARL) which is the number of sampling stage before an out-of-control is observed becomes $ARL = 12$ and the Average Number of Observations to Samples before an out-of-control is signalled is $ANOS = 78$. From Table 3, a plot of $Z_{1(j)}$ and $Z_{(j)}$ against the sampling stages (j) gives the DS \bar{X} control chart as shown in Figure 6. The black dot represents the first stage sample normalized mean $Z_{1(j)}$ while the white dot is the second stage sample normalized mean $Z_{(j)}$. The out-of-control is shown by the red marker at $j=12, 22,$ & 23 . The entire process can be automated by using the MATLAB (R2013a) codes presented by the first author.

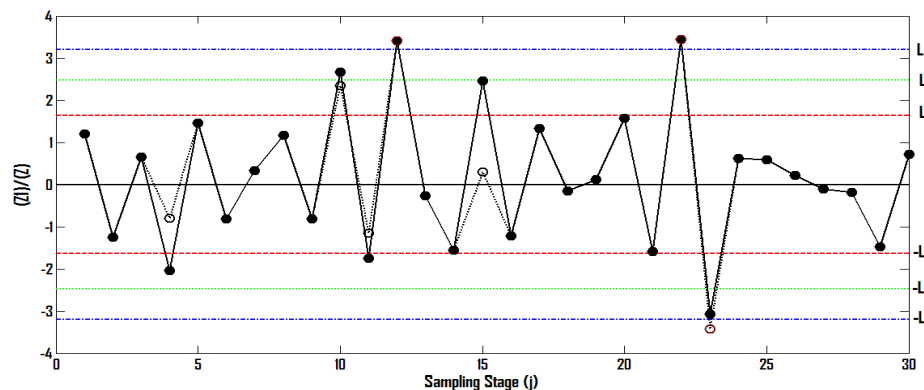


Figure 6: DS \bar{X} control chart

Table 3: Data for an illustrative example for the DS \bar{X} control chart

Sampling Stage (j)	$X_{1(1)}$	$X_{1(2)}$	$X_{1(3)}$	$X_{1(4)}$	$X_{2(1)}$	$X_{2(2)}$	$X_{2(3)}$	$X_{2(4)}$	$X_{2(5)}$	$X_{2(6)}$	$X_{2(7)}$	$X_{2(8)}$	$X_{2(9)}$	$X_{2(10)}$	$\bar{X}_{1(j)}$	$Z_{1(j)}$	$\bar{X}_{2(j)}$	\bar{X}_j	Z_j
1	1.4390	-0.9595	1.6527	0.2824	0	0	0	0	0	0	0	0	0	0	0.6037	1.2073	0	0	0
2	-0.9709	-0.8098	0.8006	-1.5232	0	0	0	0	0	0	0	0	0	0	-0.6258	-1.2517	0	0	0
3	-0.0197	0.8256	-1.0257	1.5403	0	0	0	0	0	0	0	0	0	0	0.3301	0.6603	0	0	0
4	-1.7036	0.7245	-1.9864	-1.1173	-1.9948	-2.6734	0.9277	-1.3004	1.4454	0.3955	1.6042	1.7575	-1.1153	2.0693	-1.0207	-2.0414	0.1116	-0.2119	-0.7930
5	3.4960	2.0951	-0.9405	-1.7266	0	0	0	0	0	0	0	0	0	0	0.7310	1.4620	0	0	0
6	-0.2547	-1.3046	-1.8956	1.8187	0	0	0	0	0	0	0	0	0	0	-0.4090	-0.8181	0	0	0
7	-0.2776	1.8462	1.0497	-1.9706	0	0	0	0	0	0	0	0	0	0	0.1619	0.3239	0	0	0
8	0.7202	0.8238	0.5805	0.2092	0	0	0	0	0	0	0	0	0	0	0.5834	1.1669	0	0	0
9	-1.1276	1.0895	-1.0879	-0.5165	0	0	0	0	0	0	0	0	0	0	-0.4106	-0.8213	0	0	0
10	1.5637	1.4255	-0.3903	2.7279	3.4345	1.6408	-1.6364	-0.3664	-0.6697	1.4123	-0.2304	1.6174	-1.8673	0.1297	1.3317	2.6634	0.3465	0.6280	2.3496
11	-2.8660	1.2828	-0.6539	-1.2491	2.7123	0.3846	0.1943	-1.8050	0.2109	-0.9008	-1.0340	-1.0274	-1.3834	1.8257	-0.8716	-1.7431	-0.0823	-0.3078	-1.1516
12	2.7426	1.2749	1.5130	1.2968	0	0	0	0	0	0	0	0	0	0	1.7068	3.4137	0	0	0
13	-0.5585	-1.2448	1.9952	-0.7343	0	0	0	0	0	0	0	0	0	0	-0.1356	-0.2712	0	0	0
14	0.7985	-2.5010	-1.1722	-0.2439	0	0	0	0	0	0	0	0	0	0	-0.7796	-1.5593	0	0	0
15	-0.8503	3.6650	1.0462	1.0496	0.3042	0.9907	0.5821	-1.5071	0.0176	-0.6110	-1.6314	-1.4086	-1.2073	0.6891	1.2276	2.4552	-0.3782	0.0806	0.3017
16	-0.2740	0.7776	-0.9729	-1.9610	0	0	0	0	0	0	0	0	0	0	-0.6076	-1.2152	0	0	0
17	0.1291	-0.8824	1.7849	1.6258	0	0	0	0	0	0	0	0	0	0	0.6644	1.3287	0	0	0
18	-0.4293	-1.9006	0.6857	1.3487	0	0	0	0	0	0	0	0	0	0	-0.0739	-0.1478	0	0	0
19	1.8860	-1.7723	-0.1987	0.3299	0	0	0	0	0	0	0	0	0	0	0.0612	0.1224	0	0	0
20	0.7466	0.8777	0.6002	0.9077	0	0	0	0	0	0	0	0	0	0	0.7831	1.5661	0	0	0
21	-0.5046	0.6263	-1.5355	-1.7694	0	0	0	0	0	0	0	0	0	0	-0.7958	-1.5916	0	0	0
22	1.9191	-0.8607	2.9799	2.8486	0	0	0	0	0	0	0	0	0	0	1.7217	3.4435	0	0	0
23	-1.0569	-1.2278	-2.0334	-1.8316	-1.0373	-0.7071	-1.0922	-0.5116	-0.8011	-0.7556	-0.2598	-1.0339	1.0585	-1.5561	-1.5374	-3.0748	-0.6696	-0.9176	-3.4332
24	0.2930	0.7321	-1.6738	1.8801	0	0	0	0	0	0	0	0	0	0	0.3079	0.6157	0	0	0
25	0.0790	0.2796	1.2652	-0.4551	0	0	0	0	0	0	0	0	0	0	0.2922	0.5844	0	0	0
26	0.7039	-0.4924	-0.5556	0.7843	0	0	0	0	0	0	0	0	0	0	0.1101	0.2201	0	0	0
27	-0.0652	-1.3885	-0.8554	2.0871	0	0	0	0	0	0	0	0	0	0	-0.0555	-0.1110	0	0	0
28	0.9956	0.8378	-0.4956	-1.7207	0	0	0	0	0	0	0	0	0	0	-0.0957	-0.1914	0	0	0
29	0.4819	-1.6547	-0.7410	-1.0374	0	0	0	0	0	0	0	0	0	0	-0.7378	-1.4756	0	0	0
30	-0.0028	1.2746	0.7117	-0.5412	0	0	0	0	0	0	0	0	0	0	0.3606	0.7211	0	0	0

3.3. Optimal parameters for the synthetic DS control chart

The optimization program of the DS \bar{X} control chart is incorporated with the upper bound limit L_3 , where $L_3 \leq 100$. This upper bound is considered large enough to give the optimal parameters for most small shifts of practical interest. The optimization program computes the optimal values of n_1 , n_2 , L , L_1 , L_2 and L_3 when the end user specifies the mean sample size (n), in-control $ARL_{SDS}(0)$ and magnitude of the desired mean shift $(\delta)_{opt}$ where a quick detection is desired. The program is based on standardized statistics, so $\mu_0 = 0$ & $\sigma = 1$. For illustrative purpose, if the end user specifies $n = 5$, $ARL_{SDS}(0) = 370$ & $\delta_{opt} = 0.5$. From Figure 7, the input parameters

produced 30 pairs of n_1 & n_2 based on the following inequalities $n_1 < n$ & $n_2 < 2n$ satisfying the constraint $n_1 + n_2 > n$ when the mean sample size is $n = 5$ where the $ARL_{delta} = 5.9892$ represents the minimum value of all 30 possible pairs of ARL_{delta} . The corresponding values give the optimal parameters as shown in Figure 7 as $n_1 = 3$, $n_2 = 10$, $L = 3.2052$, $L_1 = 1.2777$, $L_2 = 1.8022$ & $L_3 = 12$. Table 4 and Table 5 shows the optimal parameters of the synthetic DS control chart for combinations of $ARL_{SDS}(0) \in \{250, 370\}$, $\delta_{opt} \in \{0.2, 0.4, 0.5, 0. \dots, 2\}$ & $n \in \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Other combinations of $ARL_{DS}(0)$, δ_{opt} & n can be computed using the optimization MATLAB (R2013a) program presented in this work. This program can be requested from the first author.

n1	n2	P1	P2	L	L1	L2	L3	P4	Beta	Pdelta	ARLdelta
SOLUTIONMATRIX=											
4.0000	10.0000	0.7342	0.2521	3.2052	1.6383	1.5246	15.0000	0.3642	0.8260	0.1740	6.0944
4.0000	9.0000	0.7166	0.2696	3.2052	1.5872	1.5598	14.0000	0.4036	0.8255	0.1745	6.1487
4.0000	8.0000	0.6958	0.2905	3.2052	1.5287	1.6182	14.0000	0.4543	0.8277	0.1723	6.2475
4.0000	7.0000	0.6704	0.3159	3.2052	1.4603	1.6643	13.0000	0.5020	0.8290	0.1710	6.4055
4.0000	6.0000	0.6388	0.3474	3.2052	1.3786	1.7374	13.0000	0.5616	0.8340	0.1660	6.6499
4.0000	5.0000	0.5980	0.3883	3.2052	1.2777	1.8022	12.0000	0.6183	0.8381	0.1619	7.0173
4.0000	4.0000	0.5426	0.4437	3.2052	1.1471	1.9017	12.0000	0.6866	0.8472	0.1528	7.5812
4.0000	3.0000	0.4612	0.5251	3.2052	0.9647	2.0246	12.0000	0.7582	0.8593	0.1407	8.4811
4.0000	2.0000	0.3244	0.6619	3.2052	0.6724	2.2052	13.0000	0.8363	0.8779	0.1221	10.0353
3.0000	10.0000	0.6437	0.3466	3.2052	1.2777	1.8022	12.0000	0.4996	0.8169	0.1831	5.9892
3.0000	9.0000	0.6186	0.3717	3.2052	1.2171	1.8497	12.0000	0.5466	0.8218	0.1782	6.1997
3.0000	8.0000	0.5886	0.4017	3.2052	1.1471	1.9017	12.0000	0.5960	0.8280	0.1720	6.4880
3.0000	7.0000	0.5519	0.4384	3.2052	1.0646	1.9594	12.0000	0.6472	0.8357	0.1643	6.8827
3.0000	6.0000	0.5057	0.4846	3.2052	0.9647	2.0246	12.0000	0.6999	0.8449	0.1551	7.4289
3.0000	5.0000	0.4452	0.5451	3.2052	0.8392	2.0997	12.0000	0.7533	0.8558	0.1442	8.2016
3.0000	4.0000	0.3612	0.6291	3.2052	0.6724	2.1889	12.0000	0.8065	0.8686	0.1314	9.3322
3.0000	3.0000	0.2333	0.7570	3.2052	0.4289	2.3156	13.0000	0.8621	0.8859	0.1141	11.0585
2.0000	10.0000	0.5871	0.4066	3.2052	1.0335	1.9802	12.0000	0.5979	0.8302	0.1698	6.5967
2.0000	9.0000	0.5544	0.4393	3.2052	0.9647	2.0246	12.0000	0.6428	0.8368	0.1632	6.9453
2.0000	8.0000	0.5147	0.4790	3.2052	0.8846	2.0733	12.0000	0.6886	0.8445	0.1555	7.4083
2.0000	7.0000	0.4655	0.5282	3.2052	0.7893	2.1276	12.0000	0.7347	0.8536	0.1464	8.0311
2.0000	6.0000	0.4023	0.5914	3.2052	0.6724	2.1889	12.0000	0.7806	0.8639	0.1361	8.8854
2.0000	5.0000	0.3173	0.6764	3.2052	0.5225	2.2597	12.0000	0.8254	0.8756	0.1244	10.0896
2.0000	4.0000	0.1953	0.7985	3.2052	0.3169	2.3596	13.0000	0.8716	0.8912	0.1088	11.8399
1.0000	10.0000	0.5425	0.4540	3.2052	0.8392	2.1165	13.0000	0.6765	0.8496	0.1504	7.5592
1.0000	9.0000	0.5001	0.4964	3.2052	0.7624	2.1421	12.0000	0.7125	0.8538	0.1462	8.0460
1.0000	8.0000	0.4479	0.5486	3.2052	0.6724	2.1889	12.0000	0.7544	0.8618	0.1382	8.6926
1.0000	7.0000	0.3818	0.6147	3.2052	0.5640	2.2409	12.0000	0.7957	0.8709	0.1291	9.5665
1.0000	6.0000	0.2952	0.7013	3.2052	0.4289	2.2999	12.0000	0.8356	0.8812	0.1188	10.7766
1.0000	5.0000	0.1758	0.8207	3.2052	0.2516	2.3835	13.0000	0.8766	0.8952	0.1048	12.5028
OPTIMAL PARAMETERS											
n1	n2	P1	P2	L	L1	L2	L3	P4	Beta	Pdelta	ARLdelta
3.0000	10.0000	0.6437	0.3466	3.2052	1.2777	1.8022	12.0000	0.4996	0.8169	0.1831	5.9892

Figure 7: Possible pairs of n_1 & n_2 at $n = 5$, $ARL_{SDS}(0) = 370$ & $\delta_{opt} = 0.5$

Table 4: Optimal combinations of $n_1, n_2, L, L_1, L_2, L_3$ for synthetic DS chart for $ARL_{SDS}(0) = 250$

$ARL_{SDS}(0) = 250$											
n	$(\delta)_{opt}$										
	0.2	0.4	0.5	0.6	0.8	1	1.2	1.4	1.5	2	
2	1,4,3.0902, 1.1455, 2.1365,49	1,4,3.0902, 1.1455, 2.0211,29	1,4,3.0902, 1.1455, 1.9590,22	1,4,3.0902, 1.1455, 1.900,17	1,4,3.0902, 1.1455, 1.7981,11	1,4,3.0902, 1.1455, 1.7216,8	1,4,3.0902, 1.1455, 1.6850,7	1,4,3.0902, 1.1455, 1.7429,5	1,3,3.0902, 0.9634, 1.6891,4	1,3,3.0902, 0.9634, 1.6891,4	1,2,3.0902, 0.6713, 1.8093,3
3	2,6,3.0902, 1.3765, 1.9782,51	2,6,3.0902, 1.3765, 1.8103,25	2,6,3.0902, 1.3765, 1.7302,18	2,6,3.0902, 1.3765, 1.6675,14	2,6,3.0902, 1.3765, 1.5542,9	2,4,3.0902, 1.1455, 1.6051,5	2,3,3.0902, 0.9634, 1.6891,4	2,3,3.0902, 0.9634, 1.6182,3	2,3,3.0902, 0.9634, 1.6182,3	2,2,3.0902, 0.6713, 1.7143,2	2,2,3.0902, 0.6713, 1.7143,2
4	3,8,3.0902, 1.5260, 1.8530,51	3,8,3.0902, 1.5260, 1.6437,22	3,8,3.0902, 1.5260, 1.5607,16	3,8,3.0902, 1.5260, 1.4837,12	3,5,3.0902, 1.2759, 1.5404,6	3,4,3.0902, 1.1455, 1.5480,4	3,3,3.0902, 0.9634, 1.6182,3	3,3,3.0902, 0.9634, 1.6182,3	3,2,3.0902, 0.6713, 1.7143,2	3,2,3.0902, 0.6713, 1.7143,2	3,2,3.0902, 0.6713, 1.7143,2
5	3,10,3.0902, 1.2759, 1.9998,40	3,10,3.0902, 1.2759, 1.7859,16	3,10,3.0902, 1.2759, 1.7159,12	3,10,3.0902, 1.2759, 1.6443,9	4,5,3.0902, 1.2759, 1.4924,5	4,4,3.0902, 1.1455, 1.5480,4	4,3,3.0902, 0.9634, 1.6182,3	4,3,3.0902, 0.9634, 1.5153,3	4,2,3.0902, 0.6713, 1.7143,2	4,2,3.0902, 0.6713, 1.7143,2	4,2,3.0902, 0.6713, 1.7143,2
6	4,12,3.0902, 1.3765, 1.9099,38	4,12,3.0902, 1.3765, 1.6849,15	4,12,3.0902, 1.3765, 1.6062,11	4,11,3.0902, 1.3291, 1.5672,8	5,5,3.0902, 1.2759, 1.4325,4	5,4,3.0902, 1.1455, 1.4727,3	5,3,3.0902, 0.9634, 1.5153,2	5,2,3.0902, 0.6713, 1.7143,2	5,2,3.0902, 0.6713, 1.7143,2	5,2,3.0902, 0.6713, 1.7143,2	5,2,3.0902, 0.6713, 1.7143,2
7	5,14,3.0902, 1.4579, 1.8353,37	5,14,3.0902, 1.4579, 1.5923,14	5,13,3.0902, 1.4192, 1.5414,10	5,10,3.0902, 1.2759, 1.5803,7	5,7,3.0902, 1.0631, 1.6145,4	6,3,3.0902, 0.9634, 1.6182,3	6,3,3.0902, 0.9634, 1.5153,2	6,2,3.0902, 0.6713, 1.7143,2	6,2,3.0902, 0.6713, 1.7143,2	6,2,3.0902, 0.6713, 1.7143,2	6,2,3.0902, 0.6713, 1.7143,2
8	6,16,3.0902, 1.5260, 1.7679,36	5,16,3.0902, 1.3121, 1.6853,12	5,16,3.0902, 1.3121, 1.6128,9	6,10,3.0902, 1.2759, 1.5404,6	7,4,3.0902, 1.1455, 1.4727,3	7,3,3.0902, 0.9634, 1.5153,2	7,3,3.0902, 0.9634, 1.5153,2	7,2,3.0902, 0.6713, 1.7143,2	7,2,3.0902, 0.6713, 1.7143,2	7,2,3.0902, 0.6713, 1.7143,2	7,2,3.0902, 0.6713, 1.7143,2
9	6,18,3.0902, 1.3765, 1.8540,30	6,18,3.0902, 1.3765, 1.6062,11	6,16,3.0902, 1.3121, 1.5826,8	7,10,3.0902, 1.2759, 1.4924,5	7,7,3.0902, 1.0631, 1.5414,3	8,3,3.0902, 0.9634, 1.5153,2	8,3,3.0902, 0.9634, 1.5153,2	8,2,3.0902, 0.6713, 1.7143,2	8,2,3.0902, 0.6713, 1.7143,2	8,2,3.0902, 0.6713, 1.7143,2	8,2,3.0902, 0.6713, 1.7143,2
10	7,20,3.0902, 1.4325, 1.7984,29	7,20,3.0902, 1.4325, 1.5537,11	7,15,3.0902, 1.2759, 1.5803,7	8,9,3.0902, 1.2154, 1.5463,5	8,7,3.0902, 1.0631, 1.5414,3	9,3,3.0902, 0.9634, 1.5153,2	9,2,3.0902, 0.6713, 1.7143,2	9,2,3.0902, 0.6713, 1.7143,2	9,2,3.0902, 0.6713, 1.7143,2	9,2,3.0902, 0.6713, 1.7143,2	9,2,3.0902, 0.6713, 1.7143,2

Table 5: Optimal combinations of $n_1, n_2, L, L_1, L_2, L_3$ for synthetic DS chart for $ARL_{SDS}(0) = 370$

$ARL_{SDS}(0) = 370$											
n	$(\delta)_{opt}$										
	0.2	0.4	0.5	0.6	0.8	1	1.2	1.4	1.5	2	
2	1,4,3.2052, 1.1471, 2.2536,61	1,4,3.2052, 1.1471, 2.1366,34	1,4,3.2052, 1.1471, 2.0645,25	1,4,3.2052, 1.1471, 2.0044,19	1,4,3.2052, 1.1471, 1.9017,12	1,4,3.2052, 1.1471, 1.8357,9	1,4,3.2052, 1.1471, 1.7770,7	1,3,3.2052, 0.9647, 1.8294,5	1,3,3.2052, 0.9647, 1.7770,4	1,3,3.2052, 0.9647, 1.7770,4	1,2,3.2052, 0.6724, 1.8940,3
3	2,6,3.2052, 1.3786, 2.0967,62	2,6,3.2052, 1.3786, 1.9184,28	2,6,3.2052, 1.3786, 1.8402,20	2,6,3.2052, 1.3786, 1.7719,15	2,6,3.2052, 1.3786, 1.6468,9	2,5,3.2052, 1.2777, 1.6341,6	2,4,3.2052, 1.1471, 1.6419,4	2,3,3.2052, 0.9647, 1.7096,3	2,3,3.2052, 0.9647, 1.7096,3	2,3,3.2052, 0.9647, 1.7096,3	2,2,3.2052, 0.6724, 1.8025,2
4	3,8,3.2052, 1.5287, 1.9734,61	3,8,3.2052, 1.5287, 1.7528,24	3,8,3.2052, 1.5287, 1.6674,17	3,8,3.2052, 1.5287, 1.5786,12	3,6,3.2052, 1.3786, 1.5833,7	3,4,3.2052, 1.1471, 1.6419,4	3,3,3.2052, 0.9647, 1.7096,3	3,3,3.2052, 0.9647, 1.7096,3	3,3,3.2052, 0.9647, 1.7096,3	3,3,3.2052, 0.9647, 1.7096,3	3,2,3.2052, 0.6724, 1.8024,2
5	3,10,3.2052, 1.2777, 2.1103,47	3,10,3.2052, 1.2777, 1.8965,18	3,10,3.2052, 1.2777, 1.8022,12	3,10,3.2052, 1.2777, 1.7336,9	4,6,3.2052, 1.3786, 1.5436,6	4,4,3.2052, 1.1471, 1.6419,4	4,3,3.2052, 0.9647, 1.7096,3	4,3,3.2052, 0.9647, 1.6109,2	4,3,3.2052, 0.9647, 1.6109,2	4,3,3.2052, 0.9647, 1.6109,2	4,2,3.2052, 0.6724, 1.8025,2
6	4,12,3.2052, 1.3786, 2.0258,45	4,12,3.2052, 1.3786, 1.7873,16	4,12,3.2052, 1.3786, 1.6966,11	4,12,3.2052, 1.3786, 1.6468,9	4,8,3.2052, 1.1471, 1.6965,5	5,4,3.2052, 1.1471, 1.5699,3	5,3,3.2052, 0.9647, 1.6109,2	5,3,3.2052, 0.9647, 1.6109,2	5,2,3.2052, 0.6724, 1.8025,2	5,2,3.2052, 0.6724, 1.8025,2	5,2,3.2052, 0.6724, 1.8025,2
7	5,14,3.2052, 1.4603, 1.9503,43	5,14,3.2052, 1.4603, 1.6999,15	5,14,3.2052, 1.4603, 1.5989,10	5,11,3.2052, 1.3311, 1.6262,7	5,8,3.2052, 1.1471, 1.6419,4	6,4,3.2052, 1.1471, 1.5699,3	6,3,3.2052, 0.9647, 1.6109,2	6,3,3.2052, 0.9647, 1.6109,2	6,2,3.2052, 0.6724, 1.8025,2	6,2,3.2052, 0.6724, 1.8025,2	6,2,3.2052, 0.6724, 1.8025,2
8	6,16,3.2052, 1.5287, 1.8811,41	6,16,3.2052, 1.5287, 1.6182,14	5,16,3.2052, 1.3140, 1.7032,9	6,11,3.2052, 1.3311, 1.5872,6	6,8,3.2052, 1.1471, 1.6419,4	7,3,3.2052, 0.9647, 1.6109,2	7,3,3.2052, 0.9647, 1.6109,2	7,2,3.2052, 0.6724, 1.8024,2	7,2,3.2052, 0.6724, 1.8025,2	7,2,3.2052, 0.6724, 1.8025,2	7,2,3.2052, 0.6724, 1.8025,2
9	6,18,3.2052, 1.3786, 1.9693,35	6,18,3.2052, 1.3782, 1.7179,12	6,17,3.2052, 1.3475, 1.6451,8	7,11,3.2052, 1.3311, 1.5872,6	7,7,3.2052, 1.0646, 1.6358,3	8,3,3.2052, 0.9647, 1.6109,2	8,3,3.2052, 0.9647, 1.6109,2	8,2,3.2052, 0.6724, 1.8025,2	8,2,3.2052, 0.6724, 1.8024,2	8,2,3.2052, 0.6724, 1.8025,2	8,2,3.2052, 0.6724, 1.8025,2
10	7,20,3.2052, 1.4348, 1.9104,33	7,20,3.2052, 1.4348, 1.6460,11	7,17,3.2052, 1.3475, 1.6116,7	7,13,3.2052, 1.1949, 1.6582,5	8,7,3.2052, 1.0646, 1.6358,3	9,3,3.2052, 0.9647, 1.6109,2	9,3,3.2052, 0.9647, 1.6109,2	9,2,3.2052, 0.6724, 1.8025,2	9,2,3.2052, 0.6724, 1.8025,2	9,2,3.2052, 0.6724, 1.8025,2	9,2,3.2052, 0.6724, 1.8025,2

3.4. Construction of the synthetic DS chart

The procedures in section (2.7) are adopted to construct the synthetic DS control chart. The same random seed were used to simulate the construction of the synthetic DS control chart to allow for objective comparisons of the results produced. If the desired expected mean sample size ($n = 5$), in-control/zero state Average Run Length ($ARL_{SDS}(0) = 370$) & the magnitude of the mean shift desired is ($\delta_{opt} = 0.5$); this gives the corresponding optimal parameters as shown in Figure 7, where n_1, n_2, L, L_1, L_2 & $L_3 = (3, 10, 3.2052, 1.2777, 1.8022, 12)$.

From Table 6, considering the first sampling stage $j=1$; the first stage sample is taken with a mean sample size of $n_1 = 3$ with observations $X_{1(1)} = 1.4390, X_{1(2)} = -0.9595, X_{1(3)} = 1.6527$. Step 2, compute the first stage sample mean as $\bar{X}_{1(1)} = 0.7107$ and normalized as $Z_{1(1)} = 1.2310$. Next is Step 3, since $(Z_{1(1)} \in I_1) = 1.2310 \in (-1.2777/1.2777)$ the sampling stage $j=1$ is conforming; the process flows back to step 2. The process continues until sampling stage $j=5$ where the first stage sample $n_1 = 3$ observations are $X_{1(1)} = -1.7036, X_{1(2)} = 0.7245, X_{1(3)} = -1.9864$. Step 2, compute the first stage sample mean as $\bar{X}_{1(5)} = -0.9885$ & normalized as $Z_{1(5)} = -1.7121$, since $(Z_{1(5)} \in$

$I_2) = -1.7121 \in (-3.2052/-1.2777)$ a second stage sample is taken. Next is step 5, the second stage sample size is $n_2 = 10$ with observations $X_{2(1)} = -1.1173$, $X_{2(2)} = -1.9948$, $X_{2(3)} = -2.6734$, $X_{2(4)} = 0.9277$, $X_{2(5)} = -1.3004$, $X_{2(6)} = 1.4454$, $X_{2(7)} = 0.3955$, $X_{2(8)} = 1.6042$, $X_{2(9)} = 1.7575$ and $X_{2(10)} = -1.1153$. The second stage sample mean becomes $\bar{X}_{2(5)} = -0.2071$. Next is step 6, compute the overall sample mean $\bar{X}_{(5)} = -0.3874$ and normalized as $Z_{(5)} = -1.3968$. Next is step 7, since $(Z_{(5)} \in I_4) = -1.3968 \in (-1.8022 / 1.8022)$ the sampling stage $j=5$ is conforming, the control flows to step 2. For sampling stage $j=6$, the first stage sample normalized mean is $Z_{1(6)} = 4.4227$ hence the control flows to step 4; since $Z_{1(6)} \in I_3 = 4.4227 \in (3.2052/+∞)$ hence sampling stage $j=6$ is non-conforming the process flows to Step 8. The conforming run length ($CRL = 6$) next is step 9, since $(CRL < L_3) = (6 < 12)$ the process is out-of-control the process goes to step 10.

From Table 6, the non-conforming sampling stages were observed at $Z_{1(6)} = 4.4227$, $Z_{(10)} = 2.006$, $Z_{1(12)} = 3.3736$, $Z_{(26)} = 1.9017$ & $Z_{(30)} = 2.7801$ that is at sampling stage $J = 6, 10, 12, 26$ & 30 .

However, at sampling stage $j = 26$ the $CRL = 14$, but $L_3 = 12$ which implies that $CRL = 14 > L_3 = 12$. Therefore, the non-conforming sampling stage $j = 26$ is in-control state leaving the non-conforming sampling stages of $J = 6, 10, 12$ & 30 as an out-of-control state. The first Average Run length (ARL) which is the number of sampling stage before an out-of-control is observed becomes $ARL = 6$ and the Average Number of Observations to Samples before an out-of-control is signalled as $ANOS = 28$. The synthetic DS control chart below consists of the DS/S sub control chart in Figure 8 and CRL/S sub control chart in Figure 9. A plot of $Z_{1(j)}$ & $Z_{(j)}$ against the sampling stages (j) gives the DS/S sub chart as shown in Figure 8, the black dot represents the first stage sample normalized mean $Z_{1(j)}$ while, the white dot is the second stage sample normalized mean $Z_{(j)}$. The control run length (CRL) control chart is plotted against the sampling stages in Figure 9.

3.5. Comparison of Results

From the analysis done in section 3.2 and 3.4 between the DS \bar{X} control chart and synthetic DS control chart it can be concluded that the synthetic DS control chart detects the mean shift quicker than the DS \bar{X} control chart. The DS \bar{X} chart issues its first out-of-control signal at sampling stage $j = 12$ at the point where the process mean starts to shift. On the contrary, the synthetic DS control chart issues its first out-of-control status at sampling stage $j = 6$. The DS \bar{X} control chart has an additional lapse of 6 sampling stages before an out-of-control status is issued. In terms of Average Number of Observations to Sample (ANOS) the DS \bar{X} control chart requires 78 observations (from $j = 1$, see Table 3), while that of the synthetic DS control chart needed 28 observations (from $j = 1$, see Table 6) to signal an out-of-control situation. It is obvious that the former requires additional 50 observations before the mean shift is detected.

Table 6: Data for an illustrative example for the synthetic DS control chart

Sampling Stage (j)															$\bar{X}_{1(j)}$	$Z_{1(j)}$	$\bar{X}_{2(j)}$	\bar{X}_j	Z_j
	$X_{1(1)}$	$X_{1(2)}$	$X_{1(3)}$	$X_{2(1)}$	$X_{2(2)}$	$X_{2(3)}$	$X_{2(4)}$	$X_{2(5)}$	$X_{2(6)}$	$X_{2(7)}$	$X_{2(8)}$	$X_{2(9)}$	$X_{2(10)}$						
1	1.4390	-0.9595	1.6527	0	0	0	0	0	0	0	0	0	0	0	0.7107	1.2310	0	0	0
2	0.2824	-0.9709	-0.8098	0	0	0	0	0	0	0	0	0	0	0	-0.4994	-0.8650	0	0	0
3	0.8006	-1.5232	-0.0197	0	0	0	0	0	0	0	0	0	0	0	-0.2474	-0.4286	0	0	0
4	0.8256	-1.0257	1.5403	0	0	0	0	0	0	0	0	0	0	0	0.4467	0.7738	0	0	0
5	-1.7036	0.7245	-1.9864	-1.1173	-1.9948	-2.6734	0.9277	-1.3004	1.4454	0.3955	1.6042	1.7575	-1.1153	0	-0.9885	-1.7121	-0.2071	-0.3874	-1.3968
6	2.0693	3.4960	2.0951	0	0	0	0	0	0	0	0	0	0	0	2.5535	4.4227	0	0	0
7	-0.9405	-1.7266	-0.2547	-1.3046	-1.8956	1.8187	-0.2776	1.8462	1.0497	-1.9706	0.7202	0.8238	0.5805	0	-0.9739	-1.6869	0.1391	-0.1178	-0.4247
8	0.2092	-1.1276	1.0895	0	0	0	0	0	0	0	0	0	0	0	0.0570	0.0988	0	0	0
9	-1.0879	-0.5165	1.5637	0	0	0	0	0	0	0	0	0	0	0	-0.0136	-0.0235	0	0	0
10	1.4255	-0.3903	2.7279	3.4345	1.6408	-1.6364	-0.3664	-0.6697	1.4123	-0.2304	1.6174	-1.8673	0.1297	0	1.2544	2.1726	0.3465	0.5560	2.0046
11	-2.8660	1.2828	-0.6539	-1.2491	2.7123	0.3846	0.1943	-1.8050	0.2109	-0.9008	-1.0340	-1.0274	-1.3834	0	-0.7457	-1.2916	-0.3898	-0.4719	-1.7015
12	1.8257	2.7426	1.2749	0	0	0	0	0	0	0	0	0	0	0	1.9477	3.3736	0	0	0
13	1.5130	1.2968	-0.5585	-1.2448	1.9952	-0.7343	0.7985	-2.5010	-1.1722	-0.2439	-0.8503	3.6650	1.0462	0	0.7504	1.2998	0.0758	0.2315	0.8347
14	1.0496	0.3042	0.9907	0.5821	-1.5071	0.0176	-0.6110	-1.6314	-1.4086	-1.2073	0.6891	-0.2740	0.7776	0	0.7815	1.3536	-0.4573	-0.1714	-0.6181
15	-0.9729	-1.9610	0.1291	-0.8824	1.7849	1.6258	-0.4293	-1.9006	0.6857	1.3487	1.8860	-1.7723	-0.1987	0	-0.9349	-1.6194	0.2148	-0.0505	-0.1822
16	0.3299	0.7466	0.8777	0	0	0	0	0	0	0	0	0	0	0	0.6514	1.1283	0	0	0
17	0.6002	0.9077	-0.5046	0	0	0	0	0	0	0	0	0	0	0	0.3344	0.5793	0	0	0
18	0.6263	-1.5355	-1.7694	1.9191	-0.8607	2.9799	2.8486	-1.0569	-1.2278	-2.0334	-1.8316	-1.0373	-0.7071	0	-0.8929	-1.5465	-0.1007	-0.2835	-1.0223
19	-1.0922	-0.5116	-0.8011	-0.7556	-0.2598	-1.0339	1.0585	-1.5561	0.2930	0.7321	-1.6738	1.8801	0.0790	0	-0.8016	-1.3885	-0.1236	-0.2801	-1.0099
20	0.2796	1.2652	-0.4551	0	0	0	0	0	0	0	0	0	0	0	0.3632	0.6291	0	0	0
21	0.7039	-0.4924	-0.5556	0	0	0	0	0	0	0	0	0	0	0	-0.1147	-0.1987	0	0	0
22	0.7843	-0.0652	-1.3885	0	0	0	0	0	0	0	0	0	0	0	-0.2231	-0.3865	0	0	0
23	-0.8554	2.0871	0.9956	0.8378	-0.4956	-1.7207	0.4819	-1.6547	-0.7410	-1.0374	-0.0028	1.2746	0.7117	0	0.7424	1.2859	-0.2346	-0.0091	-0.0330
24	-0.5412	-0.6318	-1.4870	-1.1122	1.0851	-1.4422	1.8712	-0.0813	-1.4811	0.4886	2.5271	1.4105	1.3196	0	-0.8867	-1.5358	0.4585	0.1481	0.5340
25	1.0334	1.5638	-0.2920	-0.2247	2.9868	1.1079	-0.9243	0.1871	-1.6405	0.4593	-1.6034	1.0891	1.6401	0	0.7684	1.3309	0.3077	0.4140	1.4929
26	-3.4470	-2.0961	2.0191	2.1545	-1.1691	-1.7883	-0.8073	-0.4148	-0.8666	1.2236	-0.8963	-0.0062	-0.7622	0	-1.1747	-2.0346	-0.3333	-0.5274	-1.9017
27	-0.4168	1.8118	-1.7492	0	0	0	0	0	0	0	0	0	0	0	-0.1181	-0.2045	0	0	0
28	-1.2711	-0.2477	1.5625	0	0	0	0	0	0	0	0	0	0	0	0.0146	0.0252	0	0	0
29	-0.9469	1.9409	2.0792	-0.2990	2.1395	-1.3117	1.2438	0.5174	0.6846	0.1596	-0.9182	-1.2235	0.7407	0	1.0244	1.7743	0.1733	0.3697	1.3331
30	1.4097	0.8786	0.0864	1.9779	0.5659	-0.8930	0.9354	-0.5174	1.1957	2.1926	-0.7211	1.9275	0.9855	0	0.7916	1.3710	0.7649	0.7711	2.7801

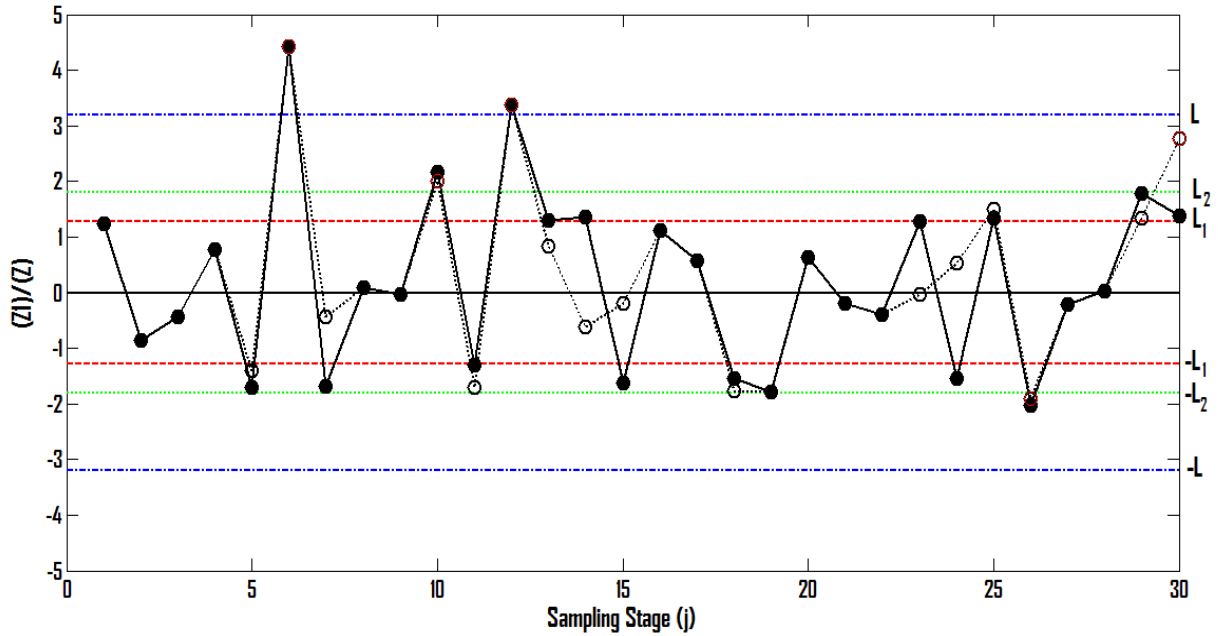


Figure 8: DS/S sub-control chart

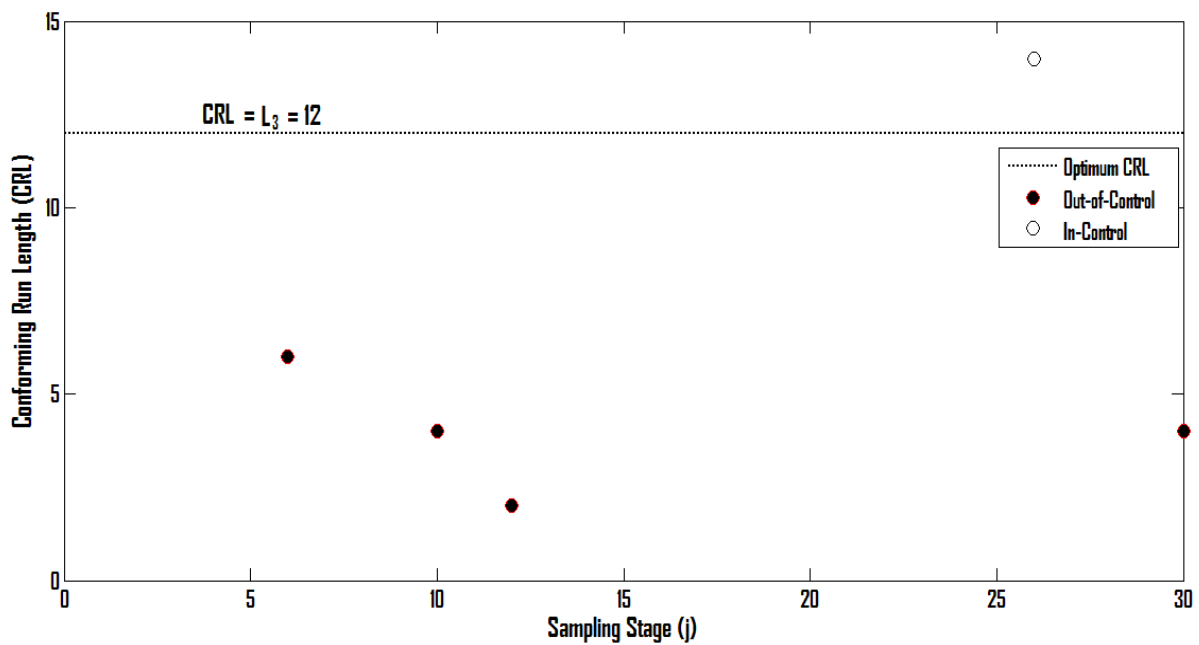


Figure 9: CRL/S sub-control chart

4. CONCLUSIONS

The optimal design parameters and the construction of the control chart were discussed extensively in this work. The results from this study showed; the DS \bar{X} chart required an additional lapse of 6 ARL before an out-of-control status is issued. In terms of ANOS; the DS \bar{X} control chart required 78 ANOS while that of the synthetic DS control chart needed 28 ANOS to signal an out-of-control situation. It is obvious that the formal requires additional 50 ANOS before the mean shift is detected. The synthetic DS control chart reduces the out-of-control ARL and ANOS values by more than half when compared with the DS \bar{X} control chart. Hence the use of the proposed control chart is justified.

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