An Approximate Solution of a Computer Virus Model with Antivirus using Modified Differential Transform Method

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Abstract **- This work extends the application of a modified differential transform method by providing an approximate analytic solution to a model that describes the effect of antivirus in the spread of computer virus over a network. The proposed method is a hybrid technique that combines differential transform method with a post treatment of Laplace-Pade resummation method. The accuracy of the solutions generated by the proposed method is established when compared with those of Runge-Kutta fourth order method. Graphs and table were presented to ascertain the simplicity and reliability of the method.**

Keywords: Virus, Antivirus, Differential Transform Method, Laplace-Pade resummation, Pade Approximant

1.0 INTRODUCTION

The excessive prevalence of computer viruses has been greatly intensified due to the fast acquisition of information through computer technologies and networks [1, 2]. These viruses are kind of computer programs or malicious code that can clone themselves and infect other computers. These viruses are grouped into file infectors, boot – sector viruses, macro viruses and Trojan horse. There are currently over 74000 different strains of computer viruses since it was first discovered in 1986 [3, 4]. The socio economic impact of computer virus includes the following, but not limited to loss of millions of dollars worth of data and loss of productivity [3, 5]. Thus the need to get a better understanding of the transmission dynamics of computer virus is very vital to increase the safety and reliability of computer network systems [6].

The similarities between biological virus and computer virus in their mode of propagation have made several researchers to adopt mathematical models as an effective strategy to study the way computer virus spreads over a network [6]. Some mathematical models for transmission dynamics of computer virus existing in literarure are found in [1, 6, 7], and the references cited there in. In particular, [1] formulated a dynamic model that provide a comprehensive study and deeper understanding of the fundamental mechanism in computer virus propagation over a network. Global stability properties of the model was established using the Lyapunov function and the analytical results obtained were validated numerically. In [7], the strength of Adams numerical method in solving a formulated epidemiological computer virus model for different case scenarios was investigated. The worth of their proposed algorithm in terms of accuracy and convergence was proved when compared with Explicit Runge Kutta Method and Implicit Backward Differentiation Method.

These computer virus models usually results in generating system of nonlinear differential equations with no exact solutions. Thus to predict the speed of computer virus propagation over a network, numerical methods that provide reliable approximate solutions are often used. These methods include Runge-Kutta fourth order [4], Adams-Bashforth-Moulton Method [3], Multi Step Generalized Differential Transform Method [8], Implicit Backward Differentiation Method [7], Euler Predictor Corrector Method [6].

In this work, a modified differential transform method will be used to provide approximate analytic solution of the nonlinear system of differential equation which describes the spread of computer virus in the presence of antivirus over a network. Although the proposed method (LPDTM) requires the application of both differential transform and Laplace –Pade resummation methods. It requires no linearisation, perturbation or discretization.

The rest of this paper is organized as follows: Section 2 presents the solution procedure of the proposed method and model formulation in Section 3. Section 4 contains application of the proposed method while Section 5 concludes the paper.

2.0 Solution Procedure of the Proposed Method

In this section, we give a brief review of the LPDTM. For convenience of the reader, we first present the basic definitions, fundamental theorems and applications of Differential Transform Method (DTM) and Pade Approximant.

2.1 Basic Idea of DTM

A semi analytic numerical method, suitable for solving linear and nonlinear differential equations, differential algebraic equations, integro- differential equation is the Differential Transformation Method (DTM) which was first proposed by Zhou (Peker et al., 2011; Akinboro et al., 2014; Adio, 2015). This method provides a highly accurate results or exact solution without linearization, discretization or perturbation.

Definition 1: The one dimensional differential transform of a function $f(t)$ at the point $t = t_0$ is defined as follows [9]

$$F(K) = \frac{1}{K!}\left(\frac{d^K}{dt^K} f(t)\right)_{t=t_0} \quad (1)$$

Definition 2: The differential inverse transform of $F(k)$ is defined as follows [9]

$$f(t) = \sum_{K=0}^{\infty} Y(K)(t-t_0)^K \quad (2)$$

Table 1: Fundamental Operations of Differential Transformation Method

Original function $f(t)$	Transformed function $F(K)$
$u(t) \pm v(t)$	$U(K) \pm V(K)$
$c\frac{du(t)}{dt}$	$\frac{c(K+n)!}{K!}U(K+n)$
t^n	$\delta(K-n) = \begin{cases} K = n \\ K \neq n \end{cases}$
$u(t)v(t)$	$\sum_{p=0}^{K} U(p)V(K-p)$

Despite numerous application of DTM in solving real life phenomena, DTM does not exhibit good approximation for large domain. Thus to improve its accuracy, DTM is usually modified either through the application of Pade approximation [12], Laplace-Pade Resummation [13], or multi step approach [14,15].

2.2 Pade Approximation

The use of polynomials to approximate truncated power series is common but not always advisable due to the singularities of polynomials which is caused by inadequate radius of convergence. Hence Pade approximants are extensively used to overcome these shortcomings. The Pade approximation of a function $f(t)$ of order $[N/M]$ is defined by [16, 17].

$$[N/M]_{f(t)} = \frac{a_0 + a_1 t + \ldots + a_N t^N}{1 + b_1 t + \ldots + b_M t^M} \quad (3)$$

Where we consider $b_0 = 1$ and the numerator and denominator have no common factors. It is important to note that the Pade approximant can be obtain with ease through the help of the in-built utilities of symbolic computational software such as Maple, Matlab and Mathematica [18].

Several application of semi analytic method that involves the use of Pade approximation includes Differential Transform Method with Pade approximant [19], Homotopy Peturbation Method and Pade Approximation [20], Laplace Adomian Decomposition Method with Pade approximation [21], Multistage Laplace Adomian Decomposition Method with Pade approximation [22], etc.

2.3 Laplace - Pade Resummation

Most of the power series solutions provided by most approximate methods fail to converge for large domains. Hence, Laplace Pade [23] resummation method is often use to increase the domain of convergence of power series solutions or inclusive to find the exact solution. Application of Laplace Pade resummation method is summarized as follows [24].

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1. First, apply Laplace transform to the power series solution generated by an approximate method.
2. Next, replaced s with $\frac{1}{t}$, in the resulting equation.
3. After that, we convert the transformed series into a meromorphic function by forming its Pade approximant of order $[N/M]$. N and M are arbitrarily chosen, but they should be smaller than the order of the power series. In this step, the Pade approximant extends the domain of the truncated series solution to obtain a better accuracy and convergence.
4. Then, t is substituted by $\frac{1}{s}$.
5. Finally, by using the inverse Laplace transformation, we obtain the exact or approximate solution.
   Applications of this method coupled with another approximate methods are found in [13, 23,24] and other references cited there in.

3.0 Model Formulation

This section considers a computer virus model in [7], that studies the effect of antivirus on computer virus infection by stratifying the total population into four classes: X denote the numbers of worms, Y denote the number of uninfected files, Z denote the number of infected files and N denote the number of antivirus agent.

The model is then governed by the following system of nonlinear differential equation.

$$\frac{dX(t)}{dt} = aZ(t) - bX(t) \tag{4}$$

$$\frac{dY(t)}{dt} = c - dY(t) - eX(t)Y(t) \tag{5}$$

$$\frac{dZ(t)}{dt} = eX(t)Y(t) - (d+f)Z(t) - gN(t) \tag{6}$$

$$\frac{dN(t)}{dt} = h - iN(t) \tag{7}$$

Subject to $X(0)=15,\ Y(0)=3,\ Z(0)=20$ and $N(0)=0.5$ $\qquad (8)$

Table 1: Parameters Description and Hypothetical Values

Parameters	Symbols	Estimated Values
Rate of infected files becoming worms	a	0.3
Death rate of worms	b	0.5
Birth rate of uninfected files by users	c	2.3
Natural death rate of uninfected file	d	0.055
Infected rate of uninfected file due to worms	e	0.015
Death rate of infected file	f	0.055
Rate of efficiency of antivirus	g	0.002
Constant rate at which antivirus run	h	2.6
Rate of inefficiency of antivirus	i	0.1

Note. Source of estimates: [7]

4.0 APPLICATION OF THE PROPOSED METHOD

Applying the differential transform method to $(4)-(8)$ to have

$$X(K+1)=\frac{1}{K+1}\left(aZ(K)-bX(K)\right) \tag{9}$$

$$Y(K+1)=\frac{1}{K+1}\left(c\delta(K)-dY(K)-e\sum_{p=0}^{K}\left(X(p)Y(K-p)\right)\right) \tag{10}$$

$$Z(K+1)=\frac{1}{K+1}\left(e\sum_{p=0}^{K}\left(X(p)Y(K-p)\right)-(d+f)Y(K)-gN(K)\right) \tag{11}$$

$$N(K+1)=\frac{1}{K+1}\left(h\delta(K)-iN(K)\right) \tag{12}$$

Where $X(0)=15,\ Y(0)=3,\ Z(0)=20$ and $N(0)=0.5$.

Without any loss of generality, it is important to state that $X(K),\ Y(K),\ Z(K)$ and $N(K)$ are the differential transform of $X(t),\ Y(t),\ Z(t)$ and $N(t)$ respectively.

Then using (2), the eleventh order solution of $(4)-(8)$ is obtained as

$$\begin{aligned}x(t)&\cong\sum_{v=0}^{11}\left(X(v)t^{v}\right)\\&=15-1.5t+0.14610t^{2}-0.0031620t^{3}-0.0017978t^{4}+0.00041466t^{5}-0.000057422t^{6}\\&\quad+0.0000059697t^{7}-4.8242\times10^{-7}t^{8}+2.7270\times10^{-8}t^{9}-2.2620\times10^{-10}t^{10}-2.1861\times10^{-10}t^{11}\end{aligned} \tag{13}$$

$$\begin{aligned}y(t)&\cong\sum_{v=0}^{11}\left(Y(v)t^{v}\right)\\&=3+1.46t-0.17065t^{2}+0.024686t^{3}-0.0034522t^{4}+0.00040922t^{5}-0.000038958t^{6}\\&\quad+0.0000025364t^{7}+8.585\times10^{-9}t^{8}-3.8134\times10^{-8}t^{9}+8.1852\times10^{-9}t^{10}-1.2409\times10^{-9}t^{11}\end{aligned} \tag{14}$$

$$\begin{aligned}z(t)&\cong\sum_{v=0}^{11}\left(Z(v)t^{v}\right)\\&=20-1.5260t+0.21188t^{2}-0.029241t^{3}+0.0039148t^{4}-0.00045732t^{5}-0.000043590t^{6}\\&\quad-0.0000029153t^{7}+1.4062\times10^{-8}t^{8}+3.7910\times10^{-8}t^{9}-8.3925\times10^{-9}t^{10}+1.2839\times10^{-9}t^{11}\end{aligned} \tag{15}$$

$$\begin{aligned}n(t)&\cong\sum_{v=0}^{11}\left(N(v)t^{v}\right)\\&=0.5+2.55t-0.1275t^{2}+0.00425t^{3}-0.00010625t^{4}+0.000002125t^{5}-3.5417\times10^{-8}t^{6}\\&\quad+5.0596\times10^{-10}t^{7}+6.3245\times10^{-12}t^{8}+7.0272\times10^{-14}t^{9}-7.0272\times10^{-16}t^{10}+6.3884\times10^{-18}t^{11}\end{aligned} \tag{16}$$

Applying the Laplace Pade Resummation technique to the power series solution $(13)-(16)$ as outlined in Section 2.3 while computing the t-Pade approximant associated to the Laplace transform of $x(t),y(t),z(t)$ and $n(t)$ at $[5/5],[6/6],[6/6]$ and $[5/4]$ respectively. Hence the approximate solutions of $(4)-(8)$ are obtained as

$$x(t)=0.001325e^{-1.5351t}-0.49391e^{-0.69273t}+2.5503e^{-0.40307t}+6.4878e^{-0.13215t}+6.4549e^{0.0069122t} \quad (17)$$

$$y(t)=(-0.000033313+0.0013152I)e^{(-1.6071-0.285931I)t}-(0.000033313+0.00131521I)e^{(-1.6071+0.285931I)t}$$
$$-0.4021e^{-0.65704t}-1.1214e^{-0.30263t}-21.311e^{-0.05779t}+25.833e^{-0.014495t} \quad (18)$$

$$z(t)=(0.0018927+0.0028918I)e^{(-1.6976-0.21909I)t}+(0.0018927-0.0028918I)e^{(-1.6976+0.219091I)t}$$
$$+0.37842e^{-0.66902t}+0.90263e^{-0.36217t}+7.8375e^{-0.12975t}+10.878e^{0.0069008t} \quad (19)$$

$$n(t)=26-25.5e^{-0.1t} \quad (20)$$

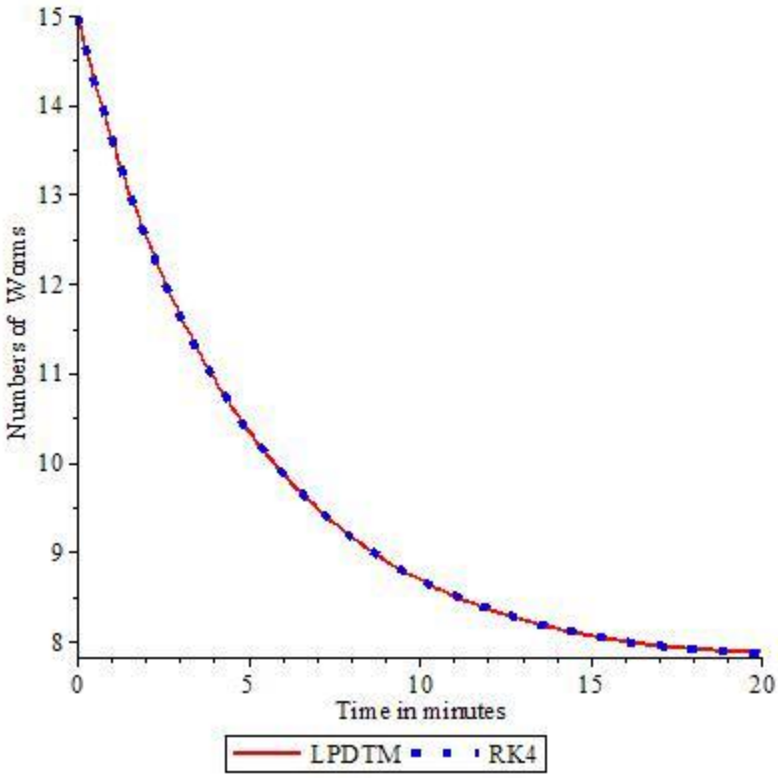

Figure 1: Graphical comparison of $X(t)$

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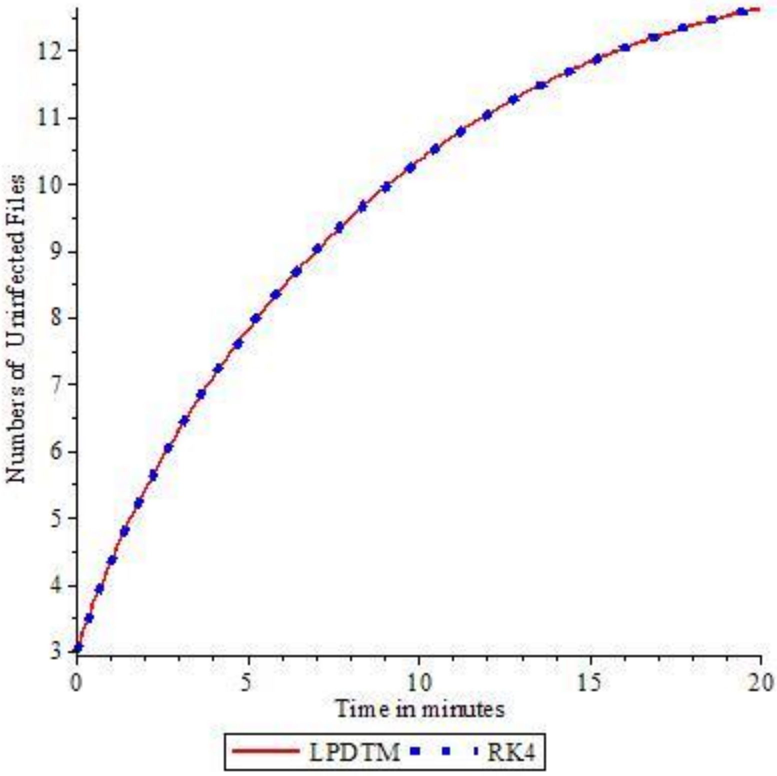

Figure 2: Graphical comparison of $Y(t)$

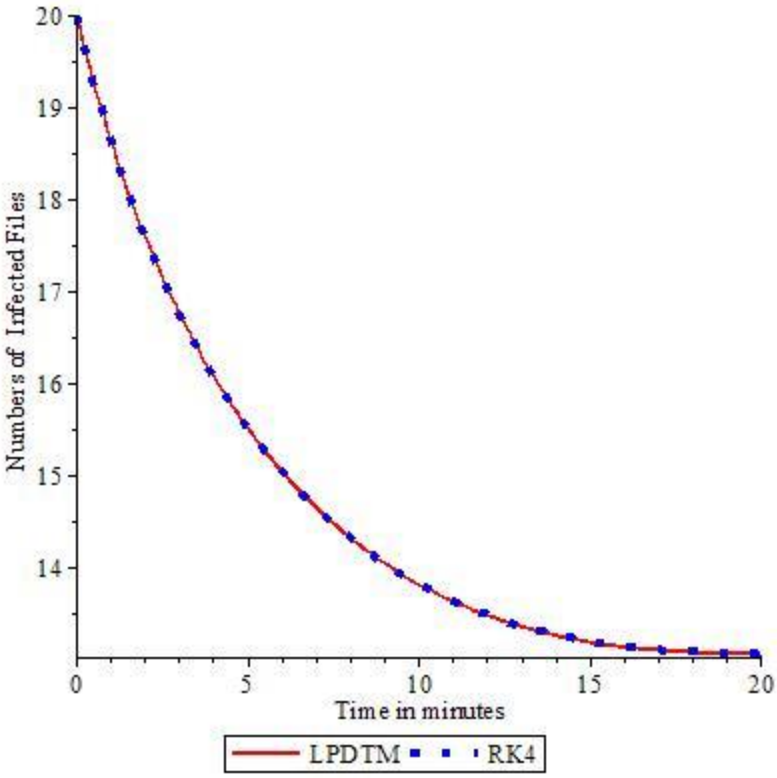

Figure 3: Graphical comparison of $Z(t)$

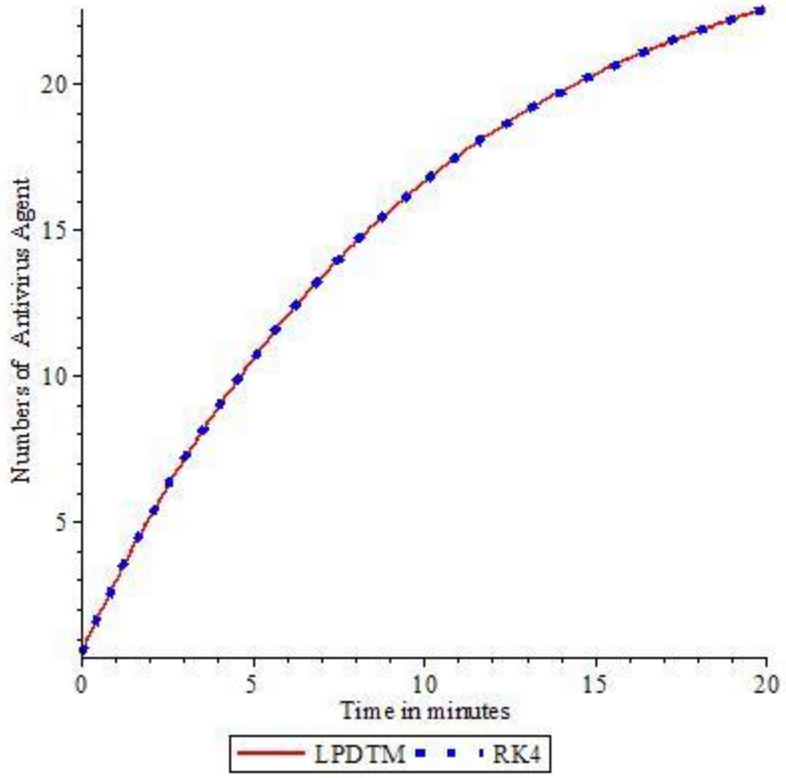

Figure 4: Graphical comparison of $N(t)$

Figure 1-4 shows that the results obtained by LPDTM is in good agreement with that of Runge – Kutta method and produces correctly the dynamics of the model . In Table 2, we present the absolute differences between LPDTM solution and RK4 solution on step size $l = 0.01$.

Table 2: Absolute differences obtained by using RK4 method of step size $l = 0.01$ and LPDTM

$$\Delta = \left| LPDTM - RK4_{\ l=0.01} \right|$$

Time	ΔX	ΔY	ΔZ	ΔN
0	0	2.0×10^{-3}	0	0
2	4.1294×10^{-4}	6.0×10^{-4}	0	3.6580×10^{-4}
4	3.6348×10^{-5}	5.0×10^{-4}	0	1.6117×10^{-4}
6	1.7385×10^{-4}	8.0×10^{-4}	0	3.0328×10^{-4}
8	8.8918×10^{-4}	1.0×10^{-4}	1.0×10^{-3}	1.1141×10^{-4}
10	9.2663×10^{-4}	1.0×10^{-3}	2.0×10^{-3}	7.425×10^{-5}
12	1.2339×10^{-3}	1.0×10^{-3}	2.0×10^{-3}	4.5240×10^{-4}
14	1.7882×10^{-3}	2.0×10^{-3}	3.0×10^{-3}	2.2258×10^{-4}
16	2.6345×10^{-3}	5.0×10^{-3}	4.0×10^{-3}	3.6121×10^{-4}
18	4.2324×10^{-3}	1.1×10^{-2}	3.0×10^{-3}	1.2165×10^{-4}
20	6.6478×10^{-3}	1.8×10^{-2}	1.0×10^{-3}	4.9722×10^{-4}

It is obvious to note that the solution provided by LPDTM in (20) is an exact solution for (8). Thus establishing the superiority of LPDTM over RK4 in providing accurate result.

5.0 CONCLUSION

In this paper, the approximate analytic solution of the computer virus model with antivirus was obtained by a modified differential transform method. The proposed method (LPDTM) improves the solution obtained by DTM by increasing its domain of convergence. This technique requires a post treatment of the power series solution of the computer virus model obtained by DTM with Laplace-Pade resummation method. LPDTM requires no discretization, perturbation or linearization and can be used to solve other nonlinear problems in science and engineering.

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