An Approach to Formulate Mathematical Model for Face Drilling in Underground Mining Operation

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Abstract

The paper details the approach to improve the productivity and conserving human energy in face drilling activity in underground mines. Face drilling is one of the primary activities and consumes a good amount of time for the mining crew in the underground mines. With formulation of the mathematical model, improvements in the present method of face drilling which can conserve human energy besides increasing the productivity and reducing the time required. This mathematical model predicts the optimization of face drilling activity. Some of the variables used to formulate this model are (1) Environment of working area such as illumination, ambient temperature and air circulation facility around the work station (2) Productivity of the face drilling activity,(3) Anthropometric data which include the ergonomic aspects; i.e. various postures of the miner etc., (4)Tools used by miner which include geometric dimensions of tool, pneumatic system parameters etc., based on the data collected of these variables, mathematical model is formulated.

Keywords: Mathematical Modelling, Face drilling, Ore productivity, Human energy, Underground mines.

1. Introduction

In the Indian mining industry, most of the work is done manually because of the limitations for mechanization such as technological, environmental and cost oriented. Considering this fact, many activities are manual in underground mines. Mine workers are exposed to all kinds of machine and environmental hazards. Ergonomics tries to achieve human comfort while accomplishing the work efficiently. Disregard for ergonomic principles and practices lead to low man-machine system efficiency, poor health and increased number of accidents.

Face drilling is one of the primary operations and consumes about 50-60% of the total time for a three member mining crew in the underground mine. It is a repetitive task that involves awkward postures and physically demanding on the neck, shoulders, back and forearms. The tools and equipments required for Face drilling operation are pneumatically (compressed air) operated Jack hammer mounted on air-leg, drill rods in lengths of 0.8m, 1.2m & 1.5 m. and flexible tube water connection for removing debris from the hole. In this face drilling operation, a three member crew (Miners) performs the task for the duration of 8 hours shift except when they are relieved for up to an hour. Jack hammer is the machine, (rested on two legs in an inclined position on the ground) utilizes 3-6 kgf/cm^2 compressed air and consumes 1000-700 c.f.m. air per hole. In this operation, a 32mm dia.,1.2m drill rod is placed into holder of a Jack hammer to drill into the face of the mine of about 1m depth. The drill rod's chisel like face edges chisels the mine face through 360^0 and the hole is formed, for this the drill rod rotates through 360^0. The drill rod is having a central longitudinal capillary, water is fed for the debris removal from drilling point and it also cools the chisel like edges of drill rod. The compressed air in Air leg along with force applied by the miners provides the necessary feed to the drill rod for obtaining progressive depth in the hole. In this process, drilling a hole takes about 5 minutes time. An approx. $1m^2$ grid will have 9 holes of about 1 m depth which takes about 1 hour time to drill these holes. Explosive sticks are inserted (explosive charging) into these holes and are remotely blasted to obtain one Cubic meter of Ore production.

In the present method, the productivity is less and requirement of human energy is substantial. Therefore, the factors influencing the face drilling have been identified, so as to optimize the productivity and conserving human energy in this activity. The generalized mathematical model has been formulated using theories of experimentation for the face drilling activity in underground mines. Therefore, present approach could be replaced with optimized techniques based on field data based modelling in which dependent and independent variables of an activity can be compared and the one most effective method for improving the present method can be evolved.

2. Problems associated with Face drilling activity in Underground mines

Strength characteristics of the underground miners including back, shoulder, arm, sitting leg strength and standing leg strength are poor when compared with other industrial workers. Most studies agree that underground miners are inclined to have lower than average aerobic capacity compared with the population norms and with the comparison groups. Occasionally, miners perform physical work in vertical space restrictions such that crawling is not even possible. While this represents an extreme case, it is not at all uncommon in the mine to be not higher than 1.2 meters. The physiological and biomechanical demands of doing manual work in such an environment are much greater, with the above constraint. Further, they have to work in humid, less airy, poor illumination & noisy environment along with vibrations. So, due to the present face drilling method the productivity is less & requirement of human energy and time required is substantial. Hence, it is required to identify the factors influencing the face drilling necessitate to formulate the Field data based model (FDBM) for this activity for increasing the productivity besides reducing the time required for face drilling and conserving human energy.

3. Need for formulation of mathematical model for identifying optimum

Indeed, a question arises before the production in-charge that in spite of the hard work done by the miner, why he fails to give the adequate productivity for complete shift of 8 hours, which reduces the efficiency of operation. Hence, this aspect in general stimulates to investigate a mathematical model, which can predict the face drilling activity performance which involves man-machine system. Indeed the model will be useful for both miners as well as for the production in-charge to work on prominent variables by which they can improve the performance of miner by deciding the strength and weakness of present method. Once weaknesses are known corrective action can be decided.

4. An approach to formulate mathematical model

Normally, the approach adopted for formulating generalized experimental data based model suggested by Schenck H. Jr., [1], to be more specific field- data based model suggested by Modak.J.P.et al[4] has been proposed in the present investigation which involves following steps:

- Identification of variables or parameters affecting the phenomenon
- Reduction of variables through Dimensional analysis
- Direct data collection for the activity from work station(Test data)
- Rejection of absurd data
- Formulation of the model

4.1. Identification of variables:

First step in this process is the identification of variables. Identification of dependent and independent variables of the phenomenon is to be done based on known qualitative physics of the phenomenon. These variables are of three types:
(1) Independent variables,
(2) Dependent variables &
(3) Extraneous variables.

The independent variables are those which can be changed without changing other variables of the phenomenon. The dependent variables are those, which can only change with any change in the independent variables. The extraneous variables change in a random and uncontrolled manner in the phenomenon. If the system involves a large number of independent variables, the experimentation becomes tedious, time consuming and costly. By deducing dimensional equation for the phenomenon, we can reduce the number of independent variables. The exact mathematical form of equation will be the targeted model. Upon getting experimental results, adopting the appropriate method for test data checking and rejection, the erroneous data be identified and removed from the gathered data. Based on the purified data as mentioned above, one has to formulate quantitative relationship between the dependant and independent π terms of the dimensional equation.

Table 1:Dependent and Independent Terms

Sr. No.	Description	Variable type	Symbol	Dimension
01	Diameter of Drill rod (Dr)	Independent	Dr	$[M^0 L T^0]$
02	Length of Drill rod(Lr)	Independent	Lr	$[M^0 LT^0]$
03	Weight of Drill rod(Wr)	Independent	Wr	$[M L T^{-2}]$
04	Hardness of Drill rod(Hr)	Independent	Hr	$[M L^{-1}T^{-2}]$
05	Diameter of Comp. air Hose(Dc)	Independent	Dc	$[M^0 LT^0]$
06	Air Velocity (Ar)	Independent	Ar	$[M^0 LT^{-1}]$
07	Length of Comp.air Hose(Lc)	Independent	Lc	$[M^0 LT^0]$
08	Weight of Comp.air hose(Wc)	Independent	Wc	$[M LT^{-2}]$
09	Rate of Water flow through hose(Qw)	Independent	Qw	$[M^0 L^3T^{-1}]$
10	Weight of Jack hammer(Wj)	Independent	Wj	$[M LT^{-2}]$
11	Illumination(I)	Independent	I	$[M^1 L^0T^{-3}]$
12	Speed of Machine(N)	Independent	N	$[M^0L^0T^{-1}]$
13	Penetration rate	Independent	R	$[M^0L^1T^{-1}]$
14	Comp.air Pressure(Pa)	Independent	Pa	$[ML^{-1} T^{-2}]$
15	Ambient temperature(θ)	Independent	Θ	$[ML^2T^{-2}]$
16	Relative Humidity(ø)	Independent	ø	$[M^0L^0T^0]$
17	Shear strength of Ore(So)	Independent	So	$[ML^{-1}T^{-2}]$
18	Shear strength Mica Schist(Ss)	Independent	Ss	$[ML^{-1}T^{-2}]$
19	Density of Ore(d'o)	Independent	Do	$[ML^{-3} T^0]$
20	Density of Mica Schist(d's)	Independent	Ds	$[ML^{-3} T^0]$
21	Ambient temperature	Independent	θ	$[ML^2T^{-2}]$
22	Stature	Independent	a	$[M^0 L T^0]$
23	Shoulder Height	Independent	b	$[M^0 L T^0]$
24	Elbow Height	Independent	c	$[M^0 L T^0]$
25	Eye Height	Independent	d	$M^0 L T^0]$
26	Finger tip Height	Independent	e	[M0 $LT^0]$
27	Shoulder Breadth	Independent	f	$[M^0 LT^0]$
28	Hip Breadth	Independent	g	$[M^0 LT^0]$
29	Head Breadth across thumb	Independent	h	$[M^0 LT^0]$
30	Walking Length	Independent	WL	$[M^0 LT^0]$
31	Walking Breadth	Independent	WW	$[M^0 LT^0]$
32	Time of drilling (Td)	Dependent	Td	$[M^0L^0T^1]$
33	Productivity of drilling(Pd)	Dependent	Pd	$[M^0L^0T^{-1}]$
34	Human energy(He)	Dependent	He	$[ML^2T^{-2}]$

4.2. Establishment of Dimensionless π terms:

These independent variables have been reduced into group of π terms. The Equation (1) shows the Dimensionless π terms of the phenomenon.

List of the Independent & Dependent π terms of the face drilling activity are:

Table 2: Independent dimensionless π terms

Sr. No.	Independent Dimensionless ratios	Nature of basic Physical Quantities
01	$\pi1=[a*c*e*g*W_L]/[b*d*f*h*W_W]$	Anthropometric dimensions of Miner
02	$\pi2=[Lr*D_C*Lc]/[Dr]$	Specifications DrillRod
03	$\pi3=[Wr*Wc*Wj/Dr^2*So]*Ss*(Do*Ar^2)*(Ds*Ar^2)*Pa*Hr/So]*[Qw/Dr^2*Ar]$	Specifications of Drilling Machine/ process parameters
04	$\pi4=[(Dr*N*R)/Ar]$	Speed& Penetration rate of Drill Machine
05	$\pi5=[\theta/(Dr^3*So)]$	Ambient temperature
06	$\pi6=ø$ %	Relative Humidity
07	$\pi7=I/[Ar*So]$	Illumination

Table 3: Dependent dimensionless π terms

Sr. No.	Dependent Dimensionless ratios or π terms	Nature of basic Physical Quantities
01	$\pi D1 = Td*Ar/Dr$	Time of drilling
02	$\pi D2 = Pd*Dr/Ar$	Productivity of drilling

03	$\pi D3 = He/ Dr^3*So$	Human energy

4.3 Formulation of Field Data Based Model

Seven independent π terms ($\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7$) and three dependent π terms ($\pi_{D1}, \pi_{D2}, \pi_{D3}$) have been identified for field study model formulation.

Each dependent π term is a function of the available independent π terms,

$Td = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7)$

$Pd = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7)$

$He = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7)$

Where,

$Td = \pi_{D1}$, First dependent π term= Td*Ar/Dr

$Pd = \pi_{D2}$, Second dependent π term= Pd*Dr/Ar

$He = \pi_{D3}$, Third dependent π term= He/ Dr^3*So

f stands for "function of ". The probable exact mathematical form for the dimensional equations of the phenomenon could be relationships assumed to be of exponential form.

$$(Z)=K\{[a*c*e*g* W_L]/[b*d*f*h* W_W]^a, [Lr* D_C* Lc]/ [Dr]^b, [(Wr*Wc*Wj/Dr^2*So)]* (Ss*(Do*Ar^2)*(Ds*Ar^2)*Pa*Hr/So)* (Qw /Dr^2*Ar)]^c, [I /(Ar*So)]^d, [(Dr *N*R)/ Ar]^e, [\theta/ (Dr^3*So)]^f, [ø]^g\} \quad –(1)$$

4.4 Model formulation by identifying the curve fitting constant & various indices of π terms:

The multiple regression analysis helps to identify the indices of the different π terms in the model aimed at, by considering seven independent π terms and one dependent π term. Let model aimed at be of the form,

To determine the regression hyper plane, determines a_1, b_1, c_1, d_1, e_1 and f_1 in equation, so that:

$$(Z_1)=K*[(\pi_1)^{a1}*(\pi_2)^{b1}*(\pi_3)^{c1}*(\pi_4)^{d1}*(\pi_5)^{e1}*(\pi_6)^{f1}*(\pi_7)^{g1}] \quad ---- \quad (2)$$
$$(Z_2)=K*[(\pi_1)^{a2}*(\pi_2)^{b2}*(\pi_3)^{c2}*(\pi_4)^{d2}*(\pi_5)^{e2}*(\pi_6)^{f2}*(\pi_7)^{g2}] \quad ---- \quad (3)$$
$$(Z_3)=K*[(\pi_1)^{a3}*(\pi_2)^{b3}*(\pi_3)^{c3}*(\pi_4)^{d3}*(\pi_5)^{e3}*(\pi_6)^{f3}*(\pi_7)^{g3}] \quad ---- \quad (4)$$

To arrive at the regression hyper plane, determination of a1, b1, c1, d1, e1, f1 and g1 in the above equations, so that:

$$\Sigma Z_1 = nK_1 + a_1*\Sigma A + b_1*\Sigma B + c_1*\Sigma C + d_1*\Sigma D + e_1*\Sigma E + f_1*\Sigma F + g_1*\Sigma G$$

$$\Sigma Z_1*A = K_1*\Sigma A + a_1*\Sigma A*A + b_1*\Sigma B*A + c_1*\Sigma C*A + d_1*\Sigma D*A + e_1*\Sigma E*A + f_1*\Sigma F*A + g_1*\Sigma G*A$$

$$\Sigma Z_1*B = K_1*\Sigma B + a_1*\Sigma A*B + b_1*\Sigma B*B + c_1*\Sigma C*B + d_1*\Sigma D*B + e_1*\Sigma E*B + f_1*\Sigma F*B + g_1*\Sigma G*B$$

$$\Sigma Z_1*C = K_1*\Sigma C + a_1*\Sigma A*C + b_1*\Sigma B*C + c_1*\Sigma C*C + d_1*\Sigma D*C + e_1*\Sigma E*C + f_1*\Sigma F*C + g_1*\Sigma G*C$$

$$\Sigma Z_1*D = K_1*\Sigma D + a_1*\Sigma A*D + b_1*\Sigma B*D + c_1*\Sigma C*D + d_1*\Sigma D*D + e_1*\Sigma E*D + f_1*\Sigma F*D + g_1*\Sigma G*D$$

$$\Sigma Z_1*E = K_1*\Sigma E + a_1*\Sigma A*E + b_1*\Sigma B*E + c_1*\Sigma C*E + d_1*\Sigma D*E + e_1*\Sigma E*E + f_1*\Sigma F*E + g_1*\Sigma G*E$$

$$\Sigma Z_1*F = K_1*\Sigma F + a_1*\Sigma A*F + b_1*\Sigma B*F + c_1*\Sigma C*F + d_1*\Sigma D*F + e_1*\Sigma E*F + f_1*\Sigma F*F + g_1*\Sigma G*F$$

$$\Sigma Z_1*G = K_1*\Sigma G + a_1*\Sigma A*G + b_1*\Sigma B*G + c_1*\Sigma C*G + d_1*\Sigma D*G + e_1*\Sigma E*G + f_1*\Sigma F*G + g_1*\Sigma G*G$$

In the above set of equations, the values of the multipliers K_1, a_1, b_1, c_1, d_1, e_1, f_1 and g1 are substituted to compute the values of the unknowns (viz. K_1, a_1, b_1, c_1, d_1, e_1, f_1 and g1). The values of the terms on L.H.S and the multipliers of K_1, a_1, b_1, c_1, d_1, e_1, f_1 and g1 in the set of equations are calculated and tabulated in the Table. After substituting these values in the equations, one will get a set of 8 equations, which are to be solved simultaneously to get the values of K_1, a_1, b_1, c_1, d_1, e_1, f_1 and g1.The above equations can be verified in the matrix form and further values of K_1, a_1, b_1, c_1, d_1, e_1, f_1 and g1 can be obtained by using matrix analysis.

$$\mathbf{X_1 = inv (W) x P_1}$$

The matrix method of solving these equations using 'MATLAB' is given below.

W = 8 x 8matrix of the multipliers of K_1, a_1, b_1, c_1, d_1, e_1, and f_1

P_1 = 8x 1 matrix of the terms on L H S and

X_1 = 8 x 1 matrix of solutions of values of K_1, a_1, b_1, c_1, d_1, e_1, and f_1

Then, the matrix obtained is given by,

Matrix

$$Z_1 \; x \begin{bmatrix} 1 \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{bmatrix} = \begin{bmatrix} n & A & B & C & D & E & F & G \\ A & A^2 & BA & CA & DA & EA & FA & GA \\ B & AB & B^2 & CB & DB & EB & FB & GB \\ C & AC & BC & C^2 & DC & EC & FC & GC \\ D & AD & BD & CD & D^2 & ED & FD & GD \\ E & AE & BE & CE & DE & E^2 & FE & GE \\ F & AF & BF & CF & DF & EF & F^2 & GF \\ G & AG & BG & CG & DG & EG & FG & G^2 \end{bmatrix} x \begin{bmatrix} K_1 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \\ f_1 \\ g1 \end{bmatrix}$$

X_1 matrix with K_1 and indices a_1, b_1, c_1, d_1, e_1, f_1, g_1 are evaluated:

In the above equations, n is the number of sets of readings, A,B,C,D,E,F and G represent the independent π terms π_1, π_2, π_3, π_4, π_5, π_6 and π_7 while, Z represents, dependent π term. Next, calculate the values of Independent π term for corresponding dependent π term, which helps to form the equation in matrix form. It is recommended to use MATLAB software for this purpose for making this process of model formulation quickest and least cumbersome.

4.5 Sensitivity of Inputs

The matrix form of derived equation is as follows:
[z]=[x]*[a]
Supposing the exact form of model is obtained as:

$$(Z1) = 28.723256*(\pi_1)^{0.0115}*(\pi_2)^{0.6607}*(\pi_3)^{0.0019}*(\pi_4)^{0.012}*(\pi_5)^{0.068}*(\pi_6)^{0.002}*(\pi_7)^{0.0018} \quad (5)$$

$$(Z2) = 8.2*(\pi_1)^{4}*(\pi_2)^{0.3}*(\pi_3)^{-1.7}*(\pi_4)^{2.1}*(\pi_5)^{0.0607}*(\pi_6)^{0.0019}*(\pi_7)^{0.0019} \quad (6)$$

$$(Z3) = 6.4*(\pi_1)^{0.0120}*(\pi_2)^{0.070}*(\pi_3)^{0.0050}*(\pi_4)^{0.060}*(\pi_5)^{0.0327}*(\pi_6)^{0.0459}*(\pi_7)^{0.0219} \quad (7)$$

In the above equations (Z_1) is relating response variable for time of face drilling activity,(Z_2) is relating response variable for productivity of face drilling and (Z_3) is relating response variable for human energy consumed in the activity.

4.6 Interpretation of model:

Interpretation of model is being reported in terms of several aspects viz. (1) Order of influence of various inputs (causes) on outputs (effects) (2) Relative influence of causes on effect (3) Interpretation of curve fitting constant K (4) Sensitivity of causes (5) optimization (6) Reliability.

4.7 Interpretation of curve fitting constant (K):

The value of curve fitting constant in this model for (Z_1) is 28.723256. This collectively represents the combined effect of all extraneous variables. Further, as it is positive, this indicates that, there are good numbers of causes, which have influence on increasing effect.

To decide the effectiveness of the present method, the influence of inputs on response variable (Z1) in the equation (5), is maximum when π_2 is as high as possible as compared to other π terms. This is so because; the index of π_2 is the highest when compared with the indices of other π terms. Similarly, for the influence of inputs on response variable (Z2) in the equation (6), the influence of π_1 is the maximum and π_3 is the minimum as their indices are 4.0 and -1.7 respectively. In the same way, the influence of other inputs on the response variables needs to be evaluated.

4.8 Optimization of the Model:

As far as the activity of face drilling is concerned any one will wish to maximize Z_2 (i.e. Productivity) whereas he would like to minimize Z_1 (i.e. Time to required for overhauling) & Z_3 (i.e. Human energy input).

Now, it is the time to apply the subject optimization technique for arriving at, at which values the inputs that Z_2 can be maximized and Z_1 & Z_3 can be minimized. This has to be the sole objective of deciding *"How to improve the method of performance of Face drilling activity"*. Thus this approach of formulation of FDBM for such a man-machine system should be looked upon as a new technique of method study. This was not possible in the absence of establishing such models. These models will help to predict the **"***Intensity of interaction of inputs on deciding Response*" of face drilling activity.

4.9 Reliability of Models:

Obviously, before taking up the step of sensitivity of inputs, it is necessary to decide the validity of the model. This is so because though, we have taken care to purify the observed data, there is a chance of some impure data entering in the mathematical processing of the data though even using MATLAB.

The approach to decide the validity would be to substitute in the model known inputs for every observation & decide the difference in response by model and actually observed response. This will give us pattern of distribution of error & frequency of its occurrence. Using this distribution & literature on reliability, we would establish the reliability of the model

5.0 Conclusions:

The postural discomfort experienced by miners while performing face drilling, became the cornerstone for this work. They are not aware as to what extent ergonomic intervention can alleviate their drudgery. Secondly, the relationship between various inputs such as anthropometry of miners, specifications of drill machine, specification of tools, surrounding environmental conditions and their responses such as time to complete drill, human energy and productivity of face drilling activity is not known to them quantitatively. Thus from these models **"***Intensity of interaction of inputs on deciding Response*" can be predicted which will help to control the variable for the desired results.

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