

# An Approach for the Unified Evaluation of Orientation Tolerances

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**Abstract** - Orientation tolerances are specified to control the parallel, perpendicular and other angular relationships between two adjacent features of manufactured parts. One feature acts as datum feature and the other as measured or controlled feature. Verification of orientation tolerances require the establishment of two parallel planes at required parallel, perpendicular or angular orientation with respect to the datum feature and encompassing all the data points of measured feature with minimum spacing. In this paper, a new approach for unified evaluation of orientation tolerances of straight line features is proposed first followed by its implementation using the random walk, simplex search and normal least squares methods. The relative performances of these methods have been studied using simulated data.

**Keywords** - Angularity; Datum Feature; Measured Feature; Normal Least Squares Method; Orientation Tolerances; Parallelism; Perpendicularity; Random Walk; Simplex Search.

## I. INTRODUCTION

The orientation tolerances are geometric tolerances that are specified with one or more datums. They are used to control the parallel (parallelism), perpendicular (perpendicularity) and other angular relationships (angularity) between two adjacent features in manufactured parts. Parallelism can be defined as the condition of a surface, median plane or axis being parallel to a datum plane or axis. Perpendicularity can be defined as the condition of a surface, median plane or axis being at right angle to a datum plane or axis. Angularity refers to the condition of a surface, median plane or axis being at some specified angle to a datum plane or axis. In all cases, the tolerance zone is defined by two parallel planes established at  $0^\circ$  for parallelism,  $90^\circ$  for perpendicularity and *specified angle* for angularity with respect to datum plane or axis. The elements of measured or controlled feature must lie between these planes [1]. Fig. 1(a-b), Fig. 1(c-d) and Fig. 1(e-f) respectively show the example specifications of angularity, perpendicularity and parallelism tolerances and their interpretations.

Verification of orientation tolerances on measured feature requires the establishment of an ideal datum feature, based on the measurement data of datum feature, using suitable methods. The standards recommend minimum zone evaluation but do not suggest any specific method for finding the minimum zone. Despite not guaranteeing the minimum zone solution, the least squares method (LSM) is commonly used for this purpose due to its sound mathematical basis [2-4]. Numerous algorithms for finding the minimum zone solutions have been reported. Such algorithms are based on some optimization, soft computing and geometry-based computational techniques. A review of some of these works is presented here.

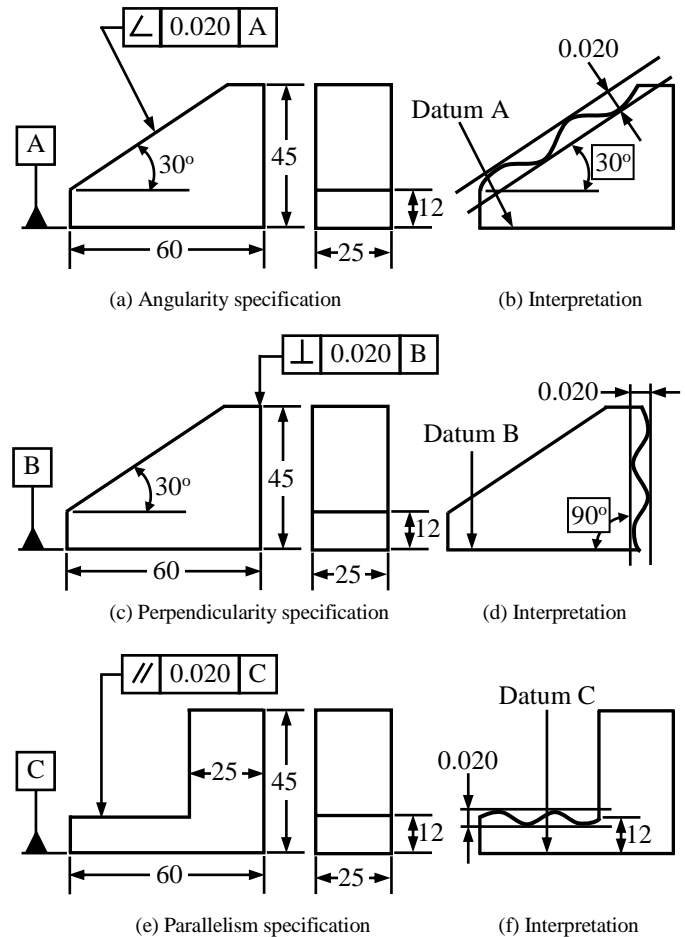


Fig. 1. Examples of orientation tolerances

Computation of minimum zone form errors using numerical methods such as the Monte-Carlo method, discrete Chebyshev approximation, min-max approximation, simplex search, spiral search, median technique, etc. [4-11], enclosing polygon based methods such as the convex hull method, Eigen-polyhedral method, etc. [10, 12-14] and other methods such as control line rotation scheme [2], nonlinear optimization approach [3-4] and linearizing nonlinear problems using combined coordinate and scaling transformations [15] have been attempted. Use of soft computing tools, such as Genetic Algorithms (GAs) have been shown to be robust in form tolerance evaluation, e.g. circularity evaluation [16]. Computational geometric techniques were also developed for dealing with datum related features [17]. Least squares method based evaluation of the geometric tolerances in relationships, viz. parallelism, run-out and concentricity, has been reported [18]. Extension of straightness evaluation using

convex hull based approach to perpendicularity evaluation has been reported [19]. An improved Particle Swarm Optimization (PSO) algorithm has been used to evaluate the perpendicularity error between two planar lines by formulating it as a linear optimization problem [20]. The authors have used maximum absolute distance as straightness error on datum feature, which violates the standards. A random walk based algorithm for the perpendicularity evaluation has also been reported [21].

There is still a need for developing effective algorithms for the evaluation of orientation tolerances. Their evaluation also lacks a unified approach. The present work attempts to address both these issues. A unified approach is proposed first for the planar straight line features. This approach is a generalization of an earlier work meant for perpendicularity evaluation [21]. The proposed approach is implemented using random walk, simplex search and normal least squares methods so as to find an effective algorithm. The effectiveness of these algorithms is tested using simulated data. The results obtained are presented and discussed.

Remainder of the paper is organized as follows. Section II pronounces the proposed unified approach. Section III briefs about the algorithms used for fitting the ideal datum features. Section IV presents the evaluation approach used and Section V presents and discusses the results obtained using different algorithms. Section VI states the conclusions and future scope.

## II. THE PROPOSED UNIFIED APPROACH

As mentioned earlier, an ideal datum feature (a straight line in present case [21]) has to be established first for evaluation of orientation tolerances. Let this feature be represented as in (1), with usual notations.

$$y = ax + b \tag{1}$$

Distances  $d_j$  ( $j = 1, 2, \dots, m$ ;  $m$  is the number of measurement points) between measured points  $P_j(x_j, y_j)$  of datum feature to the ideal datum feature may be calculated using (2).

$$d_j = \pm \frac{|y_j - ax_j - b|}{\sqrt{1 + a^2}}; j = 1, 2, \dots, m \tag{2}$$

Distance  $d_j$  is taken as positive when the measured point is above the ideal datum feature and negative when it is below. Straightness error ( $s$ ) of datum feature can be expressed as:

$$s = |d_{j_{\max}}| + |d_{j_{\min}}| \tag{3}$$

In (3),  $d_{j_{\min}}$  and  $d_{j_{\max}}$  represent the minimum and maximum values of  $d_j$  respectively. Estimation of  $a$  and  $b$  for computing the straightness error ( $s$ ) of datum feature can be stated as an unconstrained minimization problem, satisfying the minimum zone condition, as follows [20]:

$$\text{Minimize } f(a, b) = s \tag{4}$$

If  $(a^*, b^*)$  is the optimal solution for (4), equation of ideal datum feature can be written as:

$$y = a^*x + b^* \tag{5}$$

Depending on the orientation tolerance to be evaluated, two parallel lines are established at specified angular orientation, with respect to ideal datum feature, which will be the specified

angle for angularity (Fig. 2),  $90^\circ$  for perpendicularity (Fig. 3) and  $0^\circ$  for parallelism (Fig. 4).

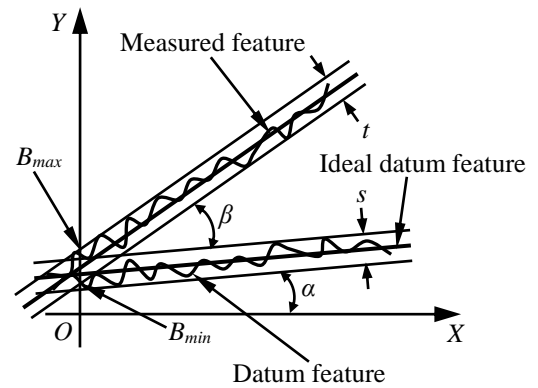


Fig. 2. Angularity analysis ( $\beta = \text{Specified angle}$ )

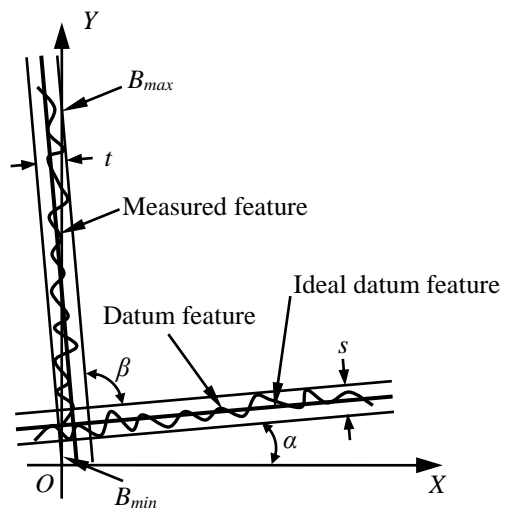


Fig. 3. Perpendicularity analysis ( $\beta = 90^\circ$ )

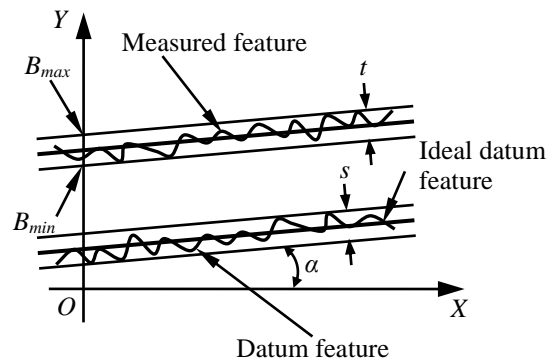


Fig. 4. Parallelism analysis ( $\beta = 0^\circ$ )

Lines are drawn through all the points  $P_j(x_j, y_j)$  ( $J = 1, 2, \dots, M$ ;  $M$  is the number of points) of the measured feature. Let the equations of such lines be described using (6).

$$Y = AX + B \tag{6}$$

In (6),  $A$  denotes the slopes and  $B$  denotes the  $y$ -intercepts of those lines. If  $B_{\max}$  and  $B_{\min}$  are the maximum (i.e. largest  $Y$ -intercept) and minimum (i.e. smallest  $Y$ -intercept) values of  $B$  respectively,  $\alpha$  is the angle between ideal datum feature and the  $x$ -axis and  $\beta$  is the specified angular orientation of measured or controlled features in relation to the datum feature, orientation tolerance ( $t$ ) can be expressed as:

$$t = (B_{max} - B_{min})\cos(\alpha + \beta) \quad (7)$$

The use of  $B_{max}$  and  $B_{min}$  in (7) ensures that all the points of measured feature lies within the tolerance zone. The tolerance value ( $t$ ) computed using (7) is therefore the minimum value, which implies that orientation tolerances obtained using this equation follows the minimum zone evaluation.

### III. ALGORITHMS FOR FITTING THE IDEAL DATUM FEATURE

#### A. Random Walk Method

In random walk, a sequence of improved approximations to the minimum value are generated based on the preceding approximation. If  $X_i$  is the approximation to minimum value obtained in  $(i-1)^{th}$  iteration, improved approximation in the  $i^{th}$  iteration is obtainable from (8).

$$X_{i+1} = X_i + \lambda u_i \quad (8)$$

where,  $\lambda$  is a prescribed scalar step length and  $u_i$  is a unit random vector generated in  $i^{th}$  stage. The detailed procedure of random walk method may be found in [22].

#### B. Simplex Search Method

Simplex is a geometric figure formed by  $(n + 1)$  points in an  $n$ -dimensional space. The simplex is gradually moved towards the optimal point by comparing the objective function values at  $(n + 1)$  vertices of the general simplex. Three operations, viz. reflection, contraction and expansion, are performed to achieve the desired movement of the simplex. More details of simplex search method and its algorithm may be found in [22].

#### C. Normal Least Squares Method

The objective of normal LSM is to obtain the best fit line (feature), as given in (1), that minimizes the sum of squares of normal distances ( $E_N$ ), between the measurement points  $P_j (x_j, y_j)$  and the best fit line (feature), as given in (9) [23].

$$E_N = \sum_{j=1}^m [(y_j - y)\cos\theta]^2 \quad (9)$$

Substituting for  $y = ax_j + b$  and  $\cos\theta = \frac{1}{\sqrt{1+a^2}}$  in (9),

we get

$$E_N = \sum_{j=1}^m \frac{(y_j - ax_j - b)^2}{(1+a^2)} \quad (10)$$

The details on normal least squares method and the method of solving the coefficients  $a$  and  $b$  can be found in [23].

### IV. APPROACH TO COMPUTATION OF ORIENTATION TOLERANCES

Flowchart for the approach used to evaluate the orientation tolerances is shown in Fig. 5. The random walk, simplex search and normal least squares method based algorithms are used only for obtaining the ideal datum feature. After this, lines are drawn through all points of measured or controller feature at specified angular orientation with reference to the ideal datum feature. If  $B_{max}$  is the  $y$ -intercept of topmost line and  $B_{min}$  is the  $y$ -intercept of bottommost line, the orientation tolerance ( $t$ ) can be computed by substituting these values in (7).

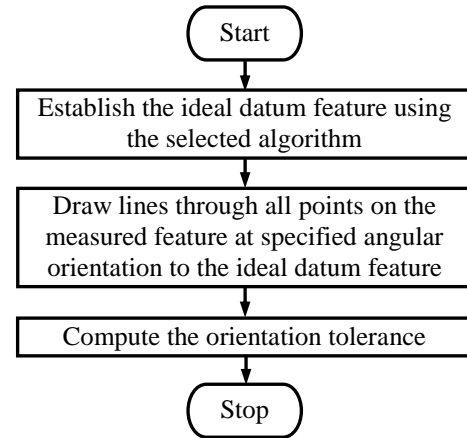


Fig. 5. Flowchart for evaluation of orientation tolerances (modified from [21])

### V. RESULTS AND DISCUSSION

Orientation tolerances refer to the geometric tolerances that are specified with datums to control the geometric deviations in manufactured parts. The verification of orientation tolerances, viz. angularity, perpendicularity and parallelism, is considered in the present work due to their importance in machine tools and coordinate measuring machines. A new unified approach for the evaluation of orientation tolerances in planar straight line features is presented in this paper. This approach has been implemented using random walk, simplex search and normal least squares algorithms using C++ language under the Visual Studio 2015 environment and run on a Microsoft Windows 10 powered PC equipped with 4 GB RAM. Performance of these algorithms were studied using the simulated datasets shown in Appendices A to C. The results of evaluation of angularity, perpendicularity and parallelism tolerances are shown in Table I, II and III respectively. As the desired criteria, evaluation algorithms are expected to yield smaller straightness values on the datum feature and *preferably* smaller values of orientation tolerances on the measured feature. The *preferably* adjective is due to the fact that these geometrical tolerances are estimated in sequence and not simultaneously. Precisely, smaller values of straightness error does not necessarily mean smaller values of orientation tolerances.

Table I reveals that the most commonly used normal least squares method overestimates the straightness (7.009  $\mu\text{m}$  for A and 8.944  $\mu\text{m}$  for B) and angularity (16.120  $\mu\text{m}$  for A and 16.323  $\mu\text{m}$  for B) errors. Random walk based algorithm yields smallest values of straightness (5.502  $\mu\text{m}$  for A and 5.287  $\mu\text{m}$  for B) and angularity (14.311  $\mu\text{m}$  for A and 15.346  $\mu\text{m}$  for B) errors and simplex search comes next. Table II reveals similar trends in perpendicularity evaluation. The results of parallelism error evaluation, shown in Table III, reveals that random walk algorithm outperforms the other algorithms, i.e. yields lowest straightness and parallelism errors. The simplex search shows a slightly poor performance in parallelism error in comparison to normal LSM, however, the straightness error is still less than that of normal LSM. Thus, the random walk based algorithm is found to perform consistently in evaluating the orientation tolerances better than the other two algorithms. As a general statement, it may be said that normal LSM overestimates the geometrical tolerances always despite its sound mathematical basis and wide application in measuring instruments. Random walk based algorithm also follows the standards.

TABLE I. RESULTS OF ANGULARITY EVALUATION

Dataset	Algorithm	Straightness error, $\mu m$	Angularity error, $\mu m$
A	Random Walk	5.502	14.311
	Simplex Search	6.737	15.794
	Normal LSM	7.009	16.120
B	Random Walk	5.287	15.346
	Simplex Search	8.668	16.020
	Normal LSM	8.944	16.323

TABLE II. RESULTS OF PERPENDICULARITY EVALUATION

Dataset	Algorithm	Straightness error, $\mu m$	Perpendicularity error, $\mu m$
C	Random Walk	5.889	12.590
	Simplex Search	7.701	14.752
	Normal LSM	7.849	14.930
D	Random Walk	4.711	19.300
	Simplex Search	7.893	22.854
	Normal LSM	8.132	23.116

TABLE III. RESULTS OF PARALLELISM EVALUATION

Dataset	Algorithm	Straightness error, $\mu m$	Parallelism error, $\mu m$
E	Random Walk	6.449	25.824
	Simplex Search	9.677	31.090
	Normal LSM	10.028	30.537
F	Random Walk	5.316	17.909
	Simplex Search	10.643	21.473
	Normal LSM	10.866	21.166

VI. CONCLUSIONS AND FUTURE SCOPE

Geometrical deviations in manufactured parts are caused by systematic and random errors that occur during manufacturing. The geometrical tolerances are used to specify and control such deviations. Their evaluation requires effective algorithms that follow the standards. The evaluation of orientation tolerances, viz. angularity, perpendicularity and parallelism in straight line features has been considered in the present work. An approach for unified evaluation orientation tolerances has been proposed and implemented using different numerical algorithms, viz. random walk, simplex search and normal least squares method. The performance of these algorithms has been evaluated using simulated data. Random walk algorithm has been found to be most effective among the three algorithms. The simplex search algorithm has also been found to yield better results. Both these algorithms outperform the most commonly used normal least squares based algorithm. The extension of proposed approach to three-dimensional features may form the future work.

APPENDIX A: ANGULARITY DATASET

S#	Dataset A				S#	Dataset B			
	Datum		Measured			Datum		Measured	
	$x_i$	$y_i$	$x_i$	$y_i$		$x_i$	$y_i$	$x_i$	$y_i$
1	0.00	2.5146	-1.1482	2.2536	1	0.00	2.5182	-2.2557	1.1493
2	0.75	2.5146	-0.4808	2.5957	2	1.00	2.5187	-1.8038	2.0414
3	1.50	2.5143	0.1861	2.9388	3	2.00	2.5187	-1.3529	2.9340
4	2.25	2.5144	0.8542	3.2795	4	3.00	2.5190	-0.8994	3.8253
5	3.00	2.5168	1.5225	3.6200	5	4.00	2.5216	-0.4455	4.7163
6	3.75	2.5165	2.1895	3.9630	6	5.00	2.5214	0.0059	5.6086
7	4.50	2.5175	2.8590	4.3009	7	6.00	2.5223	0.4627	6.4982
8	5.25	2.5149	3.5256	4.6447	8	7.00	2.5195	0.9136	7.3908
9	6.00	2.5191	4.1940	4.9849	9	8.00	2.5235	1.3683	8.2814
10	6.75	2.5174	4.8631	5.3237	10	9.00	2.5215	1.8245	9.1713
11	7.50	2.5159	5.5308	5.6654	11	10.00	2.5195	2.2781	10.0625
12	8.25	2.5170	6.1993	6.0055	12	11.00	2.5201	2.7333	10.9529
13	9.00	2.5148	6.8672	6.3465	13	12.00	2.5173	3.1877	11.8437
14	9.75	2.5182	7.5361	6.6858	14	13.00	2.5199	3.6439	12.7336

15	10.50	2.5165	8.2044	7.0261	15	14.00	2.5175	4.0992	13.6239
16	11.25	2.5161	8.8736	7.3648	16	15.00	2.5163	4.5562	14.5134
17	12.00	2.5161	9.5428	7.7035	17	16.00	2.5154	5.0132	15.4029
18	12.75	2.5153	10.2112	8.0436	18	17.00	2.5138	5.4688	16.2930
19	13.50	2.5131	10.8804	8.3823	19	18.00	2.5106	5.9259	17.1825
20	14.25	2.5145	11.5477	8.7246	20	19.00	2.5111	6.3794	18.0737
21	15.00	2.5123	12.2176	9.0620					
22	15.75	2.5111	12.8866	9.4009					
23	16.50	2.5105	13.5553	9.7407					
24	17.25	2.5090	14.2242	10.0798					
25	18.00	2.5093	14.8948	10.4157					

APPENDIX B: PERPENDICULARITY DATASET

S#	Dataset C				S#	Dataset D			
	Datum		Measured			Datum		Measured	
	$x_i$	$y_i$	$x_i$	$y_i$		$x_i$	$y_i$	$x_i$	$y_i$
1	0.00	2.5158	-2.5244	0.00	1	0.00	2.5122	-2.5292	0.00
2	0.80	2.5159	-2.5261	0.80	2	1.20	2.5124	-2.5318	1.20
3	1.60	2.5157	-2.5289	1.60	3	2.40	2.5122	-2.5353	2.40
4	2.40	2.5158	-2.5290	2.40	4	3.60	2.5122	-2.5359	3.60
5	3.20	2.5183	-2.5288	3.20	5	4.80	2.5146	-2.5358	4.80
6	4.00	2.5181	-2.5315	4.00	6	6.00	2.5141	-2.5384	6.00
7	4.80	2.5191	-2.5285	4.80	7	7.20	2.5148	-2.5350	7.20
8	5.60	2.5164	-2.5321	5.60	8	8.40	2.5118	-2.5380	8.40
9	6.40	2.5206	-2.5317	6.40	9	9.60	2.5156	-2.5366	9.60
10	7.20	2.5189	-2.5297	7.20	10	10.80	2.5134	-2.5334	10.80
11	8.00	2.5173	-2.5309	8.00	11	12.00	2.5112	-2.5332	12.00
12	8.80	2.5183	-2.5303	8.80	12	13.20	2.5116	-2.5310	13.20
13	9.60	2.5160	-2.5308	9.60	13	14.40	2.5086	-2.5297	14.40
14	10.40	2.5193	-2.5294	10.40	14	15.60	2.5112	-2.5264	15.60
15	11.20	2.5175	-2.5292	11.20	15	16.80	2.5087	-2.5241	16.80
16	12.00	2.5169	-2.5270	12.00	16	18.00	2.5075	-2.5198	18.00
17	12.80	2.5167	-2.5250	12.80	17	19.20	2.5066	-2.5156	19.20
18	13.60	2.5158	-2.5246	13.60	18	20.40	2.5051	-2.5130	20.40
19	14.40	2.5134	-2.5226	14.40	19	21.60	2.5021	-2.5090	21.60
20	15.20	2.5146	-2.5246	15.20	20	22.80	2.5028	-2.5090	22.80
21	16.00	2.5123	-2.5211	16.00					
22	16.80	2.5109	-2.5194	16.80					
23	17.60	2.5101	-2.5186	17.60					
24	18.40	2.5083	-2.5172	18.40					
25	19.20	2.5085	-2.5122	19.20					

APPENDIX C: PARALLELISM DATASET

S#	Dataset E				S#	Dataset F			
	Datum		Measured			Datum		Measured	
	$x_i$	$y_i$	$x_i$	$y_i$		$x_i$	$y_i$	$x_i$	$y_i$
1	0.00	2.5158	0.00	47.5341	1	0.00	2.5170	0.00	77.5280
2	1.00	2.5161	1.00	47.5366	2	1.25	2.5176	1.25	77.5306
3	2.00	2.5160	2.00	47.5402	3	2.50	2.5177	2.50	77.5341
4	3.00	2.5162	3.00	47.5408	4	3.75	2.5180	3.75	77.5346
5	4.00	2.5187	4.00	47.5410	5	5.00	2.5205	5.00	77.5345
6	5.00	2.5184	5.00	47.5439	6	6.25	2.5201	6.25	77.5370
7	6.00	2.5193	6.00	47.5443	7	7.50	2.5208	7.50	77.5335
8	7.00	2.5165	7.00	47.5409	8	8.75	2.5176	8.75	77.5363
9	8.00	2.5205	8.00	47.5435	9	10.00	2.5212	10.00	77.5348
10	9.00	2.5185	9.00	47.5410	10	11.25	2.5187	11.25	77.5315
11	10.00	2.5166	10.00	47.5414	11	12.50	2.5162	12.50	77.5311
12	11.00	2.5172	11.00	47.5400	12	13.75	2.5162	13.75	77.5288
13	12.00	2.5145	12.00	47.5394	13	15.00	2.5127	15.00	77.5273
14	13.00	2.5173	13.00	47.5368	14	16.25	2.5147	16.25	77.5238
15	14.00	2.5150	14.00	47.5352	15	17.50	2.5117	17.50	77.5214
16	15.00	2.5140	15.00	47.5316	16	18.75	2.5098	18.75	77.5170
17	16.00	2.5133	16.00	47.5281	17	20.00	2.5083	20.00	77.5127
18	17.00	2.5118	17.00	47.5260	18	21.25	2.5062	21.25	77.5101
19	18.00	2.5088	18.00	47.5224	19	22.50	2.5026	22.50	77.5060
20	19.00	2.5096	19.00	47.5227	20	23.75	2.5027	23.75	77.5061
21	20.00	2.5067	20.00	47.5176					
22	21.00	2.5048	21.00	47.5143					
23	22.00	2.5036	22.00	47.5120					
24	23.00	2.5015	23.00	47.5090					
25	24.00	2.5013	24.00	47.5027					

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