

An Application of Second Order Volterra Filters for Prediction of Image Sequences

Gopika Krishnan G, Leslie Raju Thomas, Ansymol R. S, Awany Roy
Department of Electronics and Communication
College of Engineering, Karunagappally, Kerala

Abstract— Prediction of image sequences is widely used in image processing and transmission schemes such as Differential Pulse Code Modulation (DPCM). Traditional linear prediction employs linear predictors for simplicity. The nonlinear Volterra predictor can be used as an alternative to linear predictor to account for the nonlinear components in the image signals. The proposed quadratic predictor is more robust to noise. Experimental results illustrate that the nonlinear predictor yields improved mean square error performance. The proposed method can be incorporated in new predictive coding schemes for high throughput.

Keywords— DPCM, image sequence, prediction error, predictor, quadratic filter, singular value decomposition, Volterra series

I. INTRODUCTION

Digital images play vital role in many applications such as biomedical signal processing, satellite imagery etc. The tremendous increases in the generation, processing, storage and transmission of digital images have created increasing demands on the storage capabilities, processing speed and bandwidth requirements. Image compression is an attractive scheme to store images with reasonable amount of storage and transmit images with acceptable speed.

Predictive coding systems have been commonly used for the encoding of speech, image and video signals. Differential Pulse Code Modulation (DPCM) is a relatively simple approach for predictive coding technique [1] which makes use of local similarities and the sequential raster scanning of images to produce a more efficient and better compressed coding of the visual signal. The main idea of the method is to encode the difference between the present sample and its predicted value so that less data elements are needed for processing. In such systems the use of an optimum predictor is critical in achieving maximum data compression. In conventional DPCM the most commonly used predictor is a linear predictor i.e. the prediction value is computed as a linearly weighted sum of samples from the past history of the signal. The linear prediction theory is well established and gives simple optimality criteria for predictor design assuming a stationary signal source.

It has been found that two dimensional linear predictors don't give much better performance when inherent nonlinearities present in the signals. As the statistics may vary, adaptive /switching methods are employed. But the adaptive methods have the drawbacks of being computationally costly

with the amount of data and timing constraints. In this paper, we use Polynomial (Volterra) series to effectively model the mild polynomial nonlinearities in the system input-output relations. Volterra series is simply a power series with a constant as the first term, a term models the linear relation between the input and output (equivalent to LTI system) and a third quadratic term corresponds to the nonlinearities. Such a represent they can model large class of non-linear systems with small number of coefficients. Moreover they enable the analysis and design as an extension of LTI systems.

In this paper, we introduce a nonlinear predictor for image sequence prediction based on the second order (polynomial) Volterra filter. Polynomial filters have the distinctive advantage that the designed quadratic predictor works in parallel with the conventional linear predictor gives improved mean square error performance between the actual signal and predicted value. The idea is extended to the prediction of fMRI frames useful to lower the bit rate needed for transmission of those images.

II. DISCRETE VOLTERRA SERIES

Although the linear systems are widely used because of its ease of design and implementation, most of the practical systems tend to deviate from the linear behavior when the signals are neither stationary nor Gaussian as in the natural images and when there is some nonlinear effects in the signal generation mechanisms. Mild polynomial nonlinearities can be better handled by Volterra series. An Nth order Volterra filter [2] with input vector $x[n]$ and output vector $y[n]$ is given by,

$$Y[n] = h_0 + \sum_{r=1}^{\infty} \sum_{n_1=1}^N \sum_{n_2=1}^N \dots \sum_{n_r=1}^N h_r[n_1, n_2, \dots, n_r] x[n-n_1]x[n-n_2] \dots x[n-n_r] \quad (1)$$

The term h_0 denotes the output offset when no input is present, r denotes the order of nonlinearity and the term $h_r[n_1, n_2, n_r]$ denotes the r^{th} order Volterra kernel which can be considered as the impulse response characterizing the nonlinear behavior of the system. With order $r = 1$, (1) defaults to a convolution between input and output, representing a linear system. For $r=2$, we get the simplest polynomial system called quadratic system which is given,

$$Y[n] = h_0 + \sum_{m_1=1}^N h_1[m_1]x[n-m_1] + \sum_{n_1=1}^N \sum_{n_2=1}^N h_2[n_1, n_2]x[n-n_1]x[n-n_2] \quad (2)$$

IV. METHODOLOGY

Better and efficient prediction of image sequences is made possible with the use of nonlinear predictors like Quadratic predictors. It works in conjunction with a linear predictor for predicting the image sequences in a DPCM system. The scheme as given in section V is proposed for predicting the image sequences. The first phase of the work is in designing the quadratic kernel H_2 . The design is based on the minimization of the mean square error (MSE) between original image sequence and its predicted version. The second phase is the computationally efficient implementation of the designed kernel based on SVD. In the last phase testing of the predictor is done with image sequences of different noise variances. The quadratic predictor is employed in the DPCM system and the performance is compared with that of linear predictors in terms of the mean square error. The experiment and results are detailed in section VI.

Or equivalently by the matrix equation,

$$y[n] = h_0 + X_1^T[n]H_1 + X_1^T[n]H_2X_1[n] \tag{3}$$

Where

$$X_1 = [x[n], x[n - 1], \dots, x[n - N + 1]]^T \tag{4}$$

$$H_1 = [h_1[0], h_1[1], \dots, h_1[N - 1]]^T \tag{5}$$

and

$$H_2 = \begin{bmatrix} h_2(0,0) & h_2(0,1) & \dots & h_2(0,N-1) \\ h_2(1,0) & h_2(1,1) & \dots & h_2(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ h_2(N-1,0) & h_2(N-1,1) & \dots & h_2(N-1,N-1) \end{bmatrix} \tag{6}$$

III. TWO DIMENSIONAL QUADRATIC FILTER

The two dimensional quadratic filter is governed by the equation:

$$y[n_1, n_2] = \sum_{m_{11}=0}^{N_1-1} \sum_{m_{12}=0}^{N_2-1} \sum_{m_{21}=0}^{N_1-1} \sum_{m_{22}=0}^{N_2-1} h_2[m_{11}, m_{12}, m_{21}, m_{22}] * x[n_1 - m_{11}, n_2 - m_{12}] x[n_1 - m_{21}, n_2 - m_{22}] \tag{7}$$

(7) is represented in the matrix form as,

$$y[n_1, n_2] = X^T[n_1, n_2]H_2X[n_1, n_2] \tag{8}$$

The quadratic kernel H_2 has $N_1N_2 \times N_1N_2$ elements and each element consists of N_2 sub-matrices $H[i, j]$ with $N_1 \times N_2$ elements given as,

$H[i, j] =$

$$\begin{bmatrix} h[0, i, 0, j] & \dots & h[0, i, N_1 - 1, j] \\ \vdots & \ddots & \vdots \\ h[N_1 - 1, i, 0, j] & \dots & h[N_1 - 1, i, N_1 - 1, j] \end{bmatrix} \tag{9}$$

Two principal issues in the quadratic systems are the identification of the quadratic kernel H_2 and its computationally efficient implementation. Unlike linear systems there are no general design methods for the quadratic kernel. Here we adopt the optimization of the mean square error using Powell's conjugate gradient method. Second phase involve the implementation of the designed kernel with minimum computational complexity. Matrix decomposition methods like SVD and LU methods are used for suitable implementation of the kernel.

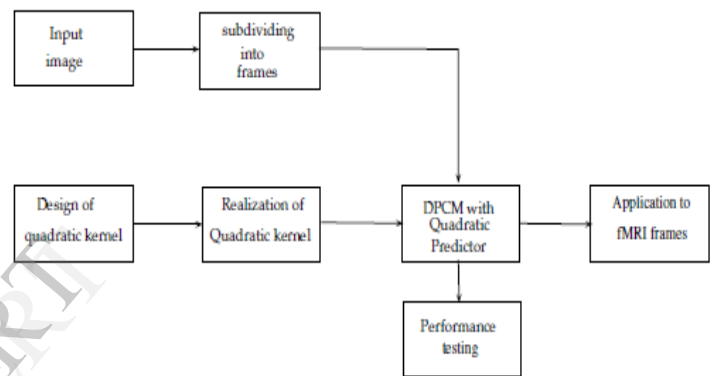


Fig.1 Flow of the Work

V. DPCM SYSTEM WITH QUADRATIC PREDICTOR

DPCM is an effective predictive coding method for the transmission and storage of digital image sequences. In the DPCM transmitter the difference between the current sample and sample value predicted from the past samples is quantized and encoded. Conventional DPCM uses a linear predictor for estimating sample values does not account for the nonlinear components. These polynomial components can be better modeled by the inclusion of Quadratic Volterra predictor as in fig. 2.

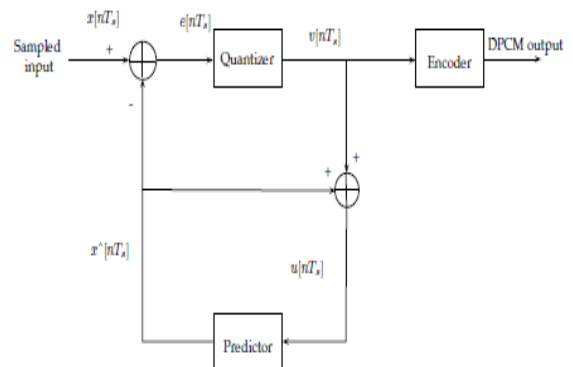


Fig 2. DPCM Transmitter

The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator estimates the prediction error, difference between quantized input and predicted signal. It is denoted by $e(nT_s)$ and can be expressed as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad (10)$$

The encoded residual informations are transmitted over AWGN channel of different noise variances. The DPCM receiver is shown in fig. 3 which incorporates an identical nonlinear predictor as in the transmitter.

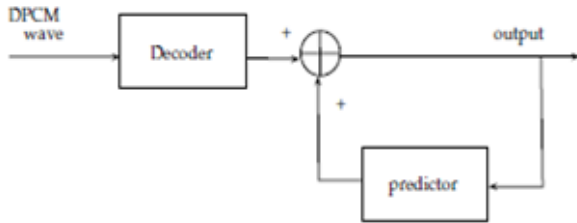


Fig.3 DPCM Receiver

VI. EXPERIMENT

First, a quadratic Volterra predictor based on minimum mean square error criterion is designed for estimating the current image sequence from its previous image sequences in a DPCM system. For the ease of computation images are subdivided into frames, then first two frames of the sequence are used for optimization technique, where the kernel coefficients are obtained. The optimization technique used here is the Powell's conjugate gradient method using built in python function "*scipy.optimize.fmin_powell*". Here the computations are done row by row. The prediction error is given by,

$$MSE = E [|Y - XH_2X^T|] \quad (11)$$

Where XH_2X^T is the predicted output Y is the desired/actual response. H_2 is plotted as in figure.4

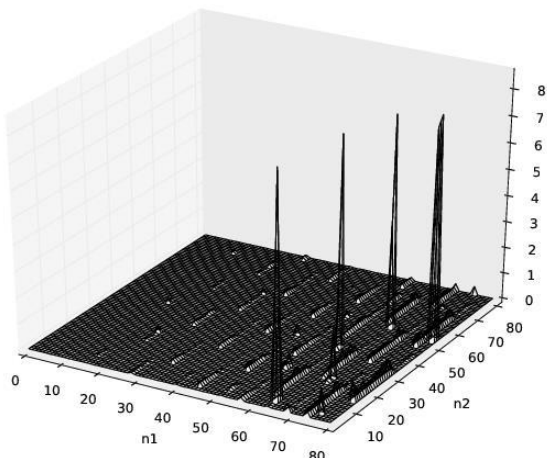


Fig.4 3D plot of Quadratic Kernel

kernel H_2 so obtained is subjected to singular value

decomposition(SVD) to yield an approximate kernel H_2 . Once the kernel is obtained, it is then used in the DPCM system as the quadratic predictor.

Four images sequence shows the rotation of a fan is taken as the image sequences at the DPCM input. Initially the first image is send as such; this is followed by the residual of difference between the first and second images. The reception of this residual at the receiver produce the second image in the sequence, similarly we get the third and fourth sequence at the receiver. The figure4 shows the transmitted images in the DPCM. The figure 5 shows the corresponding image sequence obtained at the receiver form the residual and predicted images.

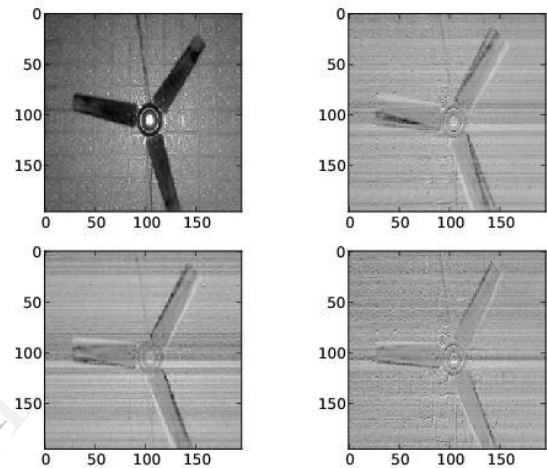


Fig.5 Image Sequence at the DPCM Transmitter

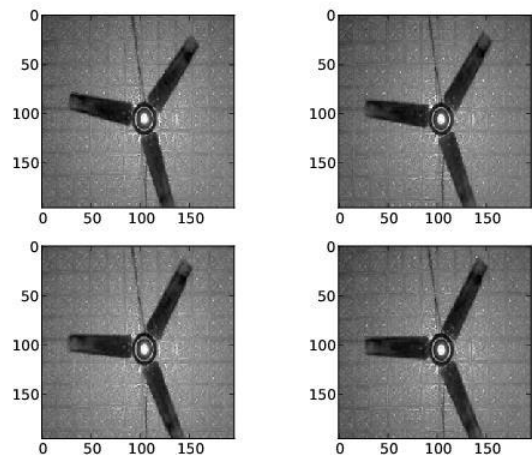


Fig.6 Image Sequence at the DPCM receiver

VII. PERFORMANCE COMPARISON

The performance of the quadratic predictor is compared with that of linear predictor in the presence of channel noise. Here we introduced a random noise of 100 in the transmission channel. The output obtained in the optimization based predictor and linear predictor is shown in Fig.6 and 7.

REFERENCES

- [1] Proakis, J. G. (2001). *Digital Communications*. Mc Graw Hill, New York.
- [2] V.J. Mathews, G.L. Sicuranza, *Polynomial Signal Processing*, John Wiley & Sons, Inc., New York, 2000.]
- [3] Sanjit K. Mitra and Giovanni L. Sicuranza, *Nonlinear Image Processing* (NJ: Academic Press, 2001)
- [4] Oppenheim, A. V. and Schaffer, R. W. (1998). *Discrete Time Signal Processing* PHI, New Delhi, 5th edition
- [5] Gonzalez, R. and Woods, R. (1992). *Digital Image Processing* Addison-Wesley, Reading,MA, 3rd edition.
- [6] N. Wiener, *Response of a nonlinear device to noise*, Report No. 129, Radiation Laboratory, MIT, Cambridge, Massachusetts, 1942.
- [7] Patrick Siarry. *Optimization in Signal and Image Processing* John Wiley Sons Ltd
- [8] Giovanni L.Sicuranza. *Quadratic Filters for Signal Processing* Proceedings of the IEEE, VOL. 80, NO. 8, AUGUST 1992.
- [9] G.Ramponi,G.L.Sicuranzaand S.Cucchi, *2-and 3-D nonlinear predictors* Proc.IEEE Int.Conf.Acoustics, Speech and Signal Processing, Dallas, Texas, pp.1079 -1082, April 1987

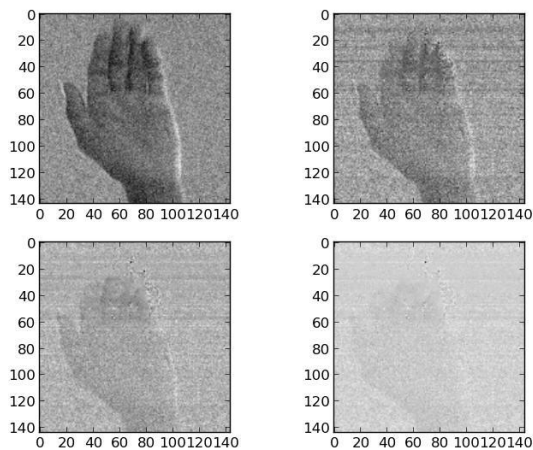


Fig.7 Reception: Linear Predictor with Noise
Variance 100

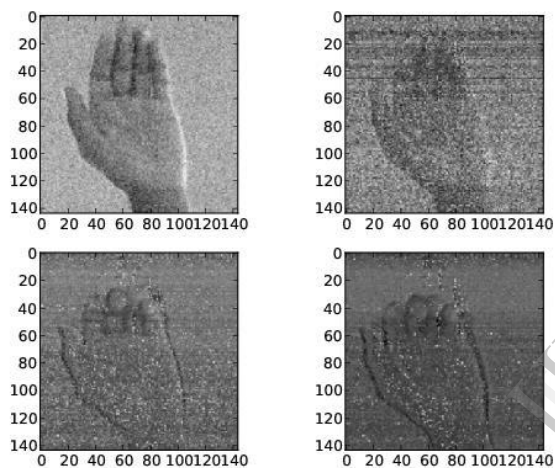


Fig.8 Reception: Optimization Predictor with Noise
Variance 100

VIII. CONCLUSION

This paper presents a new method for image sequence prediction. Conventional DPCM system uses a linear predictor as the main part which estimates the next sample based on the knowledge of the past N samples. But the linear predictor fails to correctly estimate the future pixel values. A new Volterra predictor is proposed to overcome the drawbacks of the linear predictor. A quadratic Volterra predictor is designed and an approximate predictor kernel is implemented using singular value decomposition and is used in parallel with the linear predictor in a DPCM system. The new coder is observed to have lower mean square error than the conventional DPCM system, leading to better quality output at the receiver.