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An Analytical Study of Pulse Squeezing in Fourth Harmonic Generation

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***Abstract*—Squeezing of electromagnetic field is investigated in fundamental mode in fourth harmonic generation under a short-time approximation based on quantum mechanical approach. The occurrence of squeezing in field amplitude and amplitude-squared of fundamental mode has been investigated. The degree of squeezing is found to be dependent on interaction time, coupling parameter of interacting waves and phase values of field amplitude of the fundamental mode during the process. The dependence of squeezing on photon number has also been noticed and found that it increases nonlinearly. The photon statistics of the field in the fundamental mode is found to be sub-Poissonian.**

Keywords—Nonlinear optis, squeezing, photon number, harmonic generation, sub-poissonian.

I. INTRODUCTION

The squeezed states are minimum uncertainty states with reduced fluctuation in one quadrature at the expense of increased fluctuation in the other quadrature. It is a purely quantum mechanical phenomenon without any classical counterpart.

Hong and Mandel [1] introduced the concept of *N*-th order squeezing of electromagnetic field as a generalization of second-order squeezing. Zhan [2] extended the result presented by Hillery [3] for second harmonic generation to k-th harmonic generation. Jawahar Lal and Jaiswal [4] extended the results obtained by Zhan [2] for amplitude-cubed squeezing in the fundamental mode during second and third harmonic generations to *k*-th order. Generations of squeezed states theoretically and experimentally have been investigated in a numerous non-linear processes [5-10] viz. four-wave mixing, second and third harmonic generation, parametric amplification, Raman and hyper-Raman. The importance of squeezing in gravitational wave detection [11], optical communication [12], interferometric technique [13], high precision measurement [14], dense coding [15], quantum cryptography [16] etc. is due to its low noise property [17].

In general, the two important non-classical effects, squeezing and antibunching (or Sub-Poissionian photon statistics) are not interrelated i.e. some states exist that exhibit the first but not the second and vice versa. This paper shows one of the distinguished example of non-linear process when light exhibits both squeezing and sub-Poissionian photon statistics at the same time. This paper presents the squeezing of electromagnetic field in the fundamental mode in fourth harmonic generation. The dependence of squeezing on photon number and the Poissionian behaviour of the field has also been investigated.

II. SQUEEZED STATES AND HIGHER-ORDER SQUEEZING

Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature. Normal squeezing is defined in terms of the operators

$$X_1 = \frac{1}{2}(A + A^\dagger) \tag{1a}$$

and

$$X_2 = \frac{1}{2i}(A - A^\dagger)\,, \tag{1b}$$

where X_1 and X_2 are the real and imaginary parts of the field amplitude respectively. A and $A^\dagger$ are slowly varying operators defined by

$$A = a e^{i\omega t} \tag{2a}$$

and

$$A^\dagger = a^\dagger e^{-i\omega t}\,. \tag{2b}$$

The operators X_1 and X_2 obey the commutation relation

$$[X_1, X_2] = \frac{i}{2}\,, \tag{3}$$

which leads to uncertainty relation

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}\,. \tag{4}$$

A quantum state is squeezed in X_i variable if

$$\Delta X_1 < \frac{1}{2} \text{ for } i = 1 \text{ or } 2. \tag{5}$$

Amplitude-squared squeezing is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2}(A^2 + A^{\dagger 2}) \tag{6a}$$

and

$$Y_2 = \frac{1}{2i}(A^2 - A^{\dagger 2})\,. \tag{6b}$$

The operators Y_1 and Y_2 obey the commutation relation $[Y_1, Y_2] = i(2N+1)$, where N is the usual number operator which leads to the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geq \left\langle \left(N + \frac{1}{2} \right) \right\rangle. \tag{7}$$

Amplitude-squared squeezing is said to exist in Y_i variable if

$$(\Delta Y_i)^2 < \left\langle \left(N + \frac{1}{2} \right) \right\rangle \text{ for i = 1 or 2.} \tag{8}$$

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III. FOURTHHARMONIC GENERATION

The Fourth harmonic generation model has been adopted from the works of Chen et al. [19] and is shown in Fig. 1. This process involves absorption of four photons, each having frequency ω_1 going from state 1 to 2 and emission of one photon of frequency ω_2, where $\omega_2 = 4\omega_1$.

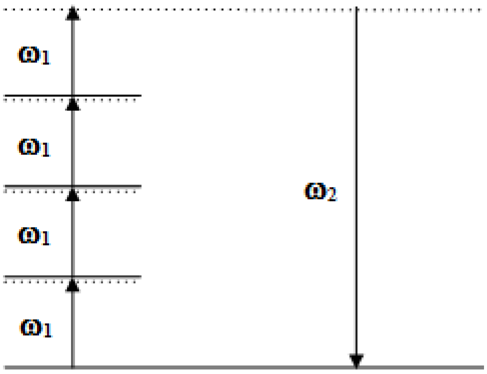

Fig. 1: Fourth harmonic generation model

We can write Hamiltonian for this process as

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + g(a^4 b^\dagger + a^{\dagger 4} b), \tag{9}$$

in which g is coupling constant for fourth harmonic generation.

$$A = a\exp(i\omega_1 t)$$

and

$$B = b\exp(i\omega_2 t)$$

are the slowly varying operators at frequency ω_1 and ω_2 respectively, where $a(a^\dagger)$ and $b(b^\dagger)$ are the usual number operator with the relation $\omega_2 = 4\omega_1$.

The Heisenberg equation of motion for fundamental mode A is as

$$\dot{A} = \frac{dA}{dt} + i[H, A]. \tag{10}$$

Using equation (9) in equation (10), we obtain

$$\dot{A} = -4igA^{\dagger 3}B \tag{11}$$

and

$$\dot{B} = -igA^4. \tag{12}$$

With the short interaction time approximation, we can write $A(t)$ in Taylor's series and retaining the terms up to g^2t^2 as

$$A(t) = A - 4igtA^{\dagger 3}B$$

$$+2g^2t^2[(12A^{\dagger 2}A^3 + 36A^\dagger A^2 + 24A)B^\dagger B - A^{\dagger 3}A^4] \tag{13}$$

For squeezing of field amplitude in fundamental mode, we can write the quadrature component as

$$X_{1A} = \frac{1}{2}[A(t) + A^\dagger(t)]. \tag{14}$$

Initially, we consider the quantum state as a product of coherent state for the fundamental mode A and the vacuum state for the harmonic mode B i.e.

$$A(0)|a\rangle = a|a\rangle;\ B(0)|\ \rangle = 0. \tag{15}$$

Using equations (13)-(15), the expectation values are derived as

$$\langle\psi|X_{1A}^2(t)|\psi\rangle = \frac{1}{4}[a^2 + a^{*2} + 2|a|^2 + 1$$
$$-2g^2t^2(2a^2|a|^6 + 3a^2|a|^4$$
$$+2a^{*2}|a|^6 + 3a^{*2}|a|^4 + 4|a|^8)] \tag{16}$$

and

$$\langle\psi|X_{1A}^2(t)|\psi\rangle^2 = \frac{1}{4}[a^2 + a^{*2} + 2|a|^2$$
$$-2g^2t^2(2a^2|a|^6 + 2a^{*2}|a|^6 + 4|a|^8] \tag{17}$$

Therefore

$$[\Delta X_{1A}(t)]^2 - \frac{1}{4} = -3g^2t^2|a|^6\cos 2\theta, \tag{18}$$

where θ is the phase angle, with $a = |a|\exp(i\theta)$ and $a^* = |a|\exp(-i\theta)$.

The right hand side of expression (18) is negative, indicating that squeezing occurs in the field amplitude of fundamental mode in fourth harmonic generation for which $\cos 2\theta > 0$.

Using equation (13), the second order amplitude in fundamental mode is expressed as

$$A^2(t) = A^2 - 4igt(2A^{\dagger 3}AB + 3A^{\dagger 2}B)$$
$$-16g^2t^2A^{\dagger 6}B^2 - 2g^2t^2(2A^{\dagger 3}A^5 + 3A^{\dagger 2}A^4) \tag{19}$$

For amplitude-squared squeezing, the real quadrature component for the fundamental mode is given as

$$Y_{1A}(t) = \frac{1}{2}[A^2(t) + A^{\dagger 2}(t)]. \tag{20}$$

Using equations (15), (19) and (20), we get the expectation value as

$$\langle\psi|Y_{1A}^2(t)|\psi\rangle = \frac{1}{4}[a^4 + a^{*4} + 2|a|^4 + 4|a|^2 + 2$$
$$-4g^2t^2\{(2|a|^6 + 9|a|^4 + 12|a|^2 + 3)(a^4 + a^{*4})$$
$$+4|a|^{10} + 10|a|^8\}] \tag{21}$$

and

$$\langle\psi|X_{1A}^2(t)|\psi\rangle^2 = \frac{1}{4}[a^4 + a^{*4} + 2|a|^4$$

$$-4g^2t^2(a^4 + a^{*4} + 2|a|^4)(2|a|^6 + 3|a|^4)]. \tag{22}$$

Therefore

$$[\Delta Y_{1A}(t)]^2 = \frac{1}{4}[4|a|^2 + 2 - 4g^2t^2$$
$$\times\{(6|a|^4 + 12|a|^2 + 3)(a^4 + a^{*4}) + 4|a|^8\}]. \tag{23}$$

The number of photons in mode A may be expressed as

$$N_{1A}(t) = A^\dagger(t)A(t) = A^\dagger A - 4igt(A^{\dagger 4}B - A^4B^\dagger)$$
$$-4g^2t^2A^{\dagger 4}A^4 + 16g^2t^2A^3A^{\dagger 3}B^\dagger B \tag{24}$$

Using equation (15), the average value of $\left\langle N_A(t) + \frac{1}{2}\right\rangle$ is given by

$$\left\langle N_A(t) + \frac{1}{2}\right\rangle = \frac{1}{4}[4|a|^2 + 2 - 16g^2t^2|a|^8]. \tag{25}$$

Subtracting equation (25) from equation (23), we get

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$$[\Delta Y_{1A}(t)]^2 - \left\langle N_{1A}(t) + \frac{1}{2} \right\rangle = -6g^2t^2(2|a|^8 + 4|a|^6 + |a|^4)\cos 4\theta . \quad (26)$$

The right hand side of above equation is negative for all values of θ for which $\cos 4\theta > 0$ and thus shows the existence of squeezing in the second order of field amplitude of the fundamental mode under short time approximation. The photon statistics of field amplitude in fundamental mode in fourth harmonic generation is sub-Poissonian, given as

$$[\Delta N_A(t)]^2 - \langle N_A(t) \rangle = -12g^2t^2 |a|^8 . \quad (27)$$

IV. RESULTS AND DISCUSSION

The results show the squeezing of field amplitude and square of the field amplitude of fundamental mode in fourth harmonic generation. To study squeezing, we denote the right hand side of equations (18) and (26) by S_x and S_y respectively. Taking $|gt|^2 = 10^{-4}$ and $\theta = 0$, the variations are shown in Figure 2 and 3. Figures show that squeezing increases nonlinearly with $|a|^2$, i.e. with the number of photons.

It is obvious that the degree of squeezing is greater in amplitude-squared states than in field amplitude states. Squeezing occurs in the fundamental mode of fourth harmonic generation obeying sub-Poissonian photon statistics.

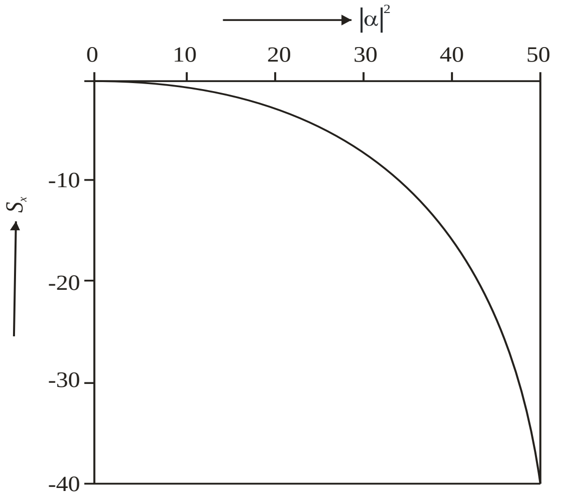

Fig. 2: Dependence of field amplitude squeezing *Sx* on $|a|^2$.

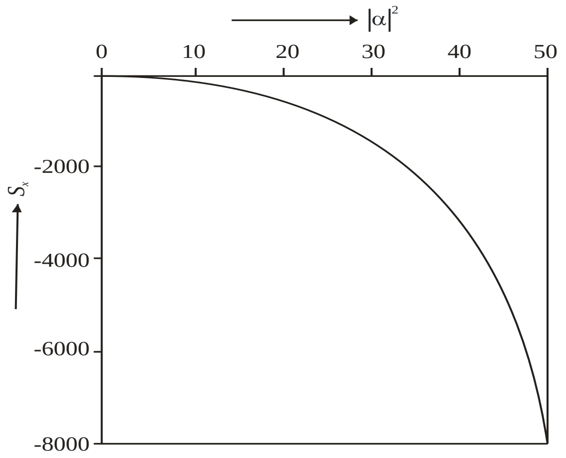

Fig. 3: Dependence of amplitude squared squeezing *Sy* on $|a|^2$.

V. CONCLUSIONS

The squeezing increases nonlinearly with $|\alpha|^2$, which directly depends on the number of photons. The squeezing in any order during stimulated interaction is higher than the squeezing in corresponding order in spontaneous interaction. The squeezing is higher in higher orders in both processes. Thus, the higher-order squeezing associated with higher order nonlinear optical processes makes it possible to achieve significant noise reduction

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