An Analysis of Combined Algorithms between Channel Estimation and ICI Reduction

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Abstract: Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme, which is used in several wireless systems for transferring data at high rate. The multi path fading channel and the frequency offset between the transmitted and received carrier frequencies introduce ICI (Inter Carrier Interference). ICI effects the OFDM symbols and degrades the system performance. This paper proposes a solution: combine channel estimation and ICI self- cancellation to combat against ICI in doubly selective fading channel. The simulation results show the effect of this solution.

Keywords: OFDM, ICI self cancellation, channel estimation, Inter Carrier Interference.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme, which is used for transferring data at high rate by using a numerous sub-carrier orthogonal to each other. With many advantages, OFDM is used in many wireless communication systems nowadays. The main disadvantage of this scheme is the inter-carrier interference (ICI), caused by Doppler shift due to relative motion between the transmitter and receiver when transferring data in multi path fading channel , or by differences between the frequencies of the local oscillators at the transmitter and receiver.

Currently, there are many different methods for reducing ICI including: pulse shaping, frequency domain equalization [1], ICI self-cancellation [2], maximum likelihood estimation ... The research of these methods is applied in the Gaussian environment with the normalized frequency offset. However the real environment is not only Gaussian noise but also the effect of the complicated multipath fading and the mismatch between the transmitter and the receiver caused by the movement of the transmitter or the receiver. In this case, channel estimation is necessary in the receiver. The channel estimation can be performed by inserting pilot tones into OFDM symbol for mobile WiMAX, [3], [4], [5]. Comb type pilot system have good performance in fading environment [6]. LS (Least Square), MMSE (Minimum Mean Squared Error) are popular estimators and used for mobile WiMAX system [7].

So we concentrate on the performance of the OFDM system in this environment and we propose the combination between channel estimation and the method of reducing ICI. In this paper, we use the channel estimation LS (Least Square) combining with 2 methods of reducing ICI: Maximum likelihood estimation and the ICI self-cancellation. The results show that the combination between channel estimation LS and ICI self-cancellation can reduce the effect of ICI and make the OFDM perform better.

II. RESEARCH METHOD

A. ICI problem

OFDM block diagram is shown in Fig. 1.



Fig. 1: OFDM block diagram.

The main disadvantage of OFDM, however, is its susceptibility to small differences in frequency at the transmitter and receiver, normally referred to as frequency offset. This frequency offset can be caused by Doppler shift due to relative motion between the transmitter and receiver, or by differences between the frequencies of the local oscillators at the transmitter and receiver. In this paper, the frequency offset is modeled as a multiplicative factor introduced in the channel, as shown in Fig. 2. In this case, we omit fading effect so that we can pay attention to frequency offset effect



Fig.2: Model for frequency offset.

The received signal in time domain could be written as

$$y(n) = x(n)e^{\frac{j2\pi n^{\frac{\varepsilon}{N}}}{N}} + w(n) \qquad (1)$$

Where ε is normalized frequency offset and $\varepsilon = \Delta f N T$, Δf is a frequency differences between the transmitted and received carrier frequencies, T_c is a subcarrier symbol period. w(n) is the AWGN introduced by the channel.

The effect of this frequency offset on the received symbol stream can be understood by considering the received symbol Y(n) on the k^{th} sub-carrier.

$$Y(k) = X(k)S(0) + \sum_{l=0}^{N-1} X(l)S(l-k) + n_k$$

$$k = 0.1, 2..., N-1$$
(2)

Where N is the total number of subcarriers, X(k) is the transmitted symbol for the k^{th} subcarrier, n_k is the FFT of w(n) and S(l-k) are the complex coefficients for the ICI components in the received signal. The ICI components are the interfering signals transmitted on sub-carriers other than the k^{th} subcarrier. The complex coefficients are given by

$$S(l-k) = \frac{\sin(\pi(l+\varepsilon-k))}{N\sin(\pi(l+\varepsilon-k)/N)} \exp\left(j\pi\left(1-\frac{1}{N}\right)(l+\varepsilon-k)\right)$$
(3)

The desired received signal power can be represented as: $E\left[\left|C(k)\right|^{2}\right] = E\left[\left|X(k)S(0)\right|^{2}\right] = E\left[\left|X(k)\right|^{2}\right]\left|S(0)\right|^{2}$

The ICI power is represented as:

$$E\left[\left|I\left(k\right)\right|^{2}\right] = E\left[\left|\sum_{l=0}^{N-1} X\left(l\right) S\left(l-k\right)\right|^{2}\right] = E\left[\left|X\left(l\right)\right|^{2}\right] \sum_{l=0}^{N-1} \left|S\left(l-k\right)\right|^{2}$$

CIR is given by below equation:

$$CIR = \frac{E[|C(k)|^{2}]}{E[|I(k)|^{2}]} = \frac{E[|X(k)|^{2}]E[|S(0)|^{2}]}{E[|X(l)|^{2}]\sum_{l=0}^{N-1}|S(l-k)|^{2}}$$
(6)

In this paper, the desired signal is transmitted on subcarrier "0", the CIR expression in Eq. 6 can be derived as:

$$CIR = \frac{|S(0)|^{2}}{\sum_{l=1}^{N-1} |S(l)|^{2}}$$
(7)

B. Maximum likelihood estimation

A method for frequency offset correction is ML estimation in OFDM systems was suggested by Moose [8]. In this approach, the frequency offset is first statistically estimated using a maximum likelihood algorithm and then cancelled at the receiver. This technique involves the replication of an OFDM symbol before transmission and comparison of the phases of each of the subcarriers between the successive symbols.

Fig. 3 shows the block diagram of the OFDM system using this method.



Fig. 3: The OFDM system using channel estimation and maximum likelihood estimation.

When an OFDM symbol of sequence length N is replicated, the receiver receives, in the absence of noise, the 2N point sequence r(n) is given by:

$$r(n) = \frac{1}{N} \left[\sum_{k=-K}^{K} X(k) H(k) e^{\frac{j2\pi n(k+\varepsilon)}{N}} \right]$$

$$k = 0, 1, \dots, N-1, N \ge 2K+1$$
(8)

Where X(k) are the 2k+1 complex modulation values used to modulate 2k+1 subcarriers. H(k) is the channel transfer function for the k^{th} carrier and ε is the normalized frequency offset of the channel. The first set of N symbols is demodulated using an N point FFT to yield the sequence $R_1(k)$ and the second set is demodulated using another N point FFT to yield the sequence $R_2(k)$. The frequency offset is the phase difference between $R_1(k)$ and $R_2(k)$, that is $R_2(k) = R_1(k)e^{j2\pi\varepsilon}$.

Adding the AWGN yields:

$$Y_{1}(k) = R_{1}(k) + W_{1}(k)$$

$$Y_{2}(k) = R_{1}(k)e^{j2\pi\varepsilon} + W_{2}(k) \qquad k = 0, 1, ..., N-1$$
(9)

The maximum likelihood estimate of the normalized frequency offset is given by:

$$\hat{\varepsilon} = \left(\frac{1}{2\pi}\right) \tan^{-1} \left\{ \frac{\left(\sum_{k=-K}^{K} Im\left[Y_{2}\left(k\right)Y_{1}^{*}\left(k\right)\right]\right)}{\left(\sum_{k=-K}^{K} Re\left[Y_{2}\left(k\right)Y_{1}^{*}\left(k\right)\right]\right)} \right\}$$
(10)

This maximum likelihood estimate is a conditionally unbiased estimate of the frequency offset and was computed using the received data .Once the frequency offset is known, the ICI distortion in the data symbols is reduced by multiplying the received symbols with a complex conjugate of the frequency shift and applying the FFT.

(5)

$$\hat{x} = FFT\left\{y(n)e^{-j\frac{2\pi\varepsilon n}{N}}\right\} \quad (11)$$

C. ICI Self-Cancellation

ICI self-cancellation [2] is a scheme that was introduced by Zhao and Sven-Gustav in 2001 to combat and suppress ICI in OFDM. Succinctly, the main idea is to modulate the input data symbol onto a group of subcarriers with predefined coefficients such that the generated ICI signals within that group cancel each other, hence the name "self-cancellation". It is seen that the difference between the ICI co-efficient of two consecutive sub-carriers are very small. This makes the basis of ICI self-cancellation.

Here one data symbol is not modulated in to one sub-carrier, rather at least in to two consecutive sub-carriers. If the data symbol "a" is modulated in to the 1st subcarrier then "-a" is modulated in to the 2nd sub-carrier. Hence the ICI generated between the two sub-carriers almost mutually cancels each other. This method is suitable for multipath fading channels as here no channel estimation is required. In multipath case, channel estimation fails as the channel changes randomly. This method is also suitable for flat channels. The method is simple, less complex and effective. The major drawback of this method is the reduction in bandwidth efficiency as same symbol occupies two sub-carrier. Fig. 4 shows the block diagram of the OFDM system using this method.



Fig. 4: The OFDM using channel estimation and ICI selfcancellation.

Assuming the transmitted symbols are such that:

$$X(1) = -X(0), X(3) = -X(2), ..., X(N-1) = -X(N-2)$$
(12)

Then the received signal on subcarrier k becomes:

$$Y'(k) = \sum_{l=0}^{N-1} X(l) S(l-k) + n_k = \sum_{\substack{l=0\\l \text{ even}}}^{N-2} X(l) S'(l-k) + n_k$$
(13)

In such a case, the ICI coefficient is denoted as: S'(l-k) = S(l-k) - S(l+1-k) (14)

Similarly the received signal on subcarrier k+1 becomes:

$$Y'(k+1) = \sum_{\substack{l=0\\l even}}^{N-2} X(l) S'(l-1-k) + n_{k+1} \quad (15)$$

To further reduce ICI, ICI cancelling demodulation is done. The demodulation is suggested to work in such a way that each signal at the $k + 1^{th}$ subcarrier (now k denotes even

number) is multiplied by "-1" and then summed with the one at the k^{th} subcarrier. Then the resultant data sequence is used for making symbol decision. It can be represented as: V''(k) - V'(k) - V'(k+1)

$$I'(k) = I(k) - I(k+1)$$

= $\sum_{l=0}^{N-2} X(l) \Big[-S(l-1-k) + 2S(l-k) - S(l+1-k) \Big] + n_k - n_{k+1}$
(16)

The corresponding ICI coefficient then becomes:

$$S''(l-k) = -S(l-1-k) + 2S(l-k) - S(l+1-k)$$
(17)

Thus, the ICI signals become smaller when applying ICI cancelling modulation. On the other hand, the ICI cancelling demodulation can further reduce the residual ICI in the received signals. This combined ICI cancelling modulation and demodulation method is called the ICI self-cancellation scheme.

In equation (16), the average power can be represented as:

$$E\left[\left|C(k)\right|^{2}\right] = E\left[\left|X(k)\right|^{2}\right]\left|S(-1) + 2S(0) + S(1)\right|^{2} (18)$$

The average power could be represented as:

$$E\left[\left|I(k)\right|^{2}\right] = E\left[\left|X(l)\right|^{2}\right] \sum_{l=0}^{N-1} \left|S(l-1-k) + 2S(l-k) - S(l+1-k)\right|^{2}$$

$$\lim_{\substack{l \neq k \\ l \text{ even}}} (19)$$

According to the definition of CIR, the CIR can be represented as:

$$CIR = \frac{E\left[\left|C(k)\right|^{2}\right]}{E\left[\left|I(k)\right|^{2}\right]} = \frac{E\left[\left|X(k)\right|^{2}\right]\left|S(-1)+2S(0)+S(1)\right|^{2}}{E\left[\left|X(l)\right|^{2}\right]\sum_{\substack{l=0\\l\neq k\\l even}}^{N-1}\left|S(l-1-k)+2S(l-k)-S(l+1-k)\right|^{2}}$$
(20)

In this paper, the desired signal is transmitted on subcarrier "0", the CIR expression can be derived as:

$$CIR = \frac{\left|S(-1) + 2S(0) + S(1)\right|^{2}}{\sum_{\substack{l=0\\l\neq k\\l \text{ even}}}^{N-1} \left|S(l-1-k) + 2S(l-k) - S(l+1-k)\right|^{2}}$$
(21)

Due to the repetition coding, the bandwidth efficiency of the ICI self-cancellation scheme is reduced by half. To fulfill the demanded bandwidth efficiency, it is natural to use a larger signal alphabet size. For example the OFDM using modulation scheme 4PSK with ICI self-cancellation has the same bandwidth efficiency as the standard OFDM (using BPSK).

D. Combine channel estimation LS with method of reduction ICI in doubly selective fading channel

The real transmitted channel is the doubly selective fading so using channel estimation is necessary. The channel estimation helps reducing the effect of the channel on the signal. In this paper we use the algorithm LS (Least Square) to estimate the channel. This algorithm does not need to know the parameters of the channel so this one is not complex but have the high variance. The response of the channel is estimated using the known pilot data and the pilot received at the receiver. Based on the estimated response, the signal after that is represented as:

$$\hat{X}(k) = \frac{Y(k)}{\hat{H}(k)}$$
, $k = 0, 1, ..., N-1$ (22)

Where $\hat{H}(k)$ is the estimated channel response.

The combination between channel estimation and the method of reducing ICI is proposed to ensure the quality of the system when we transmit the data through the real channel.

III. SIMULATION RESULTS

A. Simulation parameters

In this paper we simulate 3 systems: the standard OFDM, the OFDM system using ML estimation and the OFDM system using ICI self-cancellation. The block diagram of 3 systems was introduced in previous section.

All 3 OFDM systems use 1024 carriers with 840 of that carrying data. The CP was added to reduce the effect of ISI and has the length of 256. The OFDM system operates at frequency 2.5 GHz with the bandwidth 20 MHz

The simulation channel is the fading channel of ITU-R standard [9]. This is the standard for WiMAX system. With this standard, we have 3 type of channel: indoor, pedestrian and vehicular according to the speed between the transmitter and the receiver. The parameters of channels are shown in the table below.

Тар	Indoor		Pedestrian		Vehicular	
	Delay	Power	Delay	Power	Delay	Power
	(ns)	(dB)	(ns)	(dB)	(ns)	(dB)
1	0	0	0	0	0	0
2	100	-3.6	200	-0.9	0.8	-1
3	200	-7.2	800	-4.9	1.6	-9
4	300	-10.8	1200	-8	2.2	-10
5	500	-18	2300	-7.8	3.6	-15
6	700	-25.2	3700	-23.9	5.2	-20

Table 1: Parameters of Indoor, Pedestrian and Vehicular channel.

B. Results

Fig. 5 shows the theoretical CIR curve calculated by above CIR equation together with simulation results. As a reference, the CIR of a standard OFDM system is also shown. Such an ICI cancellation scheme gives more than 15 dB CIR improvement in the range $0 \le \varepsilon \le 0.5$. Especially for small to medium frequency offsets in the range $0 \le \varepsilon \le 0.2$ the CIR improvement can reach 17 dB



Fig. 5: CIR compare between standard OFDM and the OFDM using ICI selfcancellation.

Fig. 6 and 7 is the results of BER (Bit Error Rate) simulation in the indoor channel with 2 different frequency offset. The relative speed between the transmitter and the receiver is 1 km/h. The OFDM using channel estimation LS with ICI selfcancellation has the better performance than the other systems. It shows that this combination makes the OFDM system combat against the ICI in this environment. Channel estimation with ML estimation is effective in reducing the effect of ICI (The OFDM using this scheme has better performance than the standard OFDM system). When we increase the frequency offset from 0.2 to 0.3, the effect of ICI increases but the combination is still good for reducing the ICI.



Fig. 6: BER graph in indoor channel with frequency offset $\varepsilon = 0.2$.



Fig. 7: BER graph in indoor channel with frequency offset $\varepsilon = 0.3$.

In pedestrian channel, the speed between the transmitter and the receiver is 5 km/h. The ICI noise in this channel is much larger than in the indoor channel (the performance of these OFDM systems is not good enough as in the indoor channel). The channel estimation LS combining with the ICI reduction method is still working, the performance of this system is still better than the others, especially when the frequency offset increases. The combination with ICI self-cancellation is the best in this condition.



Fig. 8: BER graph in pedestrian channel with frequency offset $\varepsilon = 0.2$.



Fig. 9: BER graph in pedestrian channel with frequency offset $\varepsilon = 0.3$.

The last channel is the vehicular. This is the channel with the high relative speed between the transmitter and the receiver. The ICI noise in this channel affects hardly on the OFDM signal and degrades the quality of the system. For simulation, the speed of 40 km/h is chosen. The ability against the ICI of the combination is good, BER graph of the 2 OFDM systems using the combination are better than the standard OFDM system. The channel estimation combining with ML estimation helps improving the performance of the OFDM system but not better than the combination with ICI cancellation.



Fig. 10: BER graph in vehicular channel with frequency offset $\varepsilon = 0.2$.





Figure 11: BER graph in vehicular channel with frequency offset $\varepsilon = 0.3$.

IV. CONCLUSION

In this paper, we simulate the OFDM system in the fading channel and frequency offset effect at the receiver. We proposed and simulated the algorithms of combining between channel estimation and the method of ICI reducing. The ICI self-cancellation and the ML estimation is introduced to combat against the effect of ICI caused by the environment. The results in different channels and compare between the 3 OFDM systems show that the combination between ICI selfcancellation and channel estimation LS is the best. However the disadvantage of this method is the bandwidth efficiency is reduced a half because of the replication of data

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