

## Advanced Vogel's Approximation Method (AVAM): A New Approach to Determine Penalty Cost for Better Feasible Solution of Transportation Problem

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### Abstract

*In Operation Research, obtaining significant result for Transportation Problems is very important now-a-days. Vogel's Approximation Method (VAM) is the very efficient algorithm to solve the transportation problem for feasible solution which is nearer to optimal solution. In this paper we identified a computational error in VAM and approach a logical development of VAM algorithm. The main concept of VAM is to determine penalty cost which obtains from the difference of smallest and next to smallest cost in each row or column and make maximum allocation in lowest cost cell of that row or column which have largest penalty. The difficulty arises when smallest cost and next to smallest cost have same magnitude. In that case we find a very logical concept to resolve this and developed a new algorithm "Advanced Vogel's Approximation Method (AVAM)" to find a feasible solution of transportation problem which is very close to optimal solution more than VAM.*

**Keyword:** AVAM, VAM, Penalty Cost, Feasible Solution, Error Estimation, Transportation Problem (TP).

### 1. Introduction

Transportation problem is real life problem where commodities are transferred from factories to retail house so that total transportation cost should be minimized. In Operation research, TP is a special class

of Linear Programming Problem (LPP) where Vogel's Approximation Method (VAM) is known as efficient method to solve TP. The concept penalty cost (difference of smallest and next to smallest cost in row or column) makes this method more effective more than other methods such as North West Corner Rule (NWC), Least Cost Method (LCM) etc. But the way of determining penalty cost is not logical for some cases; we discuss this types of situation in [-,-] and developed an algorithm with new concept where difficulties are resolved in [-,-] and gives the feasible solution very close to optimal solution lower than VAM.

Consider a Transportation Problem with  $m$  sources and  $n$  destinations where  $C_{ij}$  is the unit transportation cost from  $i^{th}$  source to  $j^{th}$  destination. Let  $S_i$  be the supply amount of  $i^{th}$  source and  $D_j$  be the demand amount of  $j^{th}$  destination. We have to find the transported amount of commodity  $x_{ij}$  so that total transportation cost will be minimized. The above problem in LPP model can be express as follows:

**Minimize:**

$$\text{Total cost} = \sum_{i=1}^m \sum_{j=1}^n C_{ij} x_{ij}$$

**Subject to:**

$$\sum_{j=1}^n x_{ij} \leq S_i \quad \text{for } i=1,2,\dots,m \text{ (Supply constrains)}$$

$$\sum_{i=1}^m x_{ij} \geq D_j \quad \text{for } j=1,2,\dots,n \text{ (Demand constrains)}$$

$$x_{ij} \geq 0 \quad \forall i, j$$

We have to convert this LPP in the following mathematical Model and applying transportation methods to find feasible solution.

Source	Destination					Supply
	D1	D2	D3	...	Dn	
S1	C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	...	C <sub>1n</sub>	S <sub>1</sub>
S2	C <sub>21</sub>	C <sub>22</sub>	C <sub>23</sub>	...	C <sub>2n</sub>	S <sub>2</sub>
...	...	...	...	...	...	...
Sm	C <sub>m1</sub>	C <sub>m2</sub>	C <sub>m3</sub>	...	C <sub>mn</sub>	S <sub>m</sub>
Demand	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	...	D <sub>n</sub>	

## 2. Existing Algorithm: Vogel's Approximation Method (VAM)

The Vogel Approximation method is an iterative procedure for computing a basic feasible solution of a transportation problem. This method is preferred over the two methods i.e. North West Corner Rule and Least cost Method. The algorithm of VAM is given below:

**Step-1:**

- Identify the cells having minimum and next to minimum transportation cost in each row and write the difference (Penalty) along the side of the table against the corresponding row.
- Identify the cells having minimum and next to minimum transportation cost in each column and write the difference (Penalty) along the side of the table against the corresponding column.

**Step-2:** If minimum cost appear in two or more times in a row or column then select these same cost as a minimum and next to minimum cost and penalty will be zero.

**Step-3:**

- Identify the row and column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least cost in the selected row or column. Adjust the supply and demand and cross out the satisfied row or column. If a row and column are satisfied simultaneously, only one of them is crossed out and remaining row or column is assigned a zero supply or demand.
- If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

**Step-4:**

- If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.
- If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.
- If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.
- Otherwise, go to Step-1.

### 2.1. Finding Computational Error of Vogel's Approximation Method (VAM)

The main concept of VAM algorithm is the penalty cost which is determined by the difference of smallest and next to smallest cost of each row and column where highest penalty indicate that one of the value of two minimum costs is too higher than another. For that case VAM select highest penalty cost and give allocation in lowest cost cell in that row or column for avoiding the probability of selecting higher cost in next iteration.

If there are two or more cells have same smallest magnitude then VAM selects these same smallest cost as minimum and next to minimum cost and penalty will be zero [2. Step-2]. For that case penalty will be lowest in that row or column among all rows or columns so that there is no possibility to select that row or column for allocating commodities. If in that row or column have any higher cost other than same smallest costs then probability will be increased to select the higher cost in next iteration and total transportation cost may be increase.

In VAM algorithm, allocations are depends on penalty cost. For this above computational error to determine penalty in VAM, lowest cost may not be ensure in all iteration so that total cost in feasible solution has a chance to be higher.

### 3. Proposed Algorithm for Advanced Vogel's Approximation Method (AVAM)

In this section, we solved the computational error of VAM which discussed in above and proposed a new algorithm named "Advanced Vogel's Approximation Method (AVAM)". In our proposed algorithm (AVAM), when smallest cost appear in two or more times in a row or column then penalty determined by difference of two minimum cost taken one of them as a minimum and following smallest cost other than equal smallest costs as a next to minimum. As an example, if 3, 10, 3, 7, 9 are the costs of a row or column then select 3 as a smallest cost and select 7 as a next to smallest cost instead of 3 again and penalty will be 4. In that case penalty is not zero and if this penalty has the largest magnitude then probability of the chance of taking larger cost in next iteration will be decreased because of at least one more smallest cost remains. The algorithm of AVAM is given follows:

**Step-1:** Set  $S_i$  : Supply amount of the  $i^{th}$  source;

Set  $D_j$  : Demand amount of the  $j^{th}$  destination;

Set  $C_{ij}$  : Unit transportation cost of  $i^{th}$  source to  $j^{th}$  destination;

Check: if  $S_i < 0$  and  $D_j < 0$ , then Stop.

**Step-2:**

$$a. \quad f \sum_i S_i > \sum_j D_j \text{ or if } \sum_i S_i < \sum_j D_j$$

then balance the transportation problem adding dummy demand or supply.

$$b. \quad \text{Set } C_{ij} := 0 \text{ for all dummy rows or columns.}$$

**Step-3:**

- Identify the smallest and next to smallest cost of each row and column and calculate the difference between them which is called by penalty.

Set  $P_i$  : Row penalty and Set  $P_j$  : Column penalty.

$$P_i = |C_{ih} - C_{ik}| \text{ and } P_j = |C_{hj} - C_{kj}|$$

- If smallest cost appear two or more times in a row or column then select one of them as a smallest and following smaller cost other than equal smallest costs as a next to smallest cost.
- If there is no more cost other than equal smallest costs i.e. all costs are same then select smallest and next to smallest as same and penalty will be zero.

**Step-4:**

- Select  $\max(P_i, P_j)$ . Set  $x_{ij}$  : Amount of commodity from  $i^{th}$  source  $j^{th}$  to destination; Select lowest cost of that row or column which has largest penalty and allocate maximum possible amount  $x_{ij}$  i.e.  $\min(S_i, D_j)$ . If the lowest cost appear in two or more cells in that row or column then choose the extreme left or most top lowest cost cell.
- If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

**Step-5:** Adjust the supply and demand and cross out the satisfied row or column. If row and column are satisfied simultaneously then crossed out one of them and set zero supply or demand in remaining row or column.

**Step-6:**

- If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.
- If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.
- If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.
- Otherwise go to Step-3.

### 4. Numerical Simulation

Consider some special types of transportation problems where smallest cost is appear in two or more in rows or columns and solve them using Advanced Vogel's Approximation Method (AVAM) and compare these results with the solution of Vogel's Approximation Method (VAM).

#### 4.1. Example-1

Consider a Mathematical Model of a transportation problem in given below:

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6	8	10	9	50
S2	5	8	11	5	75
S3	6	9	12	5	25
Demand	20	20	50	60	

Table-1.1

Now solve this problem using AVAM and VAM respectively in below:

**Solution of Example-1 Using Advanced Vogel's Approximation Method (AVAM):**

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6	8	10	9	50
S2	5	8	11	5	75
S3	6	9	12	5	25
Demand	20	20	50	60	

Table-1.2

Total Transportation Cost:

$$(0 \times 8) + (50 \times 10) + (15 \times 5) + (60 \times 5) + (5 \times 6) + (20 \times 9) = 1085$$

**Solution of Example-1 Using Vogel's Approximation Method (VAM):**

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6	8	10	9	50
S2	5	8	11	5	75
S3	6	9	12	5	25
Demand	20	20	50	60	

Table-1.3

Total Transportation Cost:

$$(20 \times 6) + (30 \times 10) + (20 \times 8) + (20 \times 11) + (35 \times 5) + (25 \times 5) = 1100$$

**Optimal solution of Example-1:**

The optimal solution of Example-1 determined by MODI is 1060.

**4.2. Example-2**

Consider a Mathematical Model of a transportation problem in given below:

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	4	4	9	8	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

Table-2.1

Now solve this problem using AVAM and VAM respectively in below:

**Solution of Example-2 Using Advanced Vogel's Approximation Method (AVAM):**

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	4	4	9	8	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

Table-2.2

Total Transportation Cost:

$$(60 \times 4) + (40 \times 4) + (80 \times 4) + (60 \times 7) + (10 \times 6) + (0 \times 4) + (40 \times 8) + (50 \times 3) = 1670$$

**Solution of Example-2 Using Vogel's Approximation Method (VAM):**

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	4	4	9	8	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

Table-2.3

Total Transportation Cost:

$$(60 \times 4) + (0 \times 4) + (40 \times 9) + (80 \times 4) + (60 \times 7) + (10 \times 6) + (40 \times 4) + (50 \times 3) = 1710$$

#### Optimal solution of Example-2:

The optimal solution of Example-1 determined by MODI is 1670.

#### 4.3. Example-3:

Consider a Mathematical Model of a transportation problem in given below:

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	8	8	2	10	2	40
S2	11	4	10	9	4	70
S3	5	2	2	11	10	35
S4	10	6	6	5	2	90
S5	8	11	8	6	4	85
Demand	80	55	60	80	45	

Table:3.1

#### Solution of Example-3 Using Advanced Vogel's Approximation Method (AVAM):

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	8	8	2	10	2	40
S2	11	4	10	9	4	70
S3	5	2	2	11	10	35
S4	10	6	6	5	2	90
S5	8	11	8	6	4	85
Demand	80	55	60	80	45	

Table:3.2

Total Transportation Cost:

$$(2 \times 40) + (4 \times 55) + (4 \times 15) + (5 \times 15) + (2 \times 20) + (5 \times 60) + (2 \times 30) + (8 \times 65) + (6 \times 20) = 1475$$

#### Solution of Example-3 Using Vogel's Approximation Method (AVAM):

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	8	8	2	10	2	40
S2	11	4	10	9	4	70
S3	5	2	2	11	10	35
S4	10	6	6	5	2	90
S5	8	11	8	6	4	85
Demand	80	55	60	80	45	

Table:3.3

Total Transportation Cost:

$$(2 \times 40) + (4 \times 55) + (9 \times 15) + (5 \times 15) + (2 \times 20) + (5 \times 45) + (2 \times 45) + (8 \times 65) + (6 \times 20) = 1505$$

#### Optimal solution of Example-3:

The optimal solution of Example-1 determined by MODI is 1475.

#### 5. Result Analysis:

We observed in above examples that Advanced Vogel's Approximation Method (AVAM) gives the lower feasible solution other than Vogel's Approximation Method (VAM). Results using AVAM are very close to or equal to optimal solution. The comparison table of AVAM and VAM result are follows:

Methods	Transportation Problems		
	Optimal Solution	AVAM	VAM
Example-1	1060	1085	1100
Example-2	1670	1670	1710
Example-3	1475	1475	1505

Table: 5

## 6. Conclusion

In this paper we fixed the computational error of Vogel's Approximation Method (VAM) and proposed a new algorithm named "Advanced Vogel's Approximation Method (AVAM)". From the above examples it is shown that AVAM gives the lower feasible solution than VAM and it is very close to optimal solution or equal to optimal solution.

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