International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 5, July - 2012

# Adaptive Generalized Minimum Variance Control of DC Motor

Sharat Chandra Mahato
*Department of Electrical Engineering, IT BHU, Varanasi, India*
J. P. Tiwari
*Department of Electrical Engineering, IT BHU, Varanasi, India*

## Abstract

*In this paper the implicit or direct MV controller combined with the recursive least square parameter estimation method was applied to the multi-input multi-output system. The SISO system is described by linear vector difference equations with constant but unknown parameters is discussed. To obtain good tracking and control characteristics, a MV self-tuning regulator is developed. The performance of the self-tuning GMV regulator depends a great deal on the specification of weighting polynomials for output, input and the reference point.*

## 1. Introduction

Conventional controllers for such drives are designed on the basis of local linearization about an operating point. These controllers are very effective if the load change is small and the operating conditions do not force the system too far away from the linearizing equilibrium point. The development of high performance Motor drives is very important in industrial application. In the past years many techniques**,** have been developed and applied to the speed control of the DC motor drives such that very good control performance can be obtained. Among the existing techniques, PI controllers are the most commonly used. But the PI controller does not provide a good regulating performance instantaneously because the controller parameters could not be adaptively changed according to the DC motor parameters. Moreover, its control performance is sensitive to the system parameter variations. In the past few years, the modern control techniques such as optimal control, variable structure system control**,** adaptive control**,** etc., have been applied to yield better performance. Because the self-tuning regulator has the ability to tune its own parameters, it is well suited for handling the load variation of the DC motor. In this paper the implicit or direct GMV regulator combined with the recursive least squares parameter estimation method was applied to regulate the speed of the DC motor under the influence of the load variations. The generalized minimum variance regulator was developed by [3], [4], and [5] from [6]. It has several useful properties. First in its performance criteria, the control signal is weighted. Consequently, excessive actuator movements are avoided and at the same time the control of a certain class of non minimum-phase plants is made possible.

## 2. Controller design

The generalized minimum variance regulator has been discussed elsewhere **[3],** [4]**,** [5],[6], . In this section we briefly review their method. The starting point in the analysis is the single-input, single-output plant model

$$A(z^{-1})y(t) = z^{-d}u(t) + C(z^{-1})e(t) \quad (1)$$

where d is the system dead time expressed as an integer multiple of the sampling period plus 1 (to account for the zero order hold in discrete systems). The noise e(t) is the Gaussian white noise. A**,** B and ***C*** are polynomials in the backward shift operator $z^{-1}$ corresponding to the plant output, control input and system noise. Their method was designed to do one step minimization of the cost function,

$$J = E\{[P(z^{-1})y(t+d)]^2 + [Q(z^{-1})u(t)]^2\} \quad (2)$$

Where y (t) ***is*** the measured output, u(t) is the manipulated input, and ***d*** is the plant time delay $P(z^{-1})$ and $Q(z^{-1})$ are polynomials. It also show that the above minimization is equivalent to minimizing the variance **of** a "generalized output",$\varphi(t+d)$, where

$$\varphi(t+d) = P(z^{-1})y(t+d) + Q(z^{-1})u(t) \quad (3)$$

Equation **(2)** which interpreted as a generalized system
output, by adding to the original system an output filter
action term, $P(z^{-1})y(t+d)$.
Rewriting Eq. (1) we obtain

$$y(t+d) = \frac{B}{A}u(t) + \frac{C}{A}e(t+d) \quad (4)$$

The role of the forward element Qu(t) is to shift the system open-loop pole from *B* to *PB+QA*.
By substituting Eq **(4)** into Eq **(3)** yields:

$$\varphi(t+d) = \frac{PB+QA}{A}u(t) + \frac{PC}{A}e(t+d) \quad (5)$$

The method **of** argument is by splitting the Eq **(3)** into two parts [12]: the first part is to set to zero by the control action ***u(t)*** and the second part which is a function of the noise signal ***e(t*** *+ 1)*, .........., ***e(t*** *+ d)* cannot be modified by the control signal at time step *t.*

$$\frac{PC}{A}e(t+d) = Ee(t+d) + \frac{F}{A}e(t)$$
$$= Ee(t+d) + z^{-d}\frac{F}{A}e(t+d) \quad (6)$$

From eqn. (6), the following identity can be deduced:

$$PC = EA + z^{-d} \quad (7)$$

Multiplying eqn. (1) by E yields:

$$AEy(t+d) = BEu(t) + CEe(t+d)$$

Substituting for EA from eqn. (7) gives

$$PCy(t+d) = Fy(t) + Gu(t) + CEe(t+d) \quad (8)$$

Where G = BE

The optimal prediction of $py(t+d) = \varphi_y(t+d)$ given input output data up to time t is defined as

$$\varphi_y(t+d) = \varphi_y^*\left(t+{}^{d}/_{t}\right) + Ee(t+d) \quad (9)$$

Rewriting eqn. (9) gives

$$\varphi_y(t+d) = Fy(t) + Gu(t) + CEe(t+d)(1-C)\varphi_y(t+d) \quad (10)$$

Substituting eqn. (9) into eqn. (10)

$$\varphi_y(t+d) = Fy(t) + Gu(t) + (1-c)\varphi_y^*(t+d/t) + Ee(t+d) \quad (11)$$

The optimal d-step ahead weighted output predictor is then given by

$$\varphi_y^*(t+d/t) = \frac{Fy(t)+Gu(t)}{C} \quad (12)$$

The prediction of the generalized output $\varphi(t+d)$ is therefore

$$\varphi^*(t+d) = \varphi_y^*(t+d)/t) + Qu(t) \quad (13)$$

The control law, which sets this prediction to zero, is

$$u(t) = -\frac{\varphi_y^*(t+d)/t)}{Q} \quad (14)$$

## 2.1 Choice of Design Parameters

The controller design parameters for the ***GMV*** self-tuning are the P and Q polynomials of Eqn. (14). By substituting Eqn. (1 2) into Eqn. (1 **3)** yields:

$$\varphi^*(t+d) = \frac{BE}{C}u(t) + \frac{F}{C}y(t) + Qu(t)$$
$$= \frac{1}{c}[(BE+QC)u(t) + Fy(t)] \quad (15)$$

The cost function in Eqn. (2) is minimized by setting Eqn. (15) to zero, therefore

$$(G+QC)u(t) + Fy(t) = 0$$

Where G = BE

$$u(t) = -\frac{Fy(t)}{G+QC} \quad (16)$$

Substituting eqn. (16) into eqn. (5) yields:

$$y(t) = \frac{z^{-d}B}{A(G+QC)}[-Fy(t)] + \frac{C}{A}e(t) \quad (17)$$

Regrouping y (t) yields:

$$y(t)\left[1 + \frac{z^{-d}BF}{A(G+QC)}\right] = \frac{C}{A}e(t) \quad (18)$$

Therefore, the closed loop equation is given by

$$y(t) = \frac{C(G+QC)}{A(G+QC)+z^{-d}BF}e(t) \quad (19)$$

Using identity in eqn. (19), the term $A(G+QC) + z^{-d}BF$
Can be rewritten as

$$\begin{aligned} A(G+QC) + z^{-d}BF &= A(BE+QC) + z^{-d}BF \\ &= (AE)B + (QAC) + z^{-d}BF \\ &= B[AE + z^{-d}F] + (QAC) \\ &= BPC + QAC \\ &= C(PB+QA) \end{aligned}$$

From eqn. (18) and the above relationship, the closed loop equation can be expressed as:

$$y(t) = \frac{BE+QC}{PB+QA}e(t) \quad (20)$$

The minimum variance of the auxiliary output signal is obtained when the following control law is used:

$$u(t) = -\frac{Fy(t)}{G+QC}$$
$$= -\frac{F}{H}y(t)$$

Where H = G + QC

Or,
$$Hu(t) + Fy(t) = 0 \tag{21}$$

As far as closed loop stability is concerned, it can be seen that, the closed loop poles will be given by the roots of $(AQ + BP)C = 0$. By suitable choice of the polynomials P and Q, the roots of $(AQ + BP)$ can be made to lie within the unit disc in the z-domain.

Therefore, the polynomials P and Q can be calculated from the following relation.

$$AQ + BP = T \tag{22}$$
Where, T is the desired closed loop poles polynomial.

## 3. Generalized minimum variance self tuning

In the self-tuning case, the controller parameters can be estimated from (see Clarke and Gawthrop 1975)

$$\varphi(t) = Hy(t-d) + Fy(t-d) + e(t) \tag{23}$$
Where e (t) is a moving average process of order (d – 1).

Eqn. (23) can be written as
$$\varphi(t) = x^T(t)\theta + e(t) \tag{24}$$

Where,
$$x(t) = [y(t-d), y(t-d-1), \ldots, u(t-d), u(t-d-1), \ldots]^T \tag{25}$$
$$\theta = [f_0, f_1, \ldots, g_0, g_1, \ldots]^T \tag{26}$$

Note that the term e (t) is uncorrelated with the data x(t) in regression since it is a future value with respect to the time indices of y and u. Hence, Recursive Least Squares (RLS) leads directly to the required $\hat{F}$ and $\hat{H}$ parameters.

So we get a self-tuner with feedback law:
$$\hat{F}(z^{-1})y(t) + \hat{H}(z^{-1})u(t) = 0 \tag{27}$$

The procedure then, is as follows:
1) Assemble old data into the x-vector as in equation (20)
2) Use RLS to get $\hat{F}, \hat{H}$.
3) Use the estimated parameters in the feedback law of equation (21).
The following Recursive Least Squares (RLS) equations can be used.

Kalman gain vector:
$$K(t) = \frac{P(t-1)x}{\beta + x^T P(t-1)x} \tag{28}$$

Parameter update:
$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t) \tag{29}$$

Covariance update:
$$P(t) = \frac{1}{\beta}([I - K(t)x^T(t)]P(t-1)) \tag{30}$$
β: Forgetting factor

## 4. Discussion and simulation

In order to illustrate the behavior of the above presented self-tuning regulator based on minimum variance control, the simulation results on a process model are given.

The following model has been applied to describe the behavior of a separately excited DC-motor: [5]

$$A(z^{-1})\, y(t) = B(z^{-1})\, u(t-d) + C(z^{-1})\, e(t) \tag{31}$$

Where,

$A(z^{-1}) = 1 - 1.099z^{-1} + 0.1778z^{-1}$

$B(z^{-1}) = 0.01754 + 0.01718z^{-1}$

$C(z^{-1}) = 1 - 0.02578z^{-1} + 0.03439z^{-1}$

With d = 1.

The desired closed loop pole polynomial T was chosen as $T = 1 - 1.3z^{-1} + 0.42z^{-2}$

$P = 1 - 1.1268z^{-1} + 2.176z^{-2}$ and

$Q = 0.9824 - 0.21z^{-1}$

Therefore the optimal control polynomials are

E = 1

$F = -0.0535 + 2.0622z^{-1} - 0.09475z^{-2} + 0.07483z^{-3}$

$G = BE = 0.01754 + 0.01718$

$H = G + QC$

$= 1 - 0.2181z^{-1} + 0.03918z^{-2} - 0.00722z^{-3}$

The corresponding model to estimate in self-tuning is

$$y(t) = f_0 y(t-1) + f_1 y(t-2) + f_2 y(t-3) + f_3 y(t-4) + h_0 u(t-1) + h_1 u(t-2) + h_2 u(t-3) + h_3 u(t-4) + \square(t)$$

The estimator for a self-tuner will have data and parameter vectors:

$$x(t) = [y(t-1), y(t-2), y(t-3), y(t-4), u(t-1), u(t-2), u(t-3), \ u(t-4)]^T$$

$$\theta = [f_0, f_1, f_2, f_3, h_0, h_1, h_2, h_3]^T$$

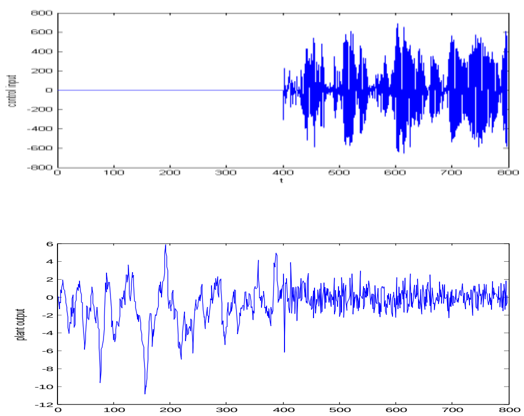

Fig. (1) Control input and Plant output

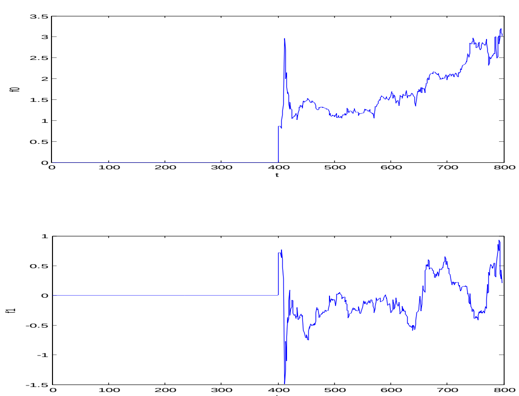

**Fig.2 Tuned controller parameters $f_0$, $f_1$**

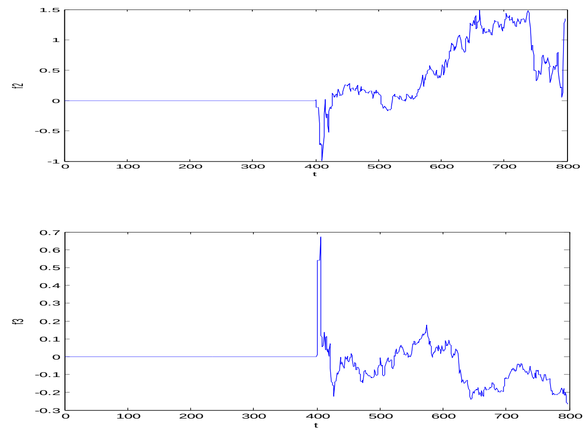

**Fig.3 Tuned controller parameters $f_2$, $f_3$**

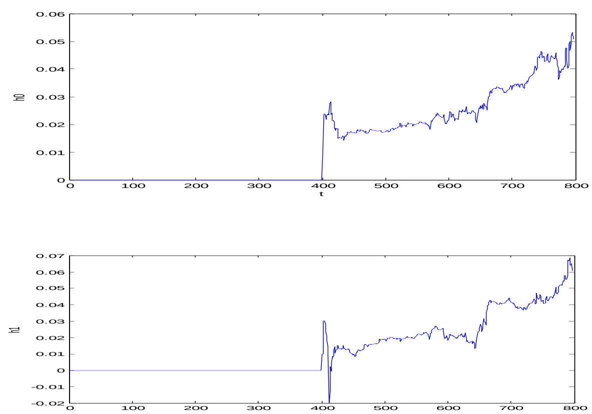

**Fig.4 Tuned controller parameters $h_0$, $h_1$**

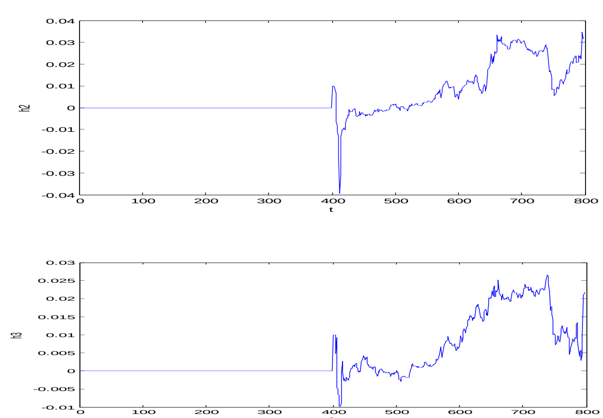

**Fig.5 Tuned controller parameters $h_2$, $h_3$**

Fig.1 shows the output response and the control input of the process (31), using the proposed algorithm. Initial condition for the covariance matrix P is set to , i.e. $P = \alpha I$, where $\alpha$ = 100 and I is the identity matrix. The noise signal is chosen as a white Gaussian noise and the forgetting factor is 0.98.

The system was simulated for 798 samples, the first 400 being open loop. At $t$ = 400 the self-tuner was switched on, giving the results in Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5. Observe how the variance has been reduced by self-tuning generalized minimum variance.

## 5. Conclusion

This paper considered the self-tuning regulator of a system with constant but unknown parameters. The analysis has been limited to single-input single-output (SISO) systems with white noise. The extension to the multi-variable (MIMO) systems case can be formulated similarly. The validity of the proposed algorithm has been demonstrated by simulation.

## 6. References

[1] K. J. Astrom, U. Borisson, L. Ljing, and B. Wittenmark, "Theory and Applications of Self-Tuning Regulators)," Automatica, Vol.13, pp .457-476. Pergamon Press. 1977. Printed in Great Britain.

[2] D. W. Clarke, Self-Tuning Control, Department of Engineering Science, Parks Road, Oxford, UK.

[3] Clark, D. W., and Gawthrop, P.J: "Self-tuning controller" Proc.IEE. 1975, 122, (9), pp. 929-934.

[4] Gawthrop, P.J.: "some interpretations of the self-tuning controller", *Proc. IEE.,* 124, (lo), pp. 889-894, 1977.

[5] Clark, D. W., and Gawthrop, P.J: "Self-tuning Controller"Proc. IEE., 126, **(6),** 633-640, 1979

[6] Astrom, K. J. B. Wittenmark, ''On Self-Tuning Regulator," *Automutica,* vol9, 185-199,1973.

[7] Zidane, Z., Lafkih, M.A., Ramzi, M. and Abounda, A. "Adaptive minimum variance control of DC motor", IEEE 2010.

[8] Sazali yaccob and Mohamed, A. "on line self tuning controller for induction motor based on generalized minimum variance method", SICE, 1998.