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Adaptive Control Strategy using Lyapunov Stability Theory

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Abstract:- **This paper present some explanations of adaptive control system which is not considered to be perfect but it leads to the consideration of what is required of a system in order that it may be said to be adaptive and the conditions needed for adaptive control system. Basic structure of adaptive control system, the performance assessment and the mechanism can be understood to be a system capable of adjusting its performance either by modifying its parameters or by modifying its input signal. For the purpose of this paper we mainly consider two classification of the adaptive control system: model reference adaptive system or control (MRAS or MRAC) and Self-tuning control system (STC). MRAC and STC can be designed using both Direct and Indirect approaches. Lyapunov stability theory is a method used to judge the stability of the system.**

Key words:- Adaptive Control System, Model Reference Adaptive System or Control (MRAS or MRAC), Self-Tuning Control System (STC), Lyapunov Stability Theory

I. INTRODUCTION

The definition of an **adaptive control system** is not considered to be perfect but it leads to the consideration of what is required of a system in order that it may be said to be adaptive. The following are some definition of adaptive control system:

An adaptive control system is one which is capable of adjusting itself [1]. Thus feedback gains and other parameters of the controller are self-adjusted in such a way that the response of the controlled process follows the desired response as closely as possible at all times.

Adaptive control is the control method used by a controller which must adapt to a controlled system with parameters which vary, or are initially uncertain [2]. For Example, as an aircraft flies, its mass will slowly decrease as a result of fuel consumption, a control law is needed that adapts itself to such changing conditions. Adaptive control does not need a priori information about the bounds on these uncertain or time-varying parameters; and is concerned with control law changing them.

A set of technique for automatic adjustment of the controllers in real time [3], in order to achieve or to maintain a desired level of performance of the control system when the parameters of the plant (disturbance) dynamic model are unknown and /or change in time.

An adaptive control is the automatic tuning of feedback controllers [4].

Adaptive control allows operating parameters to be changed continuously in response to a changing environment in order to achieve optimum performance [5].

An adaptive control system is a system where in addition to the basic (feedback) structure, explicit measures are taken to compensate for variation in the process dynamics or for variation in the disturbances in order to maintain an optimal performance of the system [6]

The need for adaptive control system arose under one or more of the following conditions [3];

- High performance control systems may require precise tuning of the controller but plant (disturbance) model parameters may be unknown or time-varying.
- Adaptive control technique provides a systematic approach for automatic on-line tuning of controller parameters.
- Adaptive control can be viewed as approximation of some nonlinear stochastic control problems (not solvable in practice).
- Objective of adaptive control is to achieve and to maintain acceptable level of performance when plant (disturbance) model are unknown or vary.

Other motivating factors to the development of adaptive control system are that the adaptive loop can provide a system that has a fixed and known transfer function. This can be an advantage when other loops are designed round the basic system.

II. BACKGROUND

During the past decades, that is, since the early 1960’s [7, 8, 9], research and development effort for adaptive control system have been ongoing. Adaptive control systems have been developed to address a common drawback of the computer numerically controlled (CNC) system. The computer numerically controlled system (CNC) operating parameters, such as speed and feed rate are prescribed by a

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part-programmer and consequently which depends on his/ her experience and knowledge.

These research efforts were primarily concentrated in the U.S., West Germany, Italy, Japan and Israel [10, 11, and 12].

The implementations of early adaptive control system were achieved by using hardware and many different strategies and approaches were developed [7, 9, 13]. Felbaum developed the dual controller in which the control action serves a dual purpose as it is directing as well as investigating in the late 1960's. Late 1960's to early 1970's, system identification approach with recursive least squares. Convergence and stability analysis was introduced in the early 1980. Professor Whitaker of MIT in the end of 1950's, first proposed a model reference adaptive control (MRAC) scheme of the plane autopilot according to requirements of flight control, and it is called MIT scheme. In the scheme the local parametric optimization theory was used when he designed the adaptive control law, but the scheme was not used in practice. Since the application of the local parametric optimization method in design of the model reference adaptive system does not consider stability of the system, after completing the design of adaptive system one must examine the stability, which restricts its application [14]. In the year 1973, a Swedish scholar, K.J.Astrom and B. Wittenmark, first proposed the self-tuning controller. D.W Clark and others, in the year 1975 also proposed a kind of self-tuning controller. In the year 1979, P.E Wellstead and K.J Atsrom proposed design scheme of pole-placement self-tuner and servo system. Mid 1990's, multiple adaptive control and iterative control was developed. In 2000's, adaptive control, fast adaptation with guaranteed robustness, performance specifications given in terms of reference model originally introduced for flight control system (MIT rule).

III. BASIC STRUCTURE OF ADAPTIVE CONTROL SYSTEM

Basic structure of adaptive control system is illustrated in fig 1. The performance assessment and mechanism in the figure can be understood to be a system capable of adjusting its performance either by modifying its parameters or by modifying its input signal.

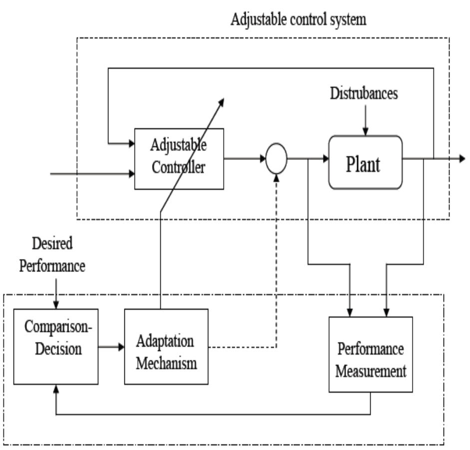

Fig.1. Basic Structure of Adaptive Control System.

In adaptive control, it is assumed that there is feedback from the system performance which adjusts the regulator parameters to compensate for the slowly varying process parameters.

For the purpose of this report we mainly consider two classification of the adaptive control system: model reference adaptive system or control (MRAS or MRAC) and Self-tuning control system (STC).

IV. CLASSIFICATION OF ADAPTIVE CONTROL SYSTEM

In general the adaptive control system can roughly be divided into two categories: Direct and Indirect. MRAC and STC can be designed using both Direct and Indirect approaches

- *Model reference Adaptive control (MRAC or MRAS)*

Classification of adaptive control scheme is based on model reference control (MRC).

In MRC, the desired plant behavior is described by reference model which is simply a linear time-invariant (LTI) system with transfer function $W_m(s)$) and is driven by a reference input [16]. The control law $C(s,\theta_c^*)$, is then developed so that the closed-loop plant has a transfer function equal to $W_m(s)$. This transfer function matching guarantees that the plant will behave like the reference model for any reference input signal.

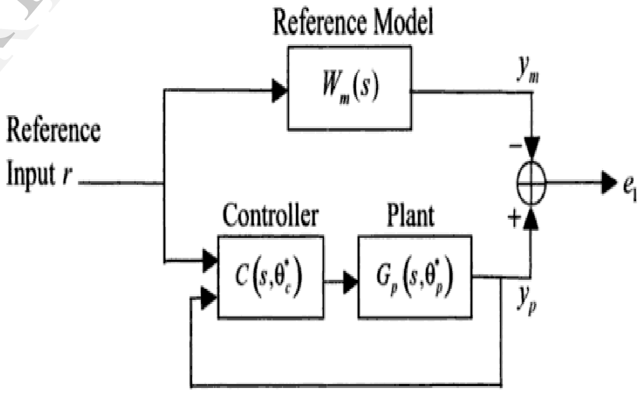

Fig.2. Model Reference Control

- *Plant*has a known structure but the parameters are unknown
- *Reference* model specifies the ideal (desired) response y_m to the external command r
- *Controller*is parameterized and provides tracking
- *Adaptive Law*is used to adjust parameters in the control law

Because this method was proposed by the scientists in MIT, it is also called MIT method.

Figure 2, shows the basic structure of MRC. The plant transfer function is$G_p(s,\theta_p^*)$, where,

θ_p^*Is a vector with the coefficient of G_p. The controller transfer function is $C(s,\theta_c^*)$, where,

θ_c^*Is a vector with the coefficient of $C(s)$.

The transfer function $C(s,\theta_c^*)$, and therefore θ_c^*, is designed so that the closed-loop transfer function of the plant from the reference input r and y_p is equal to $W_m(s)$,

That is $\frac{y_p(s)}{r(s)} = W_m(s) = \frac{y_m(s)}{r(s)}$ (1)

For this transfer matching to be possible, $G_p(s), W_m(s)$, have to satisfy certain assumptions. These assumptions enable the calculation of the controller parameter vector θ_c^* as

$\theta_c^* = F(\theta_p^*)$. (2)

Where F is a function of the plant parameter vector θ_p^*, to satisfy the matching equation (1).This transfer function matching guarantees that the tracking error $e_1 = y_p - y_m$ converges to zero for any given reference input signal r.

If the plant parameter vector θ_p^* is unknown, then the controller parameter θ_c^* can be calculated using equation (2) and the controller $C(s,\theta_c^*)$ can be implemented.

- *Indirect MRAC*

In the indirect MRAC, we are considering the case where θ_p^* is unknown. In this case the use of certainty equivalence (CE) approach, where the unknown parameters are replaced with their estimates, which leads to the adaptive control scheme referred to as indirect MRAC. Shown in fig. 3

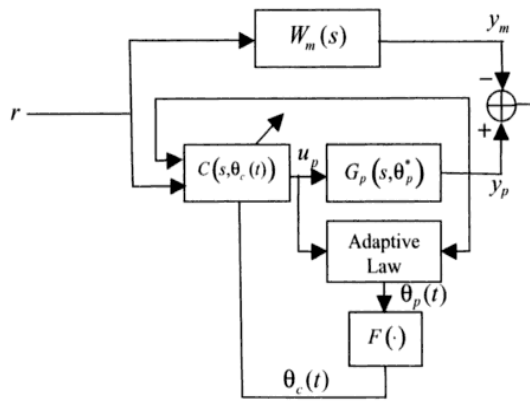

Fig.3. Indirect MRAC

The unknown plant parameter vector θ_p^* is estimated at each time t, denoted by $\theta_p^*(t)$ using an adaptive law. The plant parameter estimate θ_p^* at each time t is then used to calculate the controller parameter vector $\theta_c^*(t) = F(\theta_p^*(t)$ used in the controller $C(s,\theta_c^*)$. This class of MRAC is called ***Indirect MRAC*** because the controller parameters are not updated directly but calculated at each time t using the estimated plant parameters.

- *Direct MRAC*

In the direct scheme, the plant transfer function is parameterized in terms of the desired controller parameter vector θ_c^*.

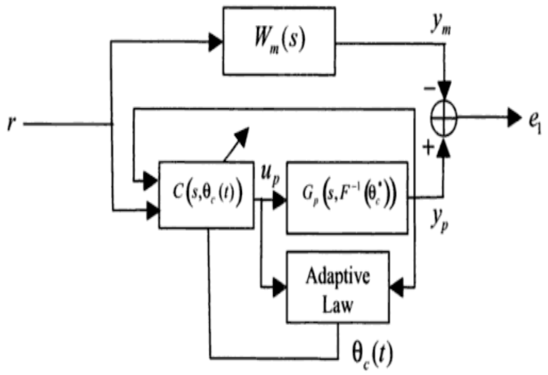

Fig.4. Direct MRAC

This is possible in the MRAC case because the structure of the MRAC law is such that we can use equation (2) to write

$\theta_p^* = F^{-1}(\theta_c^*)$ (3)

Where F^{-1} is the inverse of the mapping $F(.)$ and then express

$G_p(s,\theta_p^*) = G_p(s,F^{-1}\theta_c^*) = \bar{G}_p(s,\theta_c^*)$ (4)

The adaptive law for estimating θ_c^* online can now be developed by using

$y_p = \bar{G}_p(s,\theta_c^*)u_p$, (5)

to obtain a parametric model with θ_c^* as the unknown parameter vector. The MRAC can then be developed using the CE approach, as shown in fig. 4. In this case, the controller parameter $\theta_c^*(t)$ is updated directly without any intermediate calculation and for this reason the scheme is called ***Direct MRAC***.

- *Self-Tuning Controllers (STC)*

In control theory a self-tuning system is capable of optimizing its own internal running parameters in order to maximize or minimize the fulfillment of an objective function; typically the maximization of efficiency or error minimization. Self-tuning and auto-tuning often refer to the same concept. The self-tuning control system may adopt different control strategy in accordance with the cost function, nature and requirement of the system. Fig.5. shows the configuration of the self-tuning regulator

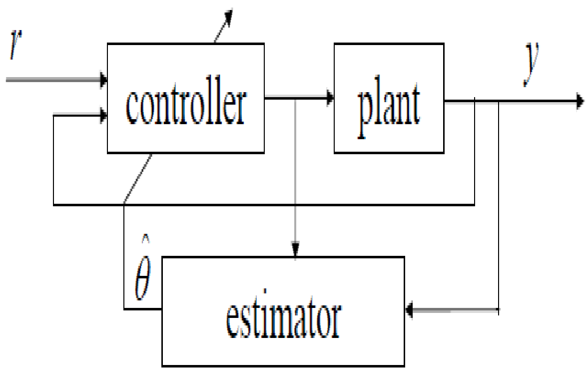

Fig.5. Configuration of the Self-Tuning Regulator
Combines a controller with an on-line (recursive) plant parameter estimator

- Reference model can be added
- Performs simultaneous parameter identification and control
  Uses *Certainty Equivalence Principle*
- controller parameters are computed from the estimates of the plant parameters as if they were the true ones

The self-tuning control system may adopt different control strategy in accordance with the cost function; nature and requirement of the system, and the strategy used universally are the minimum-variance control and the pole-placement control.

V. LYAPUNOV STABILITY THEORY

Lyapunov stability theory is a theoretical fundamental of the model reference adaptive control (MRAC). Russian scholar A.M. Lyapunov proposed the Lyapunov stability which adapts state description. It is a method used to judge the stability of system and its application domain wider than those of Algebra criterion, Nyquist criterion, Routh's criterion etc. in classical theory. Lyapunov stability can be classified into Indirect and Direct Methods. It has been widely used to analyze stability and to design the system.

- *Lyapunov Theory*

Lyapunov theory is used to make conclusions about trajectories of a system $\dot{x} = F(x)$ without finding the trajectories (i.e., solving the differential equation) a typical Lyapunov theorem has the form:

- if there exists a function $V: Rn \rightarrow R$ that satisfies some conditions on V *and* $\dot{V}$
- then, trajectories of system satisfy some property

If such a function V exists we call it a Lyapunov function (that proves the property holds for the trajectories)
Lyapunov function V can be thought of as generalized energy function for system. Some of the Lyapunov theorems are

- Lyapunov boundedness theorem,
- Lyapunov global asymptotic stability theorem,
- Lyapunov exponential stability theorem,
- Lyapunov instability theorem,
- Lyapunov divergence theorem,
- Converse Lyapunov theorems

Let us consider the application of a method of Lyapunov in the analysis of the stability for linear time-invariant system [14].
Consider a continuous linear time-invariant system described by

$$\dot{x} = Ax, \qquad x(0) = x_0, \qquad t \geq 0 \quad (6)$$

Where x is a n-dimensional state vector, and A is a $(n \times n)$ nonsingular matrix, so the origin is unique equilibrium state. Selecting positive definite quadratic function

$$V(x) = x^T P x \qquad (7)$$

As a Lyapunov function candidate, where P is a $(n \times n)$ symmetric positive definite matrix, considering the system in equation 6, and taking derivative of $V(x)$ with respect to t, we obtain

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} = x^T (A^T P + PA) x (8)$$

Let $(A^T P + PA) = -Q$ (9)
Then $\dot{V}(x) = -x^T Q x$ (10)

According to the discrimination theorem of global asymptotic stability for time-invariant system, only if the matrix Q is positive definite, the system would be globally asymptotically stable. Matrix equation (9) is called Lyapunov algebraic equation.

Consider a system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -8 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \qquad (11)$$

To determine its Lyapunov function,

Let $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$, $and p_{21} = p_{12}$ $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

According to Lyapunov equation

$$(A^T P + PA) = -Q\ , \qquad (12)$$

We obtain

$$\begin{bmatrix} 0 & -8 \\ 4 & -12 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -8 & -12 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (13)$$

$$-16p_{12} = -1,$$
$$4p_{11} - 12p_{12} - 8p_{22} = 0,$$
$$8p_{12} - 24p_{22} = -1$$

And $p_{11} = \frac{5}{16}$, $p_{21} = p_{12} = \frac{1}{16}$, $p_{22} = \frac{1}{16}$

Matrix P is given by $P = \begin{bmatrix} \frac{5}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} \end{bmatrix}$

P is a positive definite matrix.

The Lyapunov function is

$$V(x) = x^T P x = \frac{5}{16} x_1^2 + \frac{1}{8} x_1 x_2 + \frac{1}{16} x_2^2 = \frac{1}{4} x_1^2 + \frac{1}{16}(x_1 + x_2)^2 \qquad (14)$$

The derivative of $V(x)$ with respect to t *is*

$$\dot{V}(x) = \frac{1}{2} x_1 \dot{x}_2 + \frac{1}{8}(x_1 + x_2)(\dot{x}_1 + \dot{x}_2)$$
$$\dot{V}(x) = 2x_1 x_2 - (x_1 + x_2)^2$$
$$\dot{V}(x) = -(x_1^2 + x_2^2)$$

Since $V(x)$ is positive definite and $\dot{V}(x)$ is negative definite, the system is asymptotically stable.

VI. CONCLUSION

In this paper an adaptive control system can be said to be a system where in addition to the basic (feedback) structure, explicit measures are taken to compensate for variation in the process dynamics or for variation in the disturbances in order to maintain an optimal performance of the system. Beside the self-tuning control system and the model reference control system, other various types of adaptive control systems emerge endlessly, for example, variable structure control system, nonlinear adaptive control system, fuzzy adaptive control system and neural network adaptive control system etc. Lyapunov stability theorems can be used to account for the stability of the adaptive control system.

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