*A*4 Model with Five Extra Scalars for Neutrino Masses and Mixing

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Abstract

A modified neutrino mass model with five extra $SU(2)_L \times U(1)_Y$ singlet scalars is constructed using A_4 discrete symmetry group. The resultant mass matrix is able to reproduce the current neutrino masses and mixing data with good accuracy. Our model predicts the relation between the neutrinoless double-beta decay parameter $|m_{\beta\beta}|$ and oscillation parameters.

Keywords: A_4 symmetry, scalars, Tribimaximal(TBM), projection matrix, normal hierarchy, inverted hierarchy

1 Introduction

The neutrino experiments have confirmed neutrino oscillations and mixing through the observation of solar and atmospheric neutrinos indicating its mass thereby providing an important solid clue for a new physics beyond Standard Model(SM) of particle physics. At present, the neutrino oscillation experiments [1, 2, 3, 4] have measured the oscillation parameters viz: mass squared difference(Δm^2_{21} and Δm^2_{31}) and mixing angle(θ_{12}, θ_{23} and θ_{13}) to a good accuracy. The bounds on the absolute neutrino masses scale are also greatly reduced by direct and cosmological neutrino mass search experiments [5, 6]. However, the current data is still unable to explain several key issues such as the octant of θ_{23}, the neutrino mass ordering, CP violating phase etc.

The oscillation data reveals certain pattern of neutrino mixing matrix .Out of the several approaches to explain the observed pattern, the Tribimaximal(TBM) mixing[7, 8] used to be very favourable. And, it yields some interesting results like trivial value of θ_{13} and CP violating phase δ and maximal θ_{23} etc.

The A_4 flavour symmetry model proposed by Altarelli and Feruglio[9] was the first attempt to accommodate TBM mixing scheme in a neutrino mass model. Following this example, many other neutrino mass models are constructed using different non-Abelian discrete symmetry groups[10]. However, the recently observed non-zero θ_{13} disfavours Tribimaximal (TBM) and leads to the modification of several mass models constructed with TBM [11, 12, 13]. As a result, the neutrino mixing pattern like TM1[14] and TM2 mixings[15] which are proposed with slight deviation from TBM gain momentum. Currently, they can predict the observed pattern and mixing angles with good consistency.

In this present work, we proposed a model with five extra SM singlet scalars to explain neutrino mixing angles in their experimental range. The present model is constructed in T-diagonal basis. The non-zero value of θ_{13} is obtained as a consequences of specific Dirac mass matrix which is constructed using anti-symmetric part and its projection matrix arises from the product of two A_4 triplets. Here, we present a detailed analysis on the neutrino oscillation parameters and its correlation among themselves and with neutrinoless double-beta decay parameter $|m_{\beta\beta}|$.

2 Description of the model

In this work, we extended SM by adding five extra $SU(2)_L \times U(1)_Y$ singlet scalars namely ξ_1, ξ_2 and ξ_3 which are singlet under A_4 and the field ϕ_T and ϕ_S which are transformed as triplet. The standard model lepton doublets are assigned to triplet representation under A_4, right handed charge lepton e^c, μ^c, τ^c and right handed neutrino field ν^c are assigned to 1,$1^{''}$, $1^{'}$ and 3 representation respectively. The right handed neutrino field ν^c contributes to the effective neutrino mass matrix through type-I seesaw mechanism[16]. The transformation properties of the fields use in the model under $A_4 \times Z_3$ discrete group are given in the Table1

Fields	l	e^c	μ^c	τ^c	ν^c	$H_{u,d}$	ϕ_S	ϕ_T	ξ_1	ξ_2	ξ_3
A_4	3	1	$1^{''}$	$1^{'}$	3	1	3	3	1	$1^{''}$	$1^{'}$
Z_3	ω^2	ω	ω	ω	1	1	ω	1	ω	ω	ω
SM	$(2,\frac{1}{2})$	(1,1)	(1,1)	(1,1)	(1,0)	$(2,-\frac{1}{2})$	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)

Table 1: Transformation properties of various fields under $A_4 \times Z_3$ group.

The Yukawa Lagrangian term for the lepton sector which are invariant under A_4 transformation are given in the equation:

Fields	Vacuum Expectation Value(VEV)
$<\phi_S>$	(v_s, v_s, v_s)
$<\phi_T>$	$(v_T, 0, 0)$
$<H_u>$,$<H_d>$	v_u, v_d
$<\xi_1>$, $<\xi_2>$, $<\xi_3>$	u_1, u_2, u_3

Table 2: Vacuum Expectation Values of the scelar fields used in the model.

$$\begin{aligned}-L_Y =& \frac{Y_e}{\Lambda}e^c(\phi_T l)_1 H_d + \frac{Y_\mu}{\Lambda}\mu^c(\phi_T l)_{1'} H_d + \frac{Y_\tau}{\Lambda}\tau^c(\phi_T l)_{1''} H_d + \frac{y_1}{\Lambda}\xi_1(l\nu^c)_1 \\ &+\frac{y_2}{\Lambda}(\xi_2)_{1''}(l\nu^c)_{1'}+\frac{y_3}{\Lambda}(\xi_3)_{1'}(l\nu^c)_{1''} + \frac{y_a}{\Lambda}\phi_S(lH_u\nu^c)_A + \frac{y_b}{\Lambda}\phi_S(lH_u\nu^c)_S + \\ &\frac{1}{2}M(\nu^c\nu^c) + h.c.\end{aligned} \tag{1}$$

In A_4 symmetry model, the contraction of the two A_4 triplets l and ν^c into another triplet are specified by tensor product rules in two different ways. Each tensor product combination corresponds to certain form of projection matrix. Therefore, the mass terms $\frac{y_a}{\Lambda}\phi_S(lH_u\nu^c)_A$ and $\frac{y_b}{\Lambda}\phi_S(lH_u\nu^c)_S$ of Eq.(1) could have two independent new terms with different coupling constant [17]. For our model, the projection matrix P in T-diagonal basis are as follows:

$$P_A = \left[\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{pmatrix}, \begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & -1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0 & -1\\ 0 & 0 & 0\\ 1 & -1 & 0\end{pmatrix}\right] \tag{2}$$

$$P_S = \left[\begin{pmatrix}2 & 0 & 0\\ 0 & 0 & -1\\ 0 & -1 & 0\end{pmatrix}, \begin{pmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 2\end{pmatrix}, \begin{pmatrix}0 & 0 & -1\\ 0 & 2 & 0\\ -1 & 0 & 0\end{pmatrix}\right]. \tag{3}$$

The Yukawa mass matrices can be derived by using the vacuum expectation value given in Table2 in Eq.(1). The obtained charged lepton mass matrix is diagonal and has the form

$$M_l = \frac{v_d v_T}{\Lambda}\begin{pmatrix}Y_e & 0 & 0\\ 0 & Y_\mu & 0\\ 0 & 0 & Y_\tau\end{pmatrix}. \tag{4}$$

The Majorana neutrino mass matrix has the structure

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}. \qquad (5)$$

For the Dirac neutrino mass matrix, the projection matrix given in Eq.(2) and (3)are utilised.Therefore, the resultant Dirac mass matrix is in the form

$$M_D = \begin{pmatrix} 2a+c & -a+b+d & -a-b+e \\ -a-b+d & 2a+e & -a+b+c \\ -a+b+e & -a-b+c & 2a+d \end{pmatrix}. \qquad (6)$$

where $a = \frac{y_b v_u . vs}{\Lambda}$, $b = \frac{y_a v_u . vs}{\Lambda}$ and c, d, e=$\frac{y_i v_u . u_i}{\Lambda}$, i=1,2 and 3.

The effective neutrino mass matrix is obtained by using Type-I seesaw mechanism.

$$m_\nu = (M_D^T M_R^{-1} M_D) \qquad (7)$$

$$= \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}. \qquad (8)$$

where

$$m_{11} = \frac{1}{M}((2a+c)^2 + 2(a+b-d)(a-b-e))$$

$$m_{12} = m_{21} = \frac{1}{M}(-3a^2 + b^2 + 2cd + e^2 + b(-d+e) + a(6b-2c+d+e))$$

$$m_{13} = m_{31} = \frac{1}{M}(-3a^2 + b^2 + d^2 + 2ce + b(-d+e) + a(-6b-2c+d+e))$$

$$m_{22} = \frac{1}{M}((-a+b+d)^2 - 2(a+b-c)(2a+e))$$

$$m_{23} = m_{32} = \frac{1}{M}(6a^2 - 2b^2 + c^2 + 2de + b(-d+e) + a(-2c+d+e)))$$

$$m_{33} = \frac{1}{M}2(-a+b+c)(2a+d) + (a+b-e)^2).$$

3 Numerical Analysis and Results

As the light neutrino mass matrix m_ν obtained in eq.(8) is in the basis where charge lepton mass matrix is diagonal. The Pontecorvo-Maki-Nakagawa-Sakata leptonic mixing matrix U_{PMNS} which is necessary for the diagonalization of m_ν becomes a unitary matrix U. Therefore, the light neutrino mass matrix m_ν is diagonalized as:

$$m_\nu = U^* m_{diag} U^\dagger \qquad (9)$$

where $m_{diag} = diag(m_1, m_2, m_3)$ is the light neutrino mass matrix in diagonal form. The three mass eigenvalues of the neutrino can be written as $m_{diag} = diag(m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_1^2 + \Delta m_{31}^2})$ in normal hierarchy(NH) and $m_{diag} = diag(\sqrt{m_3^2 + \Delta m_{31}^2 + \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{23}^2}, m_3)$ in inverted hierarchy(IH). The upper bound on the sum of neutrino masses ($\sum m_\nu = m_1 + m_2 + m_3$) obtained by the Planck is 0.12 eV [6].

The PMNS matrix U can be parametrised in terms of neutrino mixing angles and Dirac CP phase δ. Following PDG convention, U can takes the form

$$U_{PMNS} = P_\phi \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{i\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}e^{i\delta} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{pmatrix}. P, \qquad (10)$$

where θ_{ij} (for ij= 12,13,23) are the mixing angles (with $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$) and δ is the Dirac CP phase. $P = Diag(e^{i\alpha}, e^{i\beta}, 1)$ contains two Majorana CP phases α and β, while

$P_\phi = Diag(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ consists of three unphysical phases $\phi_{1,2,3}$ that can be removed via the charged-lepton field rephasing[18]. The neutrino mixing angles θ_{12}, θ_{23} and θ_{13} in terms of the elements of U are given below:

$$S^2_{12} = \frac{|U_{12}|^2}{1-|U_{13}|^2} \quad S^2_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2} \quad S^2_{13} = |U_{13}|^2 \ . \qquad (11)$$

In order to show that the present model is in consistent with the present neutrino oscillation data. We solve the free parameters by generating sufficiently large number of random points. The points in parameters space need to satisfy Eqs.(8) and the accuracy level is determined by the experimental error on the neutrino oscillation parameters. The experimental values of the observables are given in Table3. The model predictions of the neutrino oscillations parameters in 3σ confidence level are shown in Fig. 1 and Fig. 2. One of the important prediction of model is that solar neutrino mixing angle θ_{12} lies around 35.7° in both NH and IH cases.

Parameters	Best fit$\pm1\sigma$	2σ	3σ
$\theta_{12}/°$	34.3 ± 1.0	32.3-36.4	31.4-37.4
$\theta_{13}/°$(NO)	$8.53^{+0.13}_{-0.12}$	8.27-8.79	8.20-8.97
$\theta_{13}/°$(IO)	$8.58^{+0.12}_{-0.14}$	8.30-8.83	8.17-8.96
$\theta_{23}/°$(NO)	49.26 ± 0.79	47.35-50.67	41.20-51.33
$\theta_{23}/°$(IO)	$49.46^{+0.60}_{-0.97}$	47.35-50.67	41.16-51.25
$\Delta m^2_{21}[10^{-5}eV^2]$	$7.50^{+0.22}_{-0.20}$	7.12-7.93	6.94-8.14
$\|\Delta m^2_{31}\|[10^{-3}eV^2]$(NO)	$2.55^{+0.02}_{-0.03}$	2.49-2.60	2.47-2.63
$\|\Delta m^2_{31}\|[10^{-3}eV^2]$(IO)	$2.45^{+0.02}_{-0.03}$	2.39-2.50	2.37-2.53
$\delta/°$(NO)	194^{+24}_{-22}	152-255	128-359
$\delta/°$(IO)	284^{+26}_{-28}	226-332	200-353

Table 3: The global-fit result for neutrino oscillation parameters [19].

In normal hierarchy, the Dirac CP violating phase δ is obtained in 1σ range as shown in Fig. 1a-1b. While the predicted value of θ_{13} and θ_{23} are just outside 1σ range but well within 3σ range. Fig.2 shows that the prediction of δ, θ_{23} and θ_{13} for IH are well within 3σ range. Therefore, the present model slightly prefers normal hierarchy(NH).

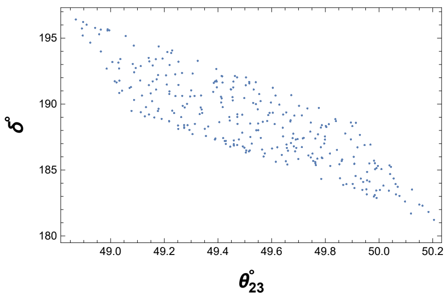
The effective neutrino mass $|m_{\beta\beta}|$ is characterized by

$$m_{\beta\beta} = |U^2_{1j} m_j|, \qquad (12)$$

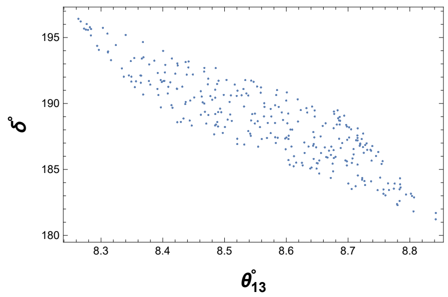
where m_j are the Majorana masses of three light neutrinos. The upper limits of the effective neutrino mass are obtained by: Gerda [5] as $|m_{\beta\beta}| < 104 - 228$ meV corresponds to $^{76}Ge(T^{0\nu}_{1/2} > 9 \times 10^{25})$, CUORE [20] as $|m_{\beta\beta}| < 75 - 350$ meV corresponds to $^{130}Te(T^{0\nu}_{1/2} > 3.2 \times 10^{25})$ and KamLAND-Zen [21] as $|m_{\beta\beta}| < 61 - 165$ meV corresponds to $^{136}Xe(T^{0\nu}_{1/2} > 1.07 \times 10^{25})$. The model prediction of the effective Majorana mass $|m_{\beta\beta}|$ vs Jarlskog invariant J in 3σ range for both NH and IH are shown in Fig.3a and Fig.[4b] respectively. The correlation between δ and $|m_{\beta\beta}|$ are depicted in Fig. 3b for NH and Fig. 4a for IH.

4 Conclusion

In conclusion, we have presented a neutrino mass model that can accommodate current neutrino oscillation data. The model has some characteristic prediction for neutrino oscillation parameters. The solar neutrino angle θ_{12} centred around 35.7° for both mass ordering. The predicted range of δ and atmospheric neutrino mixing angle θ_{23} for NH are in good agreement with their respective experimental range. The present model also predicted the correlation between δ and $m_{\beta\beta}$. The presented model slightly preferred NH data.



(a)



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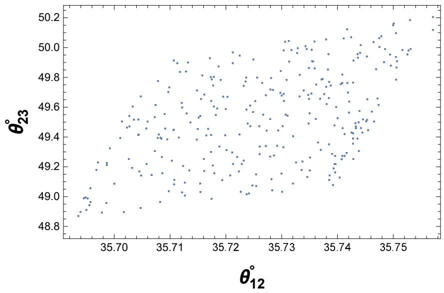

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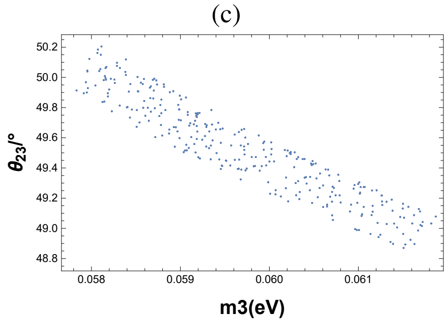
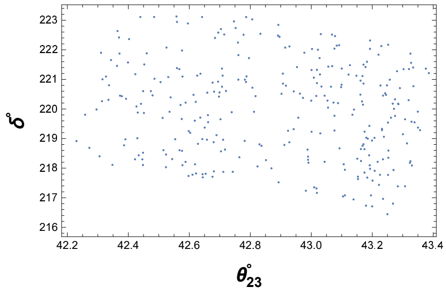
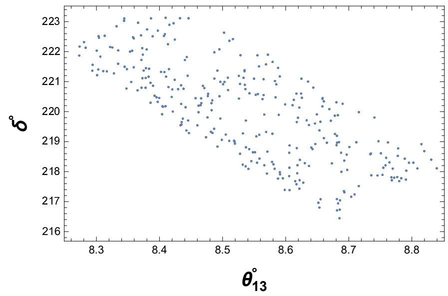


(d)

Figure 1: Correlation plot for different neutrino oscillation parameters for normal mass ordering(NH).



(a)



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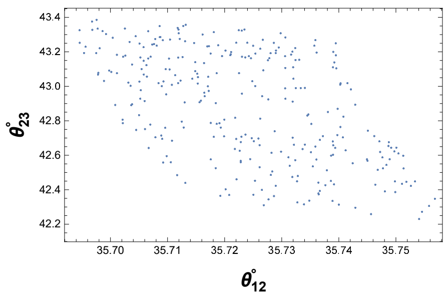

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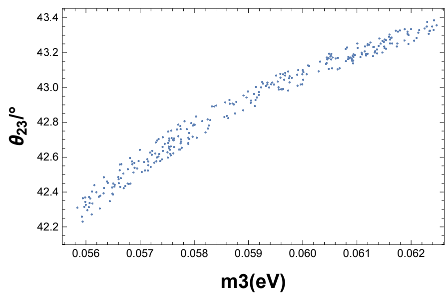

(d)

Figure 2: Correlation plot for different neutrino oscillation parameters for inverted mass ordering(IH).

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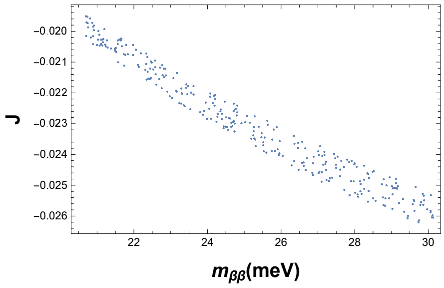
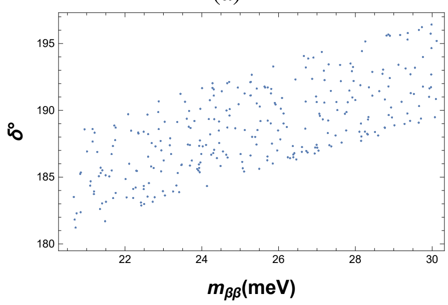
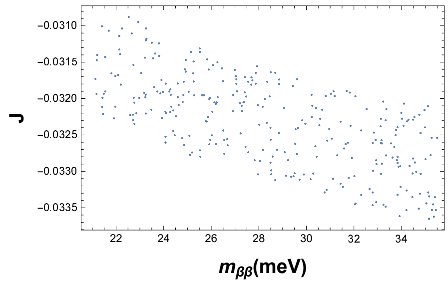


(a)

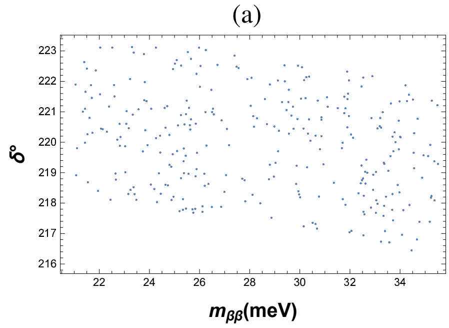


(b)

Figure 3: Model predictions for Jarlskog invariant versus effective Majorana mass and Dirac CP violating phase versus effective Majorana mass for NH.



(a)



(b)

Figure 4: Model predictions for Jarlskog invariant versus effective Majorana mass and Dirac CP violating phase versus effective Majorana mass for IH.

A APPENDIX: A_4 GROUP

A_4 is the even permutation group of 4 objects with $\frac{4!}{2}$ elements. It has four irreducible representations, namely 1, $1'$, $1''$ and 3. All the elements of the group can be generated by two elements S and T. The generators S and T satisfy the relation

$$S^2 = (ST)^3 = T^3 = 1. \tag{A.1}$$

The multiplication rules of any two irreducible representations under A_4 are given by

$$3 \otimes 3 = 1 \oplus 1^{'} \oplus 1^{''} \oplus 3_S \oplus 3_A$$

$$\begin{array}{lll} & 1 \otimes 1 = 1 & 1' \otimes 1' = 1'' \\ A1 & 1" \otimes 1'' = 1' & 1' \otimes 1'' = 1 \\ & 3 \otimes 1/1'/1'' = 3 & 1/1'/1'' \otimes 3 = 3 \end{array} . \tag{A.2}$$

The contractions of any two particle fields χ and under A_4 are specified by the tensor product rules. In our case we considered the contraction of Majorana fields:

$$\chi^T P \quad = \eta \tag{A.3}$$

where P is the projection matrix and η is result of the contraction. P is specified for each tensor product combination and it is also different in S-diagonal or T-diagonal basis. Then, the form of P for all triplet contractions in T diagonal basis for Eq.(A.3) are given below:

$$3 \times 3 = 1 : \qquad P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{A.4}$$

$$3 \times 3 = 1^{'} : \qquad P_{1'} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A.5}$$

$$3 \times 3 = 1^{''} : \qquad P_{1''} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{A.6}$$

$$3 \times 3 = 3_S : \qquad P_S = \left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix} \right] \tag{A.7}$$

$$3 \times 3 = 3_A : \qquad P_A = \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{pmatrix} \right] \tag{A.8}$$

The detailed studied on the contractions of fields under A_4 symmetry can be found in the appendix of Ref.[17].

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