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A Two-Dimensional Mathematical Model of Transportation of Contaminants in Unsaturated Porous Media with Uniform Flow

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Abstract
The main objective of this paper is to develop a mathematical model for two-dimensional flow of pollutants in unsaturated porous media with uniform flow using isotopes. The advection dispersion equation (ADE) is used to develop the model with suitable initial and boundary conditions to find the solution and compare with the other solution. The displacement experiment gives the dispersion coefficient D. The magnitude of dispersion depends on the particle size distribution and flow parameters. The average pore velocity has been considered only for the vertically downward direction. Here we have considered D_L is the longitudinal dispersion coefficient and D_T is the transverse dispersion coefficients which are perpendicular to flow directions of the fluid. The value of the D_T is more complicated to obtain than the value of D_L, because of the concentration distribution are required to be measured in vertical to the flow direction. Some studies have been performed to get the transverse dispersion coefficient on velocity of unpolluted water and porous media with distribution of various particle sizes to find out the values of longitudinal and transverse dispersion coefficient.

Keywords: **Contaminants, porous media, Diffusion, Dispersion, Advection Adsorption, Uniform flow, Integral transforms.**

1 INTRODUCTION

Mathematical models are being used extensively in groundwater studies. Groundwater modeling can be classified in to two model, they are ground water flow and solute transport models. The Solute transport models are applied in connection with groundwater quality problems. The solute transport models are extended in few cases with chemical sub models for description of the fate of non-conservative polluting species while in some case may be sufficient only to study the flow of the pollutant in the sub surface. This can be analyzed by considering two and three-dimensional model to prevent the groundwater pollution.

Previous works closely related to the work are also conducted by Yeh (1981), Domenico and Robbins (1984), Domenico (1987), Batu (1989, 1993), Leij et al (2000), and Park and Zhan (2001). Yeh (1981) provided a general framework for using Green's functions to solve transport equations without giving too many details. Domenico and Robbins (1984) and Demenico (1987) considered finite sources as boundary conditions when solving the advection-dispersion equation. They have not included the effect from the upper and lower boundaries of an aquifer. Batu (1989, 1993) provided a two-dimensional analytical solute transport model in a bounded aquifer by using the same source as the aquifer thickness along the z-axis and include the contaminant source as a boundary condition. The general solutions were derived with the help of Fourier analysis and Laplace domain. The contaminant concentration in the near field is found to be sensitive to the source geometry and anisotropy of the dispersion coefficients. The contaminant concentration in the far field is found to be less sensitive to the source geometry. All the above solutions have some restrictions. Therefore, our objective is to provide an analytical solution for two-dimensional transport problem which is applicable for both longitudinal and transverse dispersion.

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2 MATHEMATICAL MODEL

Hence, a comprehensive groundwater modeling must get the models for saturated and unsaturated porous media is integrated in the groundwater/subsurface water models. The affects of hydrodynamic processes and physical aspects that influences the mass balance on groundwater includes, advection, dispersion, diffusion, transportation, adsorption, and transformation in chemical reactions.

The infiltration is assumed that the solute transport is obtained with ADE. Let us consider the two-dimensional dispersion with one-dimensional steady state flow is of the form

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial z^2} - w\frac{\partial C}{\partial z} + D_T \frac{\partial^2 C}{\partial x^2} - \lambda C \quad (1)$$

where V is the pure water velocity [LT^{-1}], 't' is time dependent [T], C is the concentration solution [ML^{-3}], D_L and D_T are longitudinal and transversion [L^2T^{-1}] coefficient, z and x are the particular points along the Cartesian coordinate axes which is perpendicular and parallel to the direction of ground water flow [L].

For groundwater flow, a linear relation b/w the dispersion coefficient and seepage velocity is generally adopted. If the aquifer is isotropic dispersion coefficient can be characterized by a transverse and longitudinal coefficient (Fig.1).

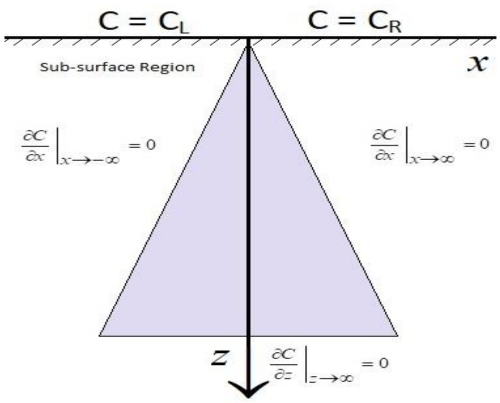

Figure 1. Physical layout of two-dimensional transport of contaminants in unsaturated porous media.

For this problem the initial and boundary conditions are as follows:

$$C(x,z,0) = f(x), 0 < z < \infty, -\infty < x < \infty \quad (2a)$$

$$\left.\frac{\partial C}{\partial z}\right|_{z\to\infty} = 0, \; -\infty < x < \infty, \; t > 0 \quad (2b)$$

$$\left.\begin{aligned} C(0,x,t) &= g(x) = C_L, x < 0, t > 0 \\ C(0,x,t) &= g(x) = \frac{C_L + C_R}{2}, x = 0, t > 0 \\ C(0,x,t) &= g(x) = C_R, x > 0, t > 0 \end{aligned}\right\} \quad (2c)$$

$$\left.\frac{\partial C}{\partial x}\right|_{x\to\pm\infty} = 0, \; 0 < z < \infty, \; t > 0 \quad (2d)$$

The initial condition (2a) is subjected to find the solution of equation (1) which can be obtained with integral transformation as, Fourier transformation for x variable and Laplace transformation for t variable. The solution is the combination of semi-infinite plane for steady state conditions considering one-dimensional advection-dispersion model and one-dimensional advection and two-dimensional dispersion model.

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The solution for the problem is solved first by using the well-known Laplace transforms applications for C and $\frac{\partial C}{\partial t}$ w.r.t to t the utilization of the initial condition (2a), for the transform equation (1) is transformed in the form

$$L\left\{C(x,z,t)\right\}=\int_0^\infty Exp(-pt)C(x,z,t)dt=C'(x,z,p) \tag{6.3}$$

$$L\left[\frac{\partial C}{\partial t}\right]=pC'(x,z,p)-C(x,z,0) \tag{3}$$

Applying the Laplace transforms to Eqns. (1) and (2), it transforms in the form

$$pC'(x,z,p)-C(x,z,0)=D_L\frac{\partial^2 C}{\partial z^2}-v\frac{\partial C}{\partial z}+D_T\frac{\partial^2 C}{\partial x^2}-\lambda C \tag{4}$$

$$pC'(x,z,p)-f=D_L\frac{\partial^2 C'}{\partial z^2}+D_T\frac{\partial^2 C'}{\partial x^2}-u_1\frac{\partial C'}{\partial x}-v_1\frac{\partial C'}{\partial z}-\lambda C' \tag{5}$$

and boundary conditions reduces to

$$\left.\frac{\partial C'}{\partial z}\right|_{z\to\infty}=0 \tag{6a}$$

$$C'(0,x,p)=\frac{g}{p} \tag{6b}$$

$$\left.\frac{\partial C'}{\partial x}\right|_{x\to\pm\infty}=0 \tag{6c}$$

For the infinite *x*-domain the Fourier transform is applied and for C' and $\frac{\partial^2 C'}{\partial t^2}$ the Fourier transform is given by

$$F\left[C'(z,x,p)\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Exp(-i\alpha x)C'(z,x,p)dx=C'(z,\alpha,p) \tag{7}$$

and

$$F\left[\frac{\partial^2 C'}{\partial x}\right]=-\alpha^2 C' \tag{8}$$

The Fourier transforms of Eqns. (5) and (6) becomes

$$D_L\frac{d^2\bar{C}'}{dz^2}-D_T\alpha^2\bar{C}'-u_1\frac{d\bar{C}'}{dx}-v_1\frac{d\bar{C}'}{dz}-\lambda\bar{C}'-p\bar{C}'(x,z,p)+f=0$$

$$D_L\frac{d^2\bar{C}'}{dz^2}-v_1\frac{d\bar{C}'}{dz}-u_1\frac{d\bar{C}'}{dx}-(D_T\alpha^2+p+\lambda)\bar{C}'+\bar{f}=0 \tag{9}$$

and the reduced boundary conditions are

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$$\left.\begin{array}{l}\left.\dfrac{d\bar{C}'}{dz}\right|_{z\to\infty}=0\\ \bar{C}'(0,x.p)=\dfrac{\bar{g}}{p}\\ \left.\dfrac{d\bar{C}'}{dx}\right|_{x\to\pm\infty}=0\end{array}\right\} \tag{10}$$

The Fourier transformation of f and g are $\bar{f}$ and $\bar{g}$.

The transport equation (9) which is an ordinary differential equation is subjected to the boundary conditions, then equation (10) is of the form

$$\bar{C}'(z,\alpha,p)=A\left[Exp(R_1z)\right]+B\left[Exp(-R_2z)\right]+\frac{\bar{f}}{D_T\alpha^2+\lambda+p} \tag{11}$$

where

$$R_1=\frac{v_1}{2D_L}+\frac{\sqrt{v_1^2+4D_L\left(D_T\alpha^2+\lambda+p\right)}}{2D_L} \tag{12a}$$

$$R_2=\frac{v_1}{2D_L}-\frac{\sqrt{v_1^2+4D_L\left(D_T\alpha^2+\lambda+p\right)}}{2D_L} \tag{12b}$$

It follows from the Eqn. (10) that $A=0$.Therefore we will use R instead of R_2. Applying the inner boundary conditions of Eqn.(10), we get

$$B=\frac{\bar{g}}{p}-\frac{\bar{f}}{D_T\alpha^2+\lambda+p} \tag{13}$$

Substituting Eqn (13) in (11), we get

$$\bar{C}'(z,\alpha,p)=\left[\frac{\bar{g}}{p}-\frac{\bar{f}}{D_T\alpha^2+\lambda+p}\right]Exp(Rz)+\frac{\bar{g}}{p}+\frac{\bar{f}}{D_T\alpha^2+\lambda+p} \tag{14}$$

Inverse Laplace transform has been applied for the given domain z,x,t in order to get the solution. This transport problem can also be solved using numerical technique but to solve using analytical method. The RHS of Eqn.(14) has divided into sum of three terms, the first integral term reduces to

$$\bar{C}'(z,\alpha,p)=L^{-1}\left[\frac{\bar{g}}{p}Exp(Rz)\right]$$

$$\bar{C}'(z,\alpha,p)=\bar{g}\,Exp\left(\frac{v_1z}{2D_L}\right)L^{-1}\left\{\frac{1}{p}Exp\left(-\frac{z}{\sqrt{D_L}}\left(\frac{v_1^2}{4D_L}+D_T\alpha^2+\lambda+p\right)^{1/2}\right)\right\} \tag{15}$$

From inverse Laplace transformation table [van Genuchten and Alves (1982)], by the application of convolution theorem it is preferable to solve the inverse Laplace transform than inverse Fourier transform. Let f and g be two continuous functions in the interval $[0,\infty]$, then convolution of f and g is denoted $f*g$ and defined by the integrals (G Zill and R cullin (2000))

$$f*g=\int_0^t f(\tau)g(t-\tau)d\tau$$

The convolution $f*g$ is a *f(t)*. The limitation of convolution is to find the Laplace transformation of two functions without evaluate the integral.

Let $h(t)$ and $k(t)$ be two functions and it's Laplace transformations are $h'(p)$ & $k'(p)$ then the convolution of these integrals are

$$L^{-1}\left[h'(p).k'(p)\right]=h*k$$

$$L^{-1}\{h'(p)k'(p)\}=\int_0^t h(\tau)k(t-\tau)d\tau=\int_0^t h(t-\tau)k(\tau)d\tau \tag{16}$$

Here τ is a variable. The difference in two functions in the Eqn.(15) is of the form

$$\left.\begin{array}{l} h'(p)=\dfrac{1}{p} \\ k(t)=Exp\left[-\dfrac{z}{\sqrt{D_L}}\left(\dfrac{v_1^2}{4D_L}+D_T\alpha^2+\lambda+p\right)^{1/2}\right] \end{array}\right\} \tag{17}$$

The values of $h(t)$ and $k(t)$ are calculate by using shift property and Laplace transformation is given by

$$\left.\begin{array}{l} h(t)=1 \\ k(t)=\dfrac{z}{t\sqrt{4\pi D_L}}Exp\left[-\left(\dfrac{v_1^2}{4D_L}+D_T\alpha^2+\lambda\right)t-\dfrac{z^2}{4D_L t}\right] \end{array}\right\} \tag{18}$$

Substitute (18) in (16) and subsequently into Eqn.(15) reduces to

$$\bar{C}'(z,\alpha,p)=\frac{\bar{g}z}{\sqrt{4\pi D_L}}\int_0^t \tau^{\frac{3}{2}}Exp\left(\frac{-(z-v_1\tau)^2}{4D_L\tau}\right)Exp\left[(D_T\alpha^2+\lambda)\tau\right]d\tau \tag{19}$$

By using Laplace transforms table, the Eqn.(19) may be transformed in the form of complimentary error function. For the integral of the error function is given by

$$erf(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-\eta^2}d\eta$$

The Complementary error function of $f(z)$ is given by

$$erfc(z)=1-erf(z)=\frac{2}{\sqrt{\pi}}\int_z^\infty e^{-\eta^2}d\eta$$

Now applying the complimentary error function to the expression (19), becomes

$$\bar{C}_1(z,\alpha,t)=\frac{\bar{g}}{2}Exp\left(\frac{v_1 z}{2D_L}\right)\left\{Exp\left[\frac{z}{\sqrt{D_L}}\left(\frac{v_1^2}{4D_L}+D_T\alpha^2+\lambda\right)^{1/2}\right]* erfc\left[\frac{z}{\sqrt{4D_LT}}+\sqrt{\frac{v_1^2 t}{4D_L}+D_T\alpha^2 t+\lambda t}\right]\right\}+\left\{Exp\left[\frac{-z}{\sqrt{D_L}}\left(\frac{v_1^2}{4D_L}+D_T\alpha^2+\lambda\right)^{1/2}\right]* erfc\left[\frac{z}{\sqrt{4D_LT}}-\sqrt{\frac{v_1^2 t}{4D_L}+D_T\alpha^2 t+\lambda t}\right]\right\} \quad (20)$$

Inverse Fourier transform of the Eqn.(20) is difficult than for Eqn.(14). The Fourier transforms variable displays in the argument of both error and exponential functions. Applying the inverse Fourier transform to the second term $\bar{C}_2(z,\alpha,p)$ of RHS of the Eqn. (14), we have

$$\bar{C}_2(z,\alpha,t)=L^{-1}\left[-\frac{\bar{f}}{D_T\alpha^2+p+\lambda}Exp(Rz)\right] \quad (21)$$

$$\bar{C}_2(z,\alpha,t)=\frac{\bar{f}}{2}Exp\left[-\left(D_T\alpha^2+\lambda\right)t\right]\left\{erfc\left(\frac{z-v_1t}{\sqrt{4D_Lt}}\right)+Exp\left(\frac{v_1z}{D_L}\right)\cdot erfc\left(\frac{z+v_1t}{\sqrt{4D_Lt}}\right)\right\} \quad (22)$$

Now applying inverse Laplace transform of the 3rdterm of the RHS of Eqn.(14) may be written the form [Oberhettinger and Baddi(1973)]

$$\bar{C}_3(z,\alpha,t)=L^{-1}\left[\frac{\bar{f}}{D_T\alpha^2+p+\lambda}\right]=\bar{f}\cdot Exp\left[-\left(D_T\alpha^2+\lambda\right)t\right] \quad (23)$$

The required Equation for $\bar{C}(z,\alpha,t)$ can be written as

$$\bar{C}(z,\alpha,t)=\bar{C}_1(z,\alpha,t)+\bar{C}_2(z,\alpha,t)+\bar{C}_3(z,\alpha,t) \quad (24)$$

The final step of the solution is in the other form, then the application of the Fourier inversion in terms of the Eqn. (24). The inverse Fourier transform of $\bar{C}(z,\alpha,t)$ is

$$C(z,x,t)=F^{-1}\left[\bar{C}(z,x,t)\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}Exp(-i\alpha x)\bar{C}(z,\alpha,t)\,d\alpha \quad (25)$$

It is left with the Fourier inversion of the 1stterm of the Eqn. (24) may be written as

$$F^{-1}\left[C_1(z,x,t)\right]=\frac{1}{\sqrt{2\pi}}\left\{\int_{-\infty}^{\infty}Exp(-i\alpha x)\frac{\bar{g}z}{\sqrt{4\pi D_L}}\cdot\int_0^t\tau^{-\frac{3}{2}}Exp\left(\frac{(z-v_1\tau)^2}{4D_L\tau}\right)Exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\right\}d\tau d\alpha$$

$$F^{-1}\left[C_1(z,x,t)\right]=\frac{z}{\sqrt{4\pi D_L}}\int_0^t\tau^{-\frac{3}{2}}Exp\left(-\frac{(z-v_1\tau)^2}{4D_L\tau}\right)* \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}Exp(-i\alpha x)\left\{\bar{g}Exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\right\}d\alpha d\tau \quad (26)$$

The convolution $h*k$ is to determine convolution of product of two functions $\bar{h}$ and $\bar{k}$ is

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$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\bar{h}(\alpha)\bar{k}(\alpha)Exp(-i\alpha x)d\alpha = h*k = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}h(x-\gamma)k(\gamma)d\gamma \tag{27}$$

Where γ is the fake variable, after verifying the last term for $C_1(z,x,t)$ in equation (26), using the convolution integral for the other two functions are

$$\left.\begin{aligned}\bar{h}(\alpha) &= \bar{g}\\ \bar{k}(\alpha) &= Exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\end{aligned}\right\} \tag{28}$$

The Fourier transform of the single step function we have $h(x)$ is equal to $g(x)$, but not determining $\bar{g}$ and to find the inverse transform of $\bar{k}(\alpha)$, we have

$$F^{-1}\left\{Exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\right\} = \frac{1}{\sqrt{2\left(D_T+\lambda\right)\tau}}Exp\left[-\frac{x^2}{4\left(D_T+\lambda\right)\tau}\right] \tag{29}$$

The resultant $h(x)$ and $k(x)$ are

$$\left.\begin{aligned}h(x) &= g(x) = C_L, \quad &: x<0\\ h(x) &= g(x) = \frac{(C_L+C_R)}{2}, &: x=0\\ h(x) &= g(x) = C_R, \quad &: x>0\end{aligned}\right] \tag{30}$$

and

$$k(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}k(\alpha)Exp(-i\alpha x)d\alpha = \frac{1}{\sqrt{2\left(D_T+\lambda\right)\tau}}Exp\left[-\frac{x^2}{4\left(D_T+\lambda\right)\tau}\right] \tag{31}$$

From Eqn. (27) is of the form

$$h*k = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{g}{\sqrt{2(D_T+\lambda)\tau}}Exp\left[-\frac{(x-\gamma)^2}{4(D_T+\lambda)\tau}\right]d\gamma \tag{32}$$

The above expression can be evaluated by

$$\rho = \frac{\gamma - x}{\sqrt{4(D_T+\lambda)\tau}}$$

The conditions of Eqn.(30) and the complimentary error function [Abramowiz and Stegan(1970 and Crank(1975))] converts to

$$h*k = \frac{C_L}{2}erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right] + \frac{C_R}{2}erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right] \tag{33}$$

The result is then substituted in (30) to get expression for $C_1(z,x,t)$, is

$$C_1\left(z,x,t\right)=\int_0^t h*k=\frac{z}{\sqrt{4\pi D_L}}\int_0^t \tau^{-\frac{3}{2}} Exp\left(-\frac{(z-v_1\tau)^2}{4D_L\tau}\right)* \left\{\frac{C_L}{2} erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right]+\frac{C_R}{2} erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right]\right\}d\tau \quad (34)$$

The inverse of the second term of the Eqn. (26), $C_2(z,x,t)$ is

$$C_2(z,\alpha,t)=-\frac{1}{2}\left\{erfc\left(\frac{z-\gamma t}{\sqrt{4D_L t}}\right)+Exp\left(\frac{\gamma z}{D_L}\right)\cdot erfc\left(\frac{z+\gamma t}{\sqrt{4D_L t}}\right)\right\}*F^{-1}\left[\bar{f}\, Exp\left[-\left(D_T+\lambda\right)t\right]\right] \quad (35)$$

Applying the convolution theorem, the inverse Fourier transform in Eqn. (35),

$$\left.\begin{aligned}\bar{h}\left(\alpha\right)&=\bar{f}\\ \bar{k}\left(\alpha\right)&=Exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\end{aligned}\right\} \quad (36)$$

Let us assume that the initial concentration C_i is constant and Eqn. (29) is utilized to find $k(x)$ may be written as

$$\left.\begin{aligned}h\left(x\right)&=C_i\\ k\left(x\right)&=\frac{1}{\sqrt{2(D_T+\lambda)t}} Exp\left[-\frac{x^2}{4(D_T+\lambda)t}\right]\end{aligned}\right\} \quad (37)$$

The inverse transformation of equation is carried out by using the properties or error function

$$F^{-1}\left[\bar{f}\, Exp\left[-\left(D_T\alpha^2+\lambda\right)t\right]\right]=h*k=\frac{C_i}{\sqrt{4\pi\left(D_T+\lambda\right)t}}\int_{-\infty}^{\infty} Exp\left[-\frac{(x-\gamma)^2}{\sqrt{4\pi\left(D_T+\lambda\right)t}}\right]d\gamma \quad (38)$$

Substitution of Eqn.(31) into Eqn.(28), then $C_2(z,x,t)$ reduces to

$$C_2(z,x,t)=-\frac{C_i}{2}\left\{erfc\left(\frac{z-\gamma t}{\sqrt{4D_L t}}\right)+Exp\left(\frac{\gamma z}{D_L}\right)\cdot erfc\left(\frac{z+\gamma t}{\sqrt{4D_L t}}\right)\right\} \quad (39)$$

The inverse Fourier transform of the last term of Eqn. (24) then $C_3(z,x,t)$ is evaluated in Eqn. (38) and we get

$$C_3(z,x,t)=F^{-1}\left[\bar{f}\cdot Exp\left[-\left(D_T\alpha^2+\lambda\right)t\right]\right]=C_i$$

$$C_3\left(z,x,t\right)=\frac{C_i}{\sqrt{4\pi(D_T+\lambda)t}}\int_{-\infty}^{\infty} Exp\left[-\frac{x-\gamma}{\sqrt{4\pi(D_T+\lambda)t}}\right]d\gamma \quad (40)$$

Substitution of $C_1(z,x,t)$, $C_2(z,x,t)$ and $C_3(z,x,t)$ in equation (24) and it reduces to

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$$C(z,x,t)=\frac{z}{\sqrt{4\pi D_L}}\int_0^t \tau^{-\frac{3}{2}}\left\{\frac{C_L}{2}erfc\left[\frac{x}{\sqrt{4(D_T+\left(\frac{1-n}{n}\right)\mathrm{K}_d)\tau}}\right]+\frac{C_R}{2}erfc\left[\frac{-x}{\sqrt{4(D_T+\left(\frac{1-n}{n}\right)\mathrm{K}_d)\tau}}\right]\right\}*$$

$$Exp\left[-\frac{(z-\gamma\tau)^2}{4D_L\tau}\right]d\tau--\frac{C_i}{2}\left\{erfc\left(\frac{z-\gamma t}{\sqrt{4D_L t}}\right)+Exp\left(\frac{\gamma z}{D_L}\right)\cdot erfc\left(\frac{z+\gamma t}{\sqrt{4D_L t}}\right)\right\}+C \qquad (41)$$

Various numerical methods can be used to evaluate Eqn.(41), among the suitable method of solution Gauss – Chebyshev quadrature. The transport of solute is linearly adsorbed using the rough layer that can be solved by splitting variables D_L, D_T and v with the factor of retardation. The solution may be different for initial and inlet conditions, like *f* and *g*.

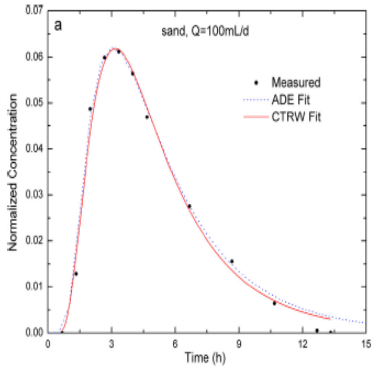

Figure 2. BTC curve with sand for Q = 100mL/d

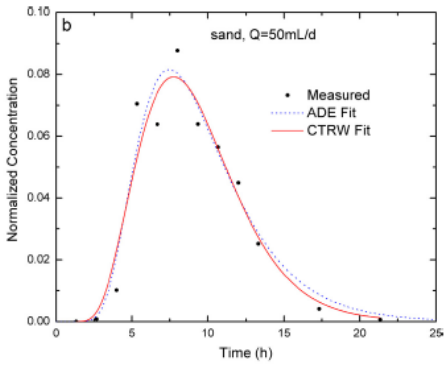

Figure 3. BTC curve with sand for Q = 50mL/d

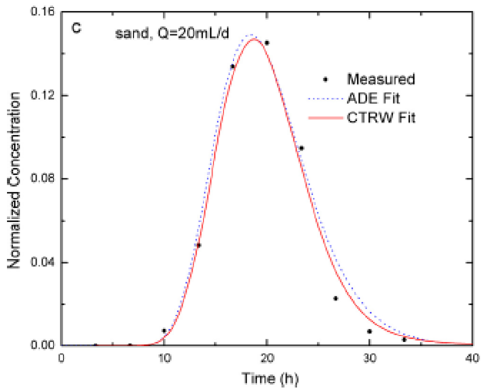

Figure 4. BTC curve with sand for Q = 20mL/d

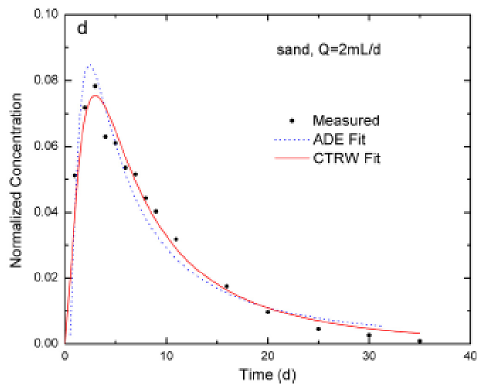

Figure 5. BTC curve with sand for Q = 2mL/d

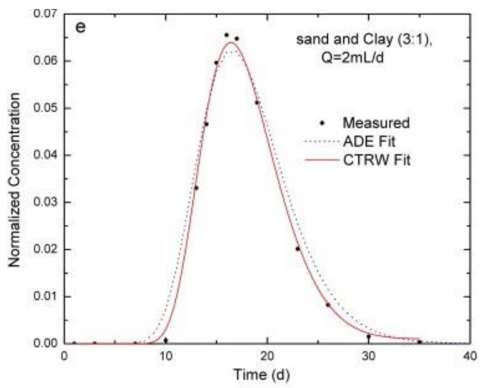

Figure 6. BTC curve with sand and clay (3:1) for Q = 20mL/d

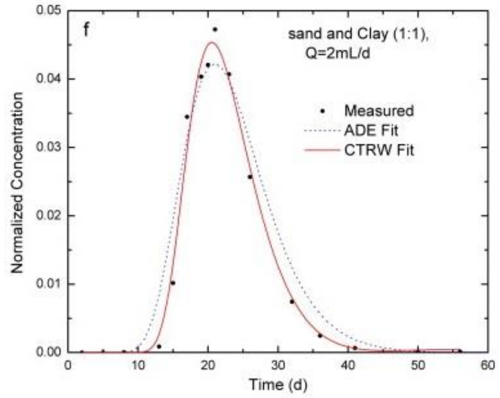

Figure 7. BTC curve with sand and clay (1:1) for Q = 2mL/d

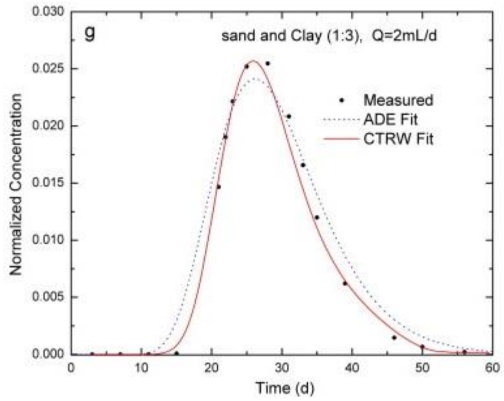

Figure 8. BTC curve sand and clay (1:3) for Q = 2mL/d

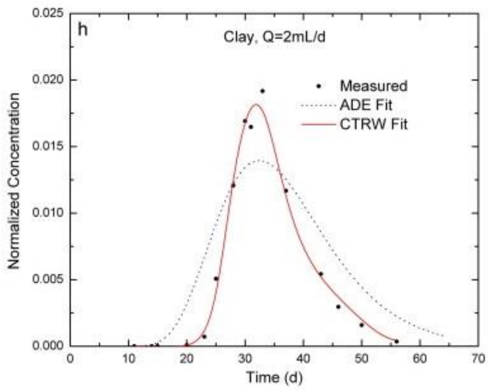

Figure 9. BTC curve with clay for Q = 2mL/d

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3 RESULTS AND CONCLUSIONS

The solution provided in Eqn. (41), wherein the mathematical integration of the first term is verified through comparing with various mathematical solutions for particular values of C_L, C_R, and C_i. First, the solution of stable condition Eqn.(1) is used below the idea that longitudinal dispersion D_L may be neglected. This solution could be obtained through using the Fourier transformation of *x* may be written as Harleman and Rumer (1963).

For the affirmation we assume that a permeable medium with the arbitrary transportation parameters are D_L=25cm^2/d, D_T=5cm^2/d, and v=50cm/d. These concentration parameters are expressed as dimensionless quantity C/Co with Co =1.

The primary concentrations are C_L=1, C_R=0, and C_1=0. Figures 2 to 9 shows that the solution for several times according to equation (41). The incursion of the solute into the medium at $x > 0$ and the consequent destruction of the solute in both directions may be observed clearly.

Figures 2 and 9 represent the Break-Through-Curves for C/C_0 verses time and intensity for specific depths *z* and *x* and *t* and *x*. It is appearing to be awareness location will boom within the starting and reaches constant nation price for consistent *z* and *x* but decreases with a growth within the radioactive decay coefficient 1. A growth in 1 will make the solute awareness lower as is evident from the bodily floor.

It should be noted that equation (41) can suitably use to determine D_T and D_L simultaneously. The awareness of these transport parameters is necessary for the prediction of two-dimensional solute transport.

In this paper we are capable of see that an analytical solution is received for the two-dimensional ADE for a semi-infinite region (half of plane) with one-dimensional waft using integral transforms. Even though the solution includes a fundamental expression, this essential can also want to effectively be evaluated using Gauss-Chebyshev quadrature.

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