A Transfer Function Model of Thrust Dynamics for Multi-Rotor Helicopters

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***Abstract*—This paper proposes a transfer function model for a dynamic thrust of a motor-driven propeller widely used for small multi-rotor helicopters. From frequency responses of a propeller speed with respect to a PWM (pulse width modulation) duty ratio of a motor speed controller, it is found that a thrust dynamics can be accurately modelled as a first-order transfer function. This frequency-domain result is compared to a time-domain method which is based on a step response of a thrust.**

Keywords—Thrust Dynamics, Transfer Function, Multi-rotor Helicopter

I. INTRODUCTION

Obtaining a reliable mathematical model of a thrust force is one of the most important and difficult steps in a controller synthesis for a multi-rotor helicopter.

There are fundamental difficulties in modeling a dynamic thrust of a propeller, as addressed in [1]. First, a commercial ESC (electronic speed controller) driving a BLDC (blushless direct current) motor itself is a microprocessor-based digital system and as such it has its own operational parameters but information on those parameters is usually undisclosed. Furthermore, with a custom firmware for an ESC, those parameters and other settings can be also modified. Second, an aerodynamic relation between a propeller speed and a thrust is highly nonlinear and complicated. Furthermore, depending on an operating condition, the relation can be significantly changed.

Because of the aforementioned difficulties, for a reliable characterization of a thrust force, it is generally unavoidable to depend on experimental procedures. This situation motivated us to develop a simple experimental method for a characterization of both a static and dynamic thrust in [2]. The proposed experimental procedure of [2] uses a load-cell type force sensor for a thrust measurement and an optical sensor for a speed measurement. The manual procedure performed in [2] for a static thrust is completely automatized with a microprocessor-based thrust measurement system in [3].

However an identification of a dynamic thrust is much harder compared to that of a static thrust from the following reason. A load-cell sensor for a thrust measurement uses a high-gain analog instrument amplifier. This high-gain amplification combined with a continual switching of large currents in a BLDC motor which needs to be installed near the load-cell (see Fig. 1), make a thrust sensor signal extremely vulnerable to electrical noises. This noise is significant and cannot be easily removed especially when a propeller speed changes abruptly. As a consequence of this, a precise measurement of a dynamic thrust is challenging in general.

Nevertheless, we have obtained a *rough* characterization of a dynamic thrust in [2], assuming that the dynamic relation between an ESC PWM command and a thrust can be described as a first-order transfer function. Parameters of the first-order transfer function were estimated from a step response of a thrust with respect to a step PWM command in [2].

However a fundamental limitation in the dynamic thrust model of [2] is that the first-order dynamic relation between a thrust and an ESC command is *presumed*, without a sufficient justification. In fact the same first-order model was also adopted in [4] for an example.

A key contribution of this paper is to experimentally substantiate the presumed first-order dynamics. To be concrete, we obtained a frequency response between an ESC command and a thrust force, and confirmed that the dynamic thrust can be rather precisely modelled as a first-order system. Furthermore, our identification method based on a frequency response was compared to the time-domain counterpart in [2].

II. DYAMIC MODELING OF THRUST

A. Experimental Setup

Fig.1 from [3] shows our thrust experiment system composed of an ESC, a BLDC motor, a load-cell for a thrust measurement and an optical sensor for a speed measurement. Components of our thrust measurement system in Fig.1 have the technical specifications in Table 1, cited from [1].

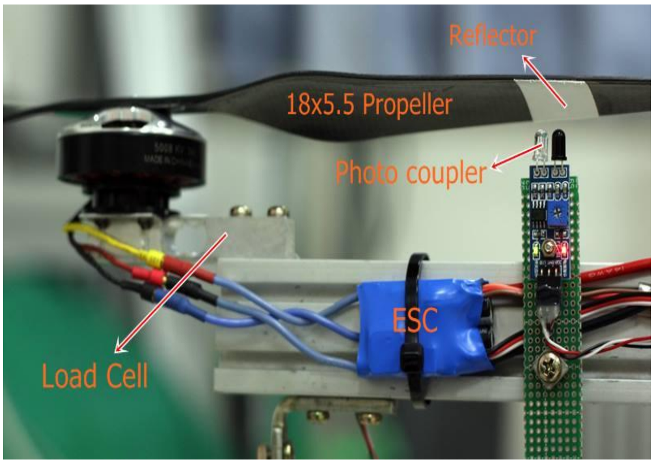

Figure 1 Sensor Configuration [3]

TABLE I. COMPONENTS SPECIFICATION [1]

BLDC Motor	Motor Outer Diameter	58.5 mm
	Stator Diameter	50.0 mm
	Speed per Volt	340 RPM /V
	Stator Number	12
	Motor Poles	14
	Weight	168 g
Propeller	Length	18 inches
	Pitch	5.5 inches
	Material	carbon fiber
	Blade Root Thickness	3.3 mm
Load Cell	Capacity	5 kg
	Resistance	1000 Ω
	Material	Aluminum
	Nonlinearity	0.05 %
ESC	Output (continuous)	40 A
Battery	Type	LiPo
	Capacity	10000 mAh
	Nominal Voltage	22.2 V
	Discharging Rate	25C

B. Time-Domain Method

In this section we will apply the time-domain identification method for a dynamic thrust proposed in [2], to our case study model.

Our ESC has a custom firmware provided by *BLheliSuite 14.2.0.1* [5]. This firmware allows us to use a 4 kHz PWM signal as a command signal. Details on this custom firmware and its driving signals can be found in [2].

From experiments, correlations between a propeller angular speed, a static thrust force and a PWM duty ratio were found to be as shown in Fig. 2-4. In addition, the step-response of a thrust with respect to step PWM command given in Fig. 5 is cited from [1]. The data with a label "*4kHz*" is to be used.

It is generally accepted that the relation between a propeller speed and a thrust is static. This static relation in our case could be found from a quadratic interpolation of the data in Fig. 4 as

$$T = 1.18 \times 10^{-6}\, \omega^2 \tag{1}$$

where T denote the thrust in Newton and ω denotes the speed in RPM unit.

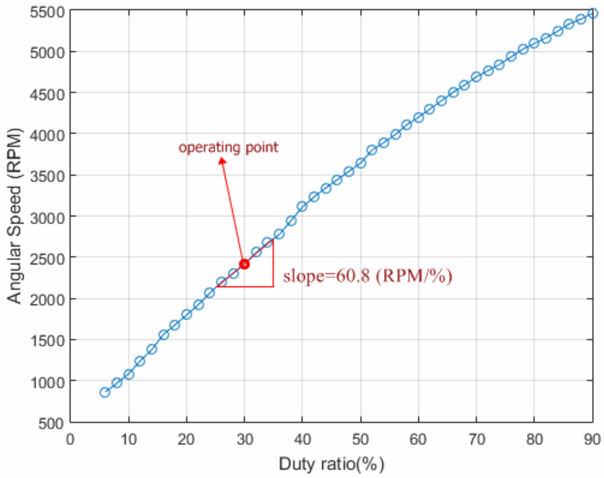

Figure 2 Speed versus Duty Ration

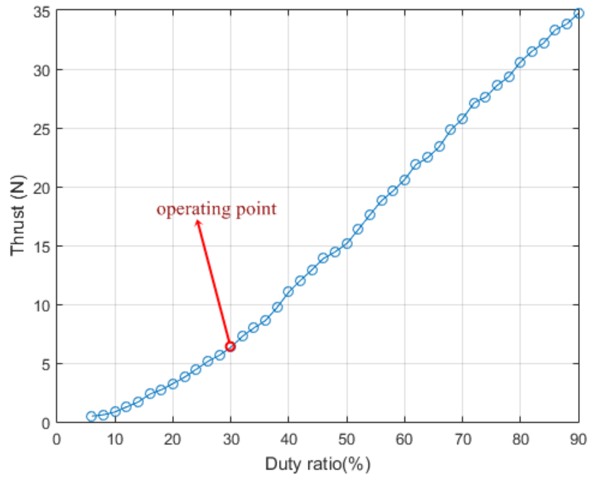

Figure 3 Thrust versus Duty Ratio

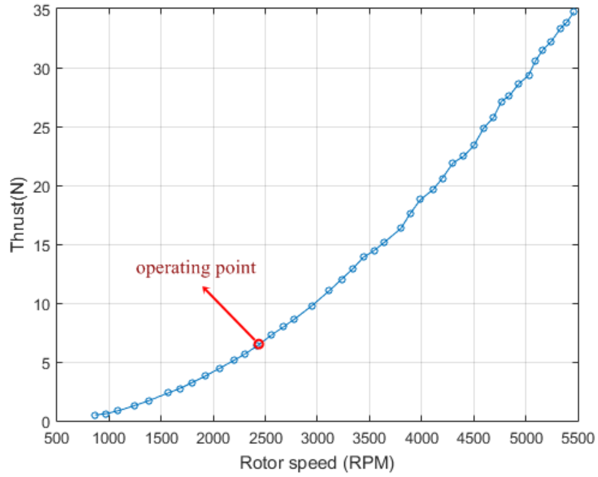

Figure 4 Thrust versus Speed

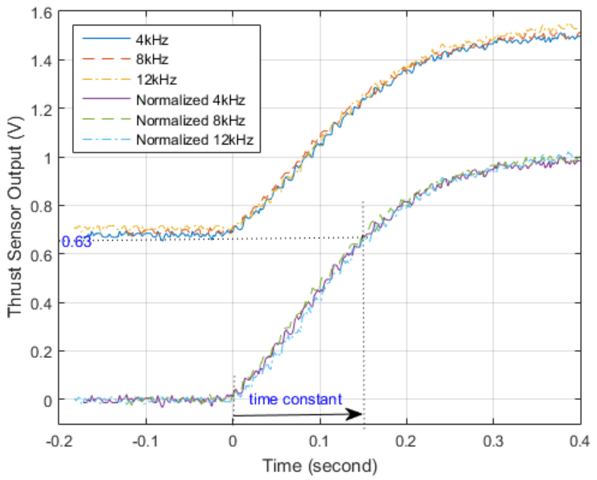

Figure 5 Thrust Step Response [1]

From the static relation (1), in principle, we have only to identify the dynamic response of either a thrust or a speed with respect to a PWM command. However the measurement of a speed is much more robust to electrical noises, compared to that of a thrust, and therefore we will investigate the dynamic property of a propeller speed first.

As a first step, we chose an operating point of the duty ratio of an ESC command input as $d_{nom} = 30$ %. Then from Fig. 2 the corresponding operating point of a propeller speed is around $\omega_{nom} = 2400$.

From a linearization

$$d = d_{nom} + \Delta d\,, \quad \omega = \omega_{nom} + \Delta\omega,$$

we need to identify two unknown parameters (k, τ) in the first-order transfer function

$$\frac{\Delta\omega(s)}{\Delta d(s)} = \frac{k}{s + 1/\tau} \qquad (2)$$

where $\Delta\omega(s), \Delta d(s)$ denote the Laplace transform of the input $\Delta d(t)$ and the output $\Delta\omega(t)$, respectively.

In (2), the unknown parameter τ can be determined from the step response shown in Fig. 5. Specifically, by reading a rising time 0.16 (sec) from Fig. 5, we obtain $\tau = 0.16$.

As a second step, for the unknown parameter k, we note that the DC (static) gain

$$\frac{\Delta\omega(0)}{\Delta d(0)} = \frac{k}{1/\tau} = k\tau$$

should be equal to the slop 60.8 (rpm/duty (%)) of a tangential line at $d_{nom} = 30$ % as illustrated in Fig. 2. This gives $k = \frac{60.8}{\tau} = 380.0$.

In summary, around an operating point $(d_{nom}, \omega_{nom}) = (30, 2400)$, the transfer function between a duty-ratio command and a propeller speed is given as

$$\frac{\Delta\omega(s)}{\Delta d(s)} = \frac{380.0}{s + 6.67} \left(\frac{RPM}{\%}\right) \qquad (3)$$

Note that at the operating point the thrust force (1) can be linearized as

$$\begin{aligned} T = T_{nom} + \Delta T &\cong \alpha\omega_{nom}^2 + 2\alpha\omega_{nom}\,\Delta\omega \\ &= 6.80 + 0.0057\,\Delta\omega \quad (\alpha = 1.18 \times 10^{-6}) \end{aligned} \qquad (4)$$

From this result, the transfer function between a duty command and a thrust at an operating point $(d_{nom}, T_{nom}) = (30, 6.80)$ is given

$$\frac{\Delta T(s)}{\Delta d(s)} = \frac{2.17}{s + 6.67} \left(\frac{N}{\%}\right). \qquad (5)$$

We note that in our experimental data in Fig. 4, the nominal thrust is around 6.34 (N) which is slightly different from the estimated value 6.80 in (4).

C. *Freqency Response Method*

In this section, without resorting to the assumption that the transfer function from a PWM duty-ratio command to a thrust is a first-order system, we measure the frequency response of a thrust and characterize its transfer function representation.

The frequency response of a thrust, with respect to a PWM duty-ratio, was obtained with a dynamic signal analyzer *Agilent 35670A* [6].

As our signal analyzer is an analog device, it was necessary to convert an analog input signal from the signal analyzer to a digital PWM signal for an ESC, and a digital pulse train from a photo sensor for a speed measurement into an analog output signal whose voltage is proportional to a propeller speed. In addition, in order to handle negative voltages from the analyzer, an operational amplifier circuit was designed for a voltage shift. Furthermore, the zero voltage of both input and output signals of the analyzer were mapped to an operating point in the previous section.

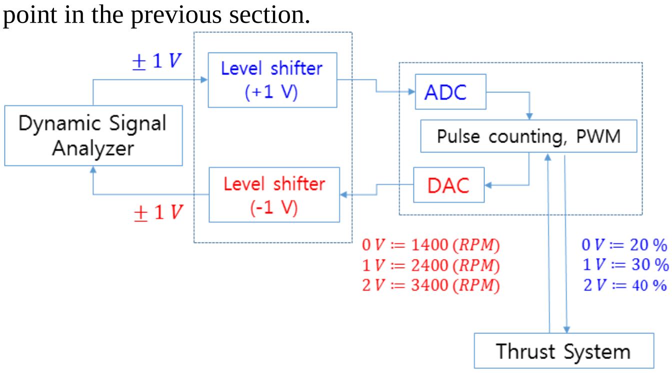

Figure 6 A Schematic Diagram

The analog-digital conversion, digital-analog conversion and the mapping between voltage signals and an operating point were implemented with a microprocessor (Arduino Due ©)-based thrust measurement board described in [3]. A schematic diagram of various signal conditioning of our experiment are illustrated in Fig. 5.

Note that in Fig. 5 the ADC has a DC gain $\frac{\Delta d}{u} = 10 \left(\frac{\%}{V}\right)$ and the DAC has a gain $\frac{y}{\Delta\omega} = \frac{1}{1000}\left(\frac{V}{RPM}\right)$ where $u(t)$ and $y(t)$ denote the output (input) and input (output) of the signal analyzer (thrust system, respectively). As a result, the relation

$$\frac{\Delta\omega(s)}{\Delta d(s)} = 100\,\frac{\mathrm{Y}(s)}{\mathrm{U}(s)} \qquad (6)$$

holds where the transfer function $\frac{\mathrm{Y}(s)}{\mathrm{U}(s)}$ corresponds to a frequency response that the signal analyzer will estimate.

The signal analyzer under the configuration of Fig. 6 gave the frequency response in Fig. 7. The frequency span was [0.1,10] (Hz) and frequency responses at 401 different frequencies were measured with a swiping sinusoidal signal.

A critical implication from both the magnitude and phase plots in Fig. 7 is that, qualitatively, the transfer function between a PWM command and a propeller speed (and a thrust, too) is a typical first-order transfer function.

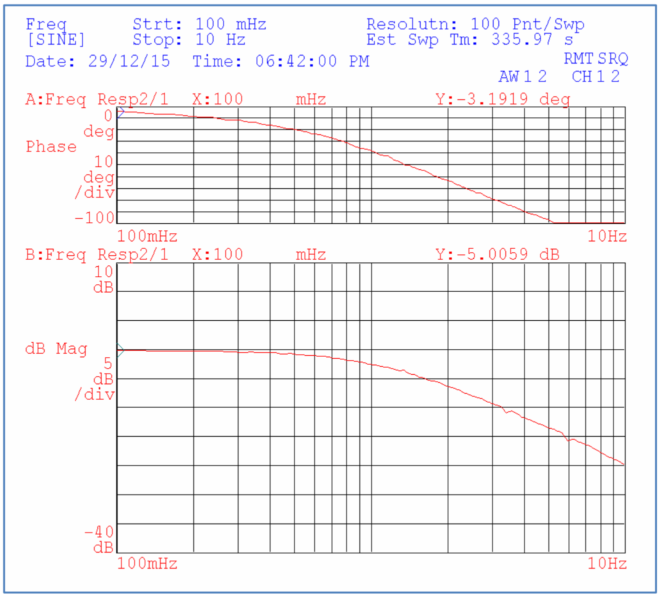

Figure 7 Speed Frequency Response

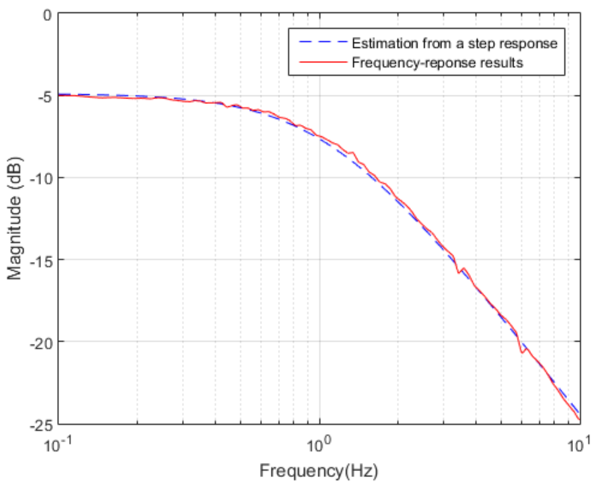

Figure 8 Comparison of Frequency Responses

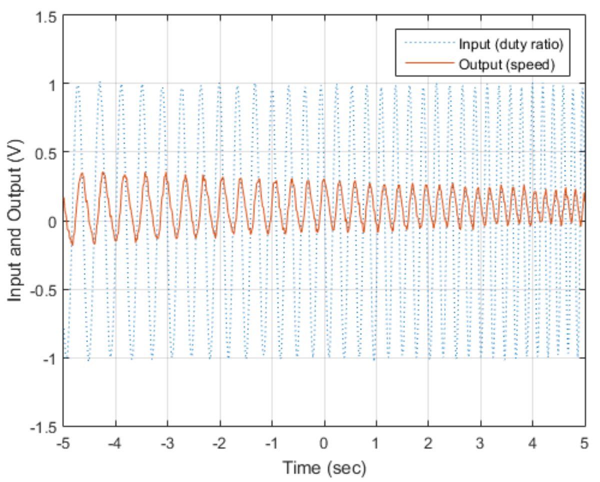

Figure 9 Input and Output Signals of Analyzer

This frequency-domain observation justifies our previous assumption of the first-order transfer function.

For a quantitative analysis of the frequency response data, we made a comparison between the transfer function in (3) which is based on a step-response of a thrust, and the frequency response data. To be concrete, in Fig. 8, the dashed blue line is the Bode plot of the transfer function $\frac{1}{100}\frac{\Delta\omega(s)}{\Delta d(s)}$ and the solid red line is the experimental data in Fig. 7. The scaling factor $\frac{1}{100}$ comes from the relation (6).

The two frequency responses in Fig. 8 show a surprising agreement in overall, even though our choice of the rising time $\tau = 0.16$ (sec) from Fig. 5 was not precise. This result also suggests that the time-domain approach in [2] for a thrust transfer function is reliable.

Finally, Fig. 9 shows an example of input-output signals of the signal analyzer during a frequency response experiment. The frequency span in this figure is around $[2.5, 5.5]$ (Hz).

III. CONCLUSION

A frequency response of a dynamic thrust with respect to a PWM duty-ratio command for an electronic speed controller was obtained with a dynamic signal analyzer. It was found that the transfer function between a PWM command and a thrust can be precisely modelled as a first-order transfer function. This transfer function gives a good agreement with another transfer function estimated from a step response of a thrust.

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