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A Three-Period Dynamic Programming Model for Optimizing Economic Order Quantity Under Stochastic Demand

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***Abstract*:- This paper demonstrates an approach to determine the Economic order quantity (EOQ) of an item under a periodic review inventory system with stochastic demand. The objective is to determine in each period of the planning horizon, an optimal EOQ so that the long run profits are maximized for a given state of demands. Using a three-period dynamic programming planning horizon with equal intervals, the decision of how much quantity to order or not to order is made. We use a numerical example to demonstrate the existence of an optimal state, determining the EOQ as well as calculating the corresponding profits.**

Keywords: Dynamic programming, EOQ, Markov chain, stochastic process.

1.0 INTRODUCTION

The EOQ model is one of the classical inventories and production scheduling models. It is a model that is used to calculate the optimal quantity that can be purchased in order to minimize the cost of both carrying the inventory and the processing of purchased orders. The EOQ model assumes that demand is constant, and that inventory is depleted at a fixed rate until it reaches zero. A specific number of items which is the EOQ arrives after a constant time, to return the inventory to its beginning level. Since the model assumes instantaneous replenishment, there are no inventory shortages or associated costs. Ordering a large amount at one time will increase the holding cost while making more frequent orders of fewer items will reduce holding costs but increase order costs. The EOQ model finds the quantity that minimizes the sum of these costs. The demand for a product in inventory is the number of units that will need to be withdrawn from inventory for some use (e.g., sales) during a specific period. If the demand in future periods can be forecast with considerable precision, it is reasonable to use an inventory policy that assumes that all forecasts will always be completely accurate. This is the case of known demand where a deterministic inventory model would be used. However, when demand cannot be predicted very well, it becomes necessary to use a stochastic inventory model where the demand in any period is a random variable rather than a known constant. The expected total rewards from the present stage until the end of the planning horizon is expressed by a value function which can be obtained through dynamic programming.

The EOQ model appeared in Harris (1913) describing a very simple deterministic inventory planning model with a tradeoff between fixed ordering cost and inventory carrying cost (Drake and Marley 2014). The EOQ lays the foundation for all kinds of extensions and real-world management applications of EOQ (Axsater 1996) Both deterministic and stochastic EOQ models were developed in (Pentico and Drake 2011). The Economic order quantity problem has been investigated by many researchers during the past decades. Most of the models presented were developed in the deterministic environment. Pentico and Drake (2011) and Khan et al. (2011) reviewed deterministic economic order quantity models. However, some of the researchers considered variations in the real-world situations and presented inventory models in the uncertain environments. These uncertain EOQ models can be classified into three general categories: fuzzy EOQ models, stochastic EOQ models, and hybrid EOQ models. This paper is concerned with the stochastic EOQ models. Eynan and Kropp (2007) presented a periodic review system under stochastic demand with variable stock out costs. Hayya *et al*. (2009) considered demand and lead time as random variables and formulated a stochastic model to obtain optimum values of reorder point and order quantity. Also, a stochastic economic order quantity model over a finite time horizon was presented by Sana (2011) in which the customer demand was assumed to be stochastic with predetermined probability distribution function. In the model, replenishment period was considered to be price dependent and selling price was assumed to be a random variable that follows a probability density function. Wang (2010) introduced a solution approach to obtain optimal values of the order quantity and the reorder point when the supplier capacity and the lead time demand are probabilistic. Yan and Kulkarni (2008) considered a single stage production-inventory system in which production and demand rates are stochastic with predetermined probability distribution function. De and Goswami (2009) presented a probabilistic inventory model for items that deteriorate at a constant rate and the demand is a random variable. Lee and Wu (2002) considered the EOQ model for inventory of an item whose deterioration follows a Weibull distribution.

Dynamic programming is a tool we use in this study and among authors who used dynamic programming to solve inventory problems come the following. Simpson (1978) generated the optimal policy structure for a finite-horizon repairable inventory system with two stocking points using a backward dynamic programming technique. The optimal policy structure, which was defined by three period dependent parameters, namely repair-up-to level, purchase-up-to level, and scrap-down to level, is valid under 0-lead time assumption for repairing and purchasing activities. Inderfurth (1997), addressed the problem of inventory optimization for a recoverable system with and without stock keeping of returned items. Inderfurth used the method of stochastic

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dynamic programming to derive optimal decision rules for procurement, remanufacturing and disposal. Inderfurth explored the impact of procurement and remanufacturing lead times to the complexity of the optimal decision rules and showed that up to a certain extent, simple rules with only a few parameters can be proved to be optimal for the stochastic recovery problem. Bai *et al*. (2016) proposed a Markov decision model to examine the desirable sizes and policies of a strategic petroleum reserve (SPR) for oil consumption countries. Oil consumers operate SPRs to cope with various market uncertainties include oil supply, oil price and disruption situations in which oil supply is highly stochastic. The decision criterion is to minimize total disruption losses and SPR costs. The output of the proposed model finds optimal SPR acquisition, drawdown and refill policies in response to different market states. Mubiru *et al*. (2017) formulated a finite state Markov decision process model where states of a Markov chain represent possible states of demand for milk powder product. The unit replenishment cost, shortage cost, demand and inventory positions are used to generate the total inventory cost matrix representing the long run measure of performance for the Markov decision process problem. The problem is to determine for each supermarket at a specific location an optimal replenishment policy so that the long run inventory costs are minimized for the given state of demand. The decisions of replenishing versus not replenishing at a given location are made using dynamic programming over e finite period planning horizon. Onggo *et al*. (2019) considered an example of a complex supply chain operation that can be viewed as an inventory routing problem with stochastic demand. They demonstrated how a simheuristic framework can be employed to solve the problem. They also illustrated the risk of not considering input uncertainty. The results showed that simheuristics can produce a good result and ignoring the uncertainty in the model, input may lead to sun optimal results. Mubiru and Ssempijja (2018) formulated a finite state Markov decision process model where states of a Markov chain represent possible states of support among voters. Using daily equal intervals, the candidate's decision of whether or not to campaign and hold a political rally at a given location were made using discrete time Markov chains and dynamic programming over a finite period planning horizon. Mubiru (2019) developed a finite state Markov decision process model where states of a Markov chain represent possible states of demand for internet service. A revenue matrix is generated representing the long run measure of performance for the Markov decision process problem. The problem is to determine an optimal bandwidth adjustment policy at cybercafés so that the long run revenue generated is maximized for a given state of demand. Using dynamic programming, the optimal bandwidth adjustment policies are determined over a finite period planning horizon. Mubiru (2010) developed an optimization model for determining the economic production lot size which minimizes production and inventory costs of a periodic review production-inventory system under stochastic demand. Mubiru (2013), developed an optimization model for determining the EOQ that minimizes inventory costs of multiple items under a periodic review system with stochastic demand. The model demonstrated ordering decision using dynamic programming to establish the existence of optimal ordering quantity and ordering policies. Mubiru (2015) considered the inventory of a stochastic system whose aim was to optimize the order quantity and profits associated with ordering and holding cost of the item. The decision of whether to order additional units of the item or not to order was modelled based on two demand states that is favorable and unfavorable states using dynamic programming. Hassan and Sani (2020) modified the work of Mubiru (2015) by changing the demand transition and the profit matrices so as to reflect what is actually happening in reality.

In this paper, we develop a model for stock replenishment under a periodic review inventory system with stochastic demand using a three-period dynamic programming technique. The aim is to determine an optimal economic order quantity so that after three periods, profits are maximized for a given state of demand. Using equal intervals, the decision of how much quantity to order or not to order is made over the three-period planning horizon. A numerical example demonstrates the existence of an optimal decision policy.

2.0 MODEL PARAMETERS

i_n, j_n = States of demand for n stage

n, N = Stages going from n=1 to N=3

f = Favorable state

u = Unfavorable state

z = Ordering policy

D^z = Demand matrix for z policy

I^z = Inventory matrix for z policy

P_{i_n, j_n} = Profit in state i_n, j_n

Q^z = Demand transition matrix for z policy

C_p = Cost price per unit

C_s = Shortage cost per unit

C_o = Ordering cost per unit

C_h = Holding cost per unit

$O^z_{i_n}$ = Economic order quantity

$e^z_{i_n}$ = Expected total profits

P_s = Selling price per unit

P^z = Profit matrix for z policy

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$Q^{z}_{i_n, j_n}$ = Demand transition probability in state i_n, j_n for z policy

$I^{z}_{i_n, j_n}$ = Quantity in inventory in state i_n, j_n for z policy

$D^{z}_{i_n, j_n}$ = Quantity demanded in state i_n, j_n for z policy

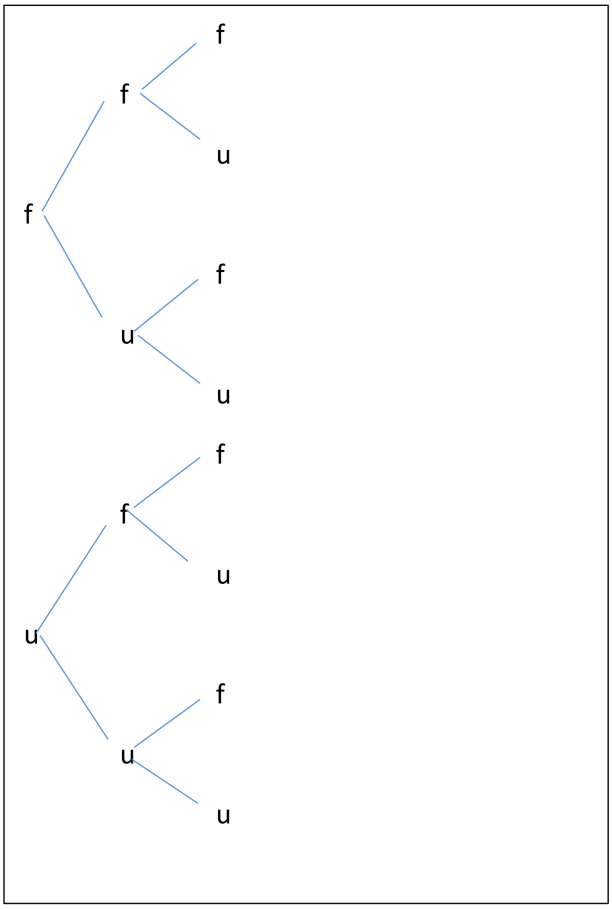
$a^{z}_{i_n}$ = Accumulated total profits at the end of the second period

$b^{z}_{i_n}$ = Accumulated total profits at the end of the third period

$g_n(i_n, p)$ = Optimal expected total inventory cost

3.0 MODEL DEVELOPMENT

We develop an inventory system of an item in which the demand is classified into favorable state (f) and unfavorable state (u). A three-period planning horizon is considered, the transition probabilities over the planning horizon are assumed to follow a Markov process. An optimal course of action is obtained between a decision to order additional units (z=1) or not to order additional units denoted by (z=0). This decision is to be made during each period over the three-period planning horizon, where z is the decision variable. The maximum expected profits are summed up together at the end of the period so as to obtain optimality. The tree diagram for the system is as follows:



Key:
f = favorable state
u = unfavorable state

Figure 1: The tree diagram for the three-period planning horizon

With the scenario above, the change in demand is modeled as a Markov Chain with the following possibilities:

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$$Q^z_{i_n,j_n} \Leftrightarrow \begin{bmatrix} Q^z{}_{fff} & Q^z{}_{ffu} & Q^z{}_{fuf} & Q^z{}_{fuu} \\ Q^z{}_{uff} & Q^z{}_{ufu} & Q^z{}_{uuf} & Q^z{}_{uuu} \end{bmatrix}$$

$$D^z_{i_n,j_n} \Leftrightarrow \begin{bmatrix} D^z{}_{fff} & D^z{}_{ffu} & D^z{}_{fuf} & D^z{}_{fuu} \\ D^z{}_{uff} & D^z{}_{ufu} & D^z{}_{uuf} & D^z{}_{uuu} \end{bmatrix}$$

$$I^z_{i_n,j_n} \Leftrightarrow \begin{bmatrix} I^z{}_{fff} & I^z{}_{ffu} & I^z{}_{fuf} & I^z{}_{fuu} \\ I^z{}_{uff} & I_{ufu} & I^z{}_{uuf} & I^z{}_{uuu} \end{bmatrix}$$

$$P^z_{i_n,j_n} \Leftrightarrow \begin{bmatrix} P^z{}_{fff} & P^z{}_{ffu} & P^z{}_{fuf} & P^z{}_{fuu} \\ P^z{}_{uff} & P_{ufu} & P^z{}_{uuf} & P^z{}_{uuu} \end{bmatrix}$$

These are converted to multiple transition matrices as follows:

$$Q^z_{i_n,j_n} = \begin{bmatrix} Q^z_{ff} & Q^z_{fu} \\ Q^z_{uf} & Q^z_{uu} \end{bmatrix}\begin{bmatrix} Q^z_{ff} & Q^z_{fu} \\ Q^z_{uf} & Q^z_{uu} \end{bmatrix}\begin{bmatrix} Q^z_{ff} & Q^z_{fu} \\ Q^z_{uf} & Q^z_{uu} \end{bmatrix} = \begin{bmatrix} Q^z_{FF} & Q^z_{FU} \\ Q^z_{UF} & Q^z_{UU} \end{bmatrix} \text{ say}$$

$$D^z_{i_n,j_n} = \begin{bmatrix} D^z_{ff} & D^z_{fu} \\ D^z_{uf} & D^z_{uu} \end{bmatrix}\begin{bmatrix} D^z_{ff} & D^z_{fu} \\ D^z_{uf} & D^z_{uu} \end{bmatrix}\begin{bmatrix} D^z_{ff} & D^z_{fu} \\ D^z_{uf} & D^z_{uu} \end{bmatrix} = \begin{bmatrix} D^z_{FF} & D^z_{FU} \\ D^z_{UF} & D^z_{UU} \end{bmatrix} \text{ say}$$

$$I^z_{i_n,j_n} = \begin{bmatrix} I^z_{ff} & I^z_{fu} \\ I^z_{uf} & I^z_{uu} \end{bmatrix}\begin{bmatrix} I^z_{ff} & I^z_{fu} \\ I^z_{uf} & I^z_{uu} \end{bmatrix}\begin{bmatrix} I^z_{ff} & I^z_{fu} \\ I^z_{uf} & I^z_{uu} \end{bmatrix} = \begin{bmatrix} I^z_{FF} & I^z_{FU} \\ I^z_{UF} & I^z_{UU} \end{bmatrix} \text{ say}$$

$$P^z_{i_n,j_n} = \begin{bmatrix} P^z_{ff} & P^z_{fu} \\ P^z_{uf} & P^z_{uu} \end{bmatrix}\begin{bmatrix} P^z_{ff} & P^z_{fu} \\ P^z_{uf} & P^z_{uu} \end{bmatrix}\begin{bmatrix} P^z_{ff} & P^z_{fu} \\ P^z_{uf} & P^z_{uu} \end{bmatrix} = \begin{bmatrix} P^z_{FF} & P^z_{FU} \\ P^z_{UF} & P^z_{UU} \end{bmatrix} \text{ say}$$

In carrying out the work, we apply the modification in Hassan and Sani (2020) done to the work of Mubiru (2015).

4.0 DYNAMIC PROGRAMMING FORMULATION

The demand for the item can either be in favorable state (f) or unfavorable state (u) depending on the market situation. To obtain an optimal EOQ, we can express the problem as a three-period dynamic programming model.

We denote $g_n(i_n)$ as the expected total profit accumulated during the n periods for $n \le 3$ given that the state of the system at the beginning of period n is $i_n \in \{f, u\}$

The recursive equation relating g_n and g_{n+1} is as follows

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$$g_n(i_n) = \max_z\{Q^z_{i_n f}(P^z_{i_n f} + g_{n+1}(f), Q^z_{i_n u}(P^z_{i_n u} + g_{n+1}(u))\} \quad (1)$$

$$z \in \{0,1\}$$

$$i_n \in \{f, u\}$$

$$n = 1,2,3$$

for $$g_4(f) = g_4(u) = 0 \quad (2)$$

The recursive relationship can be justified by taking note of the following cumulative total profit

$$P^z_{i_n, j_n} + g_{n+1}(j_n) \quad (3)$$

which results from getting to state $j_n \in \{f, u\}$ at the start of period n+1 from state $i_n \in \{f, u\}$ at the beginning of period n occuring with the probability $Q^z_{i_n, j_n}$.

which shows that $e^z_{i_n} = Q^z (P^z)^T$ (4)

Then dynamic recursive equation becomes

$$g_n(i_n) = \max_z\{e^z_{i_n} + Q^z_{i_n f} g_{n+1}(f) + Q^z_{i_n u} g_{n+1}(u)\} \quad (5)$$

or $$g_N(i_n) = \max_z \left[e^z_{i_n} \right] \quad (6)$$

5.0 THE COMPUTATION OF DEMAND TRANSITION MATRIX, PROFIT MATRIX AND EOQ

For ordering policy, $z \in \{0,1\}$, the demand transition probability from state i_n *to* j_n can be expressed as the amount of quantity demanded when the demand is initially in state i_n and then changing to state j_n divided by the total number of quantity demanded over all states. This can therefore be given by the following equation

$$Q^z_{i_n, j_n} = \frac{D^z_{i_n, j_n}}{[D^z_{i_n f} + D^z_{i_n u}]} \quad (7)$$

$$i_n, j_n \in \{f, u\}$$

$$z \in \{0,1\}$$

If the demand is greater than the inventory at hand, the profit matrix P^z can be computed by the following equations:

$$P_{i_n, j_n} = P_s - C_p \quad (8)$$

and

$$P^z = P_{i_n, j_n}(D^z) - (C_o + C_h) I^z - (C_o + C_s)[D^z - I^z] \quad (9)$$

On the other hand, if the demand is less than or equal to the inventory at hand, then

$$P^z = P_{i_n, j_n} D^z_{i_n, j_n} - (C_o + C_h) D^z_{i_n, j_n} \quad (10)$$

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$$\Rightarrow P^z = \begin{cases} P_{i_n,j_n} D^z_{i_n,j_n} - (C_o + C_h) I^z_{i_n,j_n} - (C_o + C_s)[D^z_{i_n,j_n} - I^z_{i_n,j_n}] & if \;\; D^z_{i_n,j_n} > I^z_{i_n,j_n} \\ P_{i_n,j_n} D^z_{i_n,j_n} - (C_o + C_h) D^z_{i_n,j_n} \quad if \;\; D^z_{i_n,j_n} \le I^z_{i_n,j_n} \end{cases} \quad (11)$$

$$\forall \;\; i_n, j_n \in \{f, u\} \;,$$

$$z \in \{0,1,\}$$

For the first part of equation (11) to be justified, we note that $D^z_{i_n,j_n} - I^z_{i_n,j_n}$ units must be ordered immediately to meet the excess demand. The units ordered should attract shortage cost but not holding cost since they will not be stored. On the other hand, when demand is less than or equal to on hand inventory, no order will be placed. In that case there will be no shortage cost but holding cost since the units are in store. The economic order quantity when demand is in state $i_n \in \{f, u\}$ with ordering policy $z \in \{0,1\}$ is

$$O^z_{i_n} = (D^z_{i_n f} - I^z_{i_n f}) + (D^z_{i_n u} - I^z_{i_n u}) \quad \text{provided } D^z_{i_n,j_n} > I^z_{i_n,j_n} \quad (12)$$

$$i_n, j_n \in \{f, u\}, \; z \in \{0,1\}$$

Otherwise, $O^z_{i_n} = 0$

6.0 COMPUTING THE ECONOMIC ORDER QUANTITY FOR THE TWO PLANNING HORIZONS

We consider a three-period (N=3) planning horizon in this paper.

6.1 Optimization during period 1 (First planning horizon)

The ordering policy during period 1 when demand is favorable

$$z = \begin{cases} 1, & if \; e^1_f > e^0_f \\ 0, & if \; e^1_f \le e^0_f \end{cases} \quad (13)$$

The associated total profit and EOQ are

$$g_1(f) = \begin{cases} e^1_f, \; if \; z = 1 \\ e^0_f, \; if \; z = 0 \end{cases} \quad (14)$$

and $$O^z_f = \begin{cases} (D^1_{ff} - I^1_{ff}) + (D^1_{fu} - I^1_{fu}); \;\; if \; z = 1 \\ 0; \;\; if \; z = 0 \end{cases} \quad (15)$$

with the proviso in (12)

The ordering policy during period 1 when demand is unfavorable

$$z = \begin{cases} 1, \;\; if \; e^1_u > e^0_u \\ 0, \; if \; e^1_u \le e^0_u \end{cases} \quad (16)$$

The associated total profit and EOQ are

$$g_1(u) = \begin{cases} e^1_u, \; if \; z = 1 \\ e^0_u, \; if \; z = 0 \end{cases} \quad (17)$$

and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{18}$$

with the proviso in (12)

6.2 Optimization during period 2 (Second planning horizon)

Recalling that $a_{i_n}^z$ represent the accumulated profits at the end of period 1 as a result of decisions made using recursive equation (1), it follows that:

$$a_{i_n}^z = e_{i_n}^z + Q_{i_n f}^z g_1(f) + Q_{i_n u}^z g_1(u) \tag{19}$$

Therefore, the ordering policy when the demand is favorable is

$$z = \begin{cases} 1, & if\ a_f^1 > a_f^0 \\ 0, & if\ a_f^1 \le a_f^0 \end{cases} \tag{20}$$

The associated total profits and EOQ are

$$g_2(f) = \begin{cases} a_f^1, & if\ z = 1 \\ a_f^0, & if\ z = 0 \end{cases} \tag{21}$$

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{22}$$

With the proviso in (12)

Also, the ordering policy when demand is unfavorable is

$$z = \begin{cases} 1, & if\ a_u^1 > a_u^0 \\ 0, & if\ a_u^1 \le a_u^0 \end{cases} \tag{23}$$

The associated total profits and EOQ are

$$g_2(u) = \begin{cases} a_u^1, & if\ z = 1 \\ a_u^0, & if\ z = 0 \end{cases} \tag{24}$$

and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{25}$$

with the proviso in (12)

6.3 Optimization during period 3 (Third planning horizon)

The accumulated profits at the end of period 2 as a result of decisions made using recursive equation (1), is as follows:

$$b_{i_n}^z = a_{i_n}^z + Q_{i_n f}^z g_2(f) + Q_{i_n u}^z g_2(u) \tag{26}$$

Therefore, the ordering policy when the demand is favorable is

$$z = \begin{cases} 1, & if\ b_f^1 > b_f^0 \\ 0, & if\ b_f^1 \le b_f^0 \end{cases} \tag{27}$$

The associated total profits and EOQ are

$$g_3(f) = \begin{cases} b_f^1, & if\ z = 1 \\ b_f^0, & if\ z = 0 \end{cases} \tag{28}$$

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and
$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); \ if\ z = 1 \\ 0; \ if\ z = 0 \end{cases} \quad (29)$$

With the proviso in (12)

Also, the ordering policy when demand is unfavorable is

$$z = \begin{cases} 1, \ if\ b_u^1 > b_u^0 \\ 0, \ if\ b_u^1 \le b_u^0 \end{cases} \quad (30)$$

The associated total profits and EOQ are

$$g_3(u) = \begin{cases} b_u^1, \ if\ z = 1 \\ b_u^0, \ if\ z = 0 \end{cases} \quad (31)$$

and
$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); \ if\ z = 1 \\ 0; \ if\ z = 0 \end{cases} \quad (32)$$

with the proviso in (12)

7.0 NUMERICAL EXAMPLE

We consider a sample of customers with the following demand patterns and inventory levels over state transitions collected in past 30 weeks in respect of favorable and unfavorable demand of a particular item. For ordering policy z=0, the data is in Table 1 and for ordering policy z=1, the data is in Table 2 below:

Table 1: demand and inventory at transitions for ordering policy z=1

State transition (i_n, j_n)	Demand $D_{i_n j_n}^1$	Inventory $I_{i_n j_n}^1$
ff	20	10
fu	15	10
uf	14	11
uu	10	05

Table 2: demand and inventory at state transitions for ordering policy z=0

State transition (i_n, j_n)	Demand $D_{i_n j_n}^0$	Inventory $I_{i_n j_n}^0$
ff	20	15
fu	09	06
uf	07	05
uu	15	10

We can break the above tables into the following matrices:
When additional units are ordered, z=1

$$D^1 = \begin{pmatrix} 20 & 15 \\ 14 & 10 \end{pmatrix}$$

$$I^1 = \begin{pmatrix} 10 & 10 \\ 11 & 05 \end{pmatrix}$$

When additional units are not ordered, z=0

$$D^0 = \begin{pmatrix} 20 & 09 \\ 07 & 15 \end{pmatrix}$$

$$I^0 = \begin{pmatrix} 15 & 06 \\ 05 & 10 \end{pmatrix}$$

The above matrices need to be expressed as 3-multiple transition matrices because the problem involves a three-period planning horizon. Note however that for a two-period planning horizon, there is only one transition so no need for any multiple transition program. The transition matrices for the three-period are therefore as follows:

When additional units are ordered, z=1

$$D_F^1 = \begin{bmatrix} 20 & 15 \\ 14 & 10 \end{bmatrix}\begin{bmatrix} 20 & 15 \\ 14 & 10 \end{bmatrix}\begin{bmatrix} 20 & 15 \\ 14 & 10 \end{bmatrix} = \begin{bmatrix} 18,500 & 13,650 \\ 12,740 & 9,400 \end{bmatrix}$$

$$I_F^1 = \begin{bmatrix} 10 & 10 \\ 11 & 05 \end{bmatrix}\begin{bmatrix} 10 & 10 \\ 11 & 05 \end{bmatrix}\begin{bmatrix} 10 & 10 \\ 11 & 05 \end{bmatrix} = \begin{bmatrix} 3,750 & 2,850 \\ 3,135 & 2,325 \end{bmatrix}$$

When additional units are not ordered, z=0

$$D_F^0 = \begin{bmatrix} 20 & 09 \\ 07 & 15 \end{bmatrix}\begin{bmatrix} 20 & 09 \\ 07 & 15 \end{bmatrix}\begin{bmatrix} 20 & 09 \\ 07 & 15 \end{bmatrix} = \begin{bmatrix} 11,465 & 8,892 \\ 6,916 & 6,525 \end{bmatrix}$$

$$I_F^0 = \begin{bmatrix} 15 & 06 \\ 05 & 10 \end{bmatrix}\begin{bmatrix} 15 & 06 \\ 05 & 10 \end{bmatrix}\begin{bmatrix} 15 & 06 \\ 05 & 10 \end{bmatrix} = \begin{bmatrix} 4,575 & 3,030 \\ 2,525 & 2,050 \end{bmatrix}$$

Suppose in each of the cases, the unit selling price (P_s) is ₦2000, the ordering cost (C_o) is ₦300, the holding cost (C_h) is ₦100, the shortage cost (C_s) is ₦200 and the cost price (C_p) is ₦1000

7.1 Computation of Model Parameters

$$P_{i_n j_n} = 2000 - 1000 = 1000$$

The matrices for the demand transition and profit are computed using equations (7) and (11).

When additional units are ordered, z=1 we get

$$Q^1 = \begin{bmatrix} \dfrac{D_{FF}^1}{D_{FF}^1 + D_{FU}^1} & \dfrac{D_{FU}^1}{D_{FF}^1 + D_{FU}^1} \\ \dfrac{D_{UF}^1}{D_{UF}^1 + D_{UU}^1} & \dfrac{D_{UU}^1}{D_{UF}^1 + D_{UU}^1} \end{bmatrix}$$

$$Q^1 = \begin{bmatrix} \dfrac{18500}{32150} & \dfrac{13650}{32150} \\ \dfrac{12740}{22140} & \dfrac{9400}{22140} \end{bmatrix}$$

$$\Rightarrow Q^1 = \begin{bmatrix} 0.575 & 0.425 \\ 0.575 & 0.425 \end{bmatrix}$$

$$P^1 = 1000\begin{bmatrix} 18500 & 13650 \\ 12740 & 9400 \end{bmatrix} - 400\begin{bmatrix} 3750 & 2850 \\ 3135 & 2325 \end{bmatrix} - 500\begin{bmatrix} 14750 & 10800 \\ 9605 & 7075 \end{bmatrix}$$

$$P^1 = \begin{bmatrix} 18,500,000 & 13,650,000 \\ 12,740,000 & 9,400,000 \end{bmatrix} - \begin{bmatrix} 1,500,000 & 1,140,000 \\ 1,254,000 & 930,000 \end{bmatrix} - \begin{bmatrix} 7,375,000 & 5,400,000 \\ 4,802,500 & 3,537,500 \end{bmatrix}$$

$$\Rightarrow P^1 = \begin{bmatrix} 9,625,000 & 7,110,000 \\ 6,683,500 & 4,932,500 \end{bmatrix}$$

When additional units are not ordered z=0

$$Q^0 = \begin{bmatrix} \frac{D^0_{FF}}{D^0_{FF}+D^0_{FU}} & \frac{D^0_{FU}}{D^0_{FF}+D^0_{FU}} \\ \frac{D^0_{UF}}{D^0_{UF}+D^0_{UU}} & \frac{D^0_{UU}}{D^0_{UF}+D^0_{UU}} \end{bmatrix}$$

$$\Rightarrow Q^0 = \begin{bmatrix} \frac{11465}{20357} & \frac{8892}{20357} \\ \frac{6916}{13441} & \frac{6525}{13441} \end{bmatrix}$$

$$\Rightarrow Q^0 = \begin{bmatrix} 0.563 & 0.437 \\ 0.515 & 0.485 \end{bmatrix}$$

$$P^0 = 1000\begin{bmatrix} 11465 & 8892 \\ 6916 & 6525 \end{bmatrix} - 400\begin{bmatrix} 11465 & 8892 \\ 6916 & 6525 \end{bmatrix}$$

$$\Rightarrow P^0 = \begin{bmatrix} 11,465,000 & 8,892,000 \\ 6,916,000 & 6,525,000 \end{bmatrix} - \begin{bmatrix} 4,586,000 & 3,556,800 \\ 2,766,400 & 2,610,000 \end{bmatrix}$$

$$\Rightarrow P^0 = \begin{bmatrix} 6,879,000 & 5,335,200 \\ 4,149,600 & 3,915,000 \end{bmatrix}$$

7.2 Computation of expected total profit for first planning horizon

For z=1, that is when additional units are ordered, the matrices Q^1 and P^1 yield profits as follows:

$$e^1 = Q^1P^1 = \begin{bmatrix} 0.575 & 0.425 \\ 0.575 & 0.425 \end{bmatrix}\begin{bmatrix} 9,625,000 & 7,110,000 \\ 6,683,500 & 4,932,500 \end{bmatrix}$$

so that $e^1_f = 9,625,000(0.575) + 7,110,000(0.425) = 8,556,125$

$$\Rightarrow e^1_u = 6,683,500(0.575) + 4,932,500(0.425) = 5,939,325$$

Similarly, for z=0, that is when additional units are not ordered, the matrices Q^0 and P^0 yield profits as follows:

$$e^0 = Q^0P^0 = \begin{bmatrix} 0.563 & 0.437 \\ 0.515 & 0.485 \end{bmatrix}\begin{bmatrix} 6,879,000 & 5,335,200 \\ 4,149,600 & 3,915,000 \end{bmatrix}$$

$$e^0_f = 6,879,000(0.563) + 5,335,200(0.437) = 6,204,359.4$$

$$\Rightarrow e^0_u = 4,149,600(0.515) + 3,915,000(0.485) = 4,035,819$$

This shows that z=1 is the optimal ordering policy for favorable state since ₦8,556,125 is greater than ₦6,204,359.4 with associated total profit of ₦8,556,125 and EOQ of 15units. Also, it shows that z=1 is the optimal ordering policy for unfavorable state since ₦5,939,325 is greater than ₦4,035,819 with associated profit of ₦5,939,325 and EOQ of 8units for the first planning horizon.

7.3 computation of expected profit for second planning horizon

The accumulated profits for favorable and unfavorable demand for the second planning horizon are computed using the equation:

$$a^z_{i_n} = e^z_{i_n} + Q^z_{i_n f} g_1(f) + Q^z_{i_n u} g_1(u) \quad for \quad z = \{0,1\} \ and \ i_n = \{f,u\},$$

For favorable demand, we have:

$$a^1_f = 8,556,125 + 8,556,125(0.575) + 5,939,325(0.425) = 16,000,110$$

$$a^0_f = 6,204359.4 + 8,556,125(0.575) + 5,939,325(0.425) = 13,648,344.4$$

Which shows that z=1 is the optimal ordering policy for favorable demand since ₦16,000,110 is greater than ₦13,648,344.4 with the accumulated profit of ₦16,000,110 and EOQ of 15 units.

On the other hand, the accumulated profit for unfavorable demand is as follows:

$$a_u^1 = 5,939,325 + 8,556,125(0.563) + 5,939,325(0.437) = 13,351,908.4$$

$$a_u^0 = 4,035,819 + 8,556,125(0.515) + 5,939,325(0.485) = 11,322,796$$

It is clear that z=1 is the optimal ordering policy for the unfavorable demand since ₦13,351,908.4 is greater than ₦11,322,796 with accumulated profit of ₦13,351,908.4 with an EOQ of 8units.

7.4 computation of expected total profit for third planning horizon

The accumulated profits for favorable and unfavorable demand for the third planning are computed using the equation:

$$b_{i_n}^z = a_{i_n}^z + Q_{i_n f}^z g_2(f) + Q_{i_n u}^z g_2(u) \quad for \ \ z = \{0,1\} \ \ and \ \ i_n = \{f, u\}$$

For favorable demand, we have:

$$b_f^1 = 16,000,110 + 16,000,110(0.575) + 13,351,908.4(0.425) = 30,874,734.32$$

$$b_f^0 = 13,648,344.4 + 16,000,110(0.575) + 13,351,908.4(0.425) = 28,522,968.72$$

Which shows that z=1 is the optimal ordering policy for favorable demand since ₦30,874,734.32 is greater than ₦28,522,968.72 with the accumulated profit of ₦30,874,734.32 and an EOQ of 15 units.

On the other hand, the accumulated profit for unfavorable demand is as follows:

$$b_u^1 = 13,351,908.4 + 16,000,110(0.563) + 13,351,908.4(0.437) = 28,194,754.30$$

$$b_u^0 = 11,322,796 + 16,000,110(0.515) + 13,351,908.4(0.485) = 26,038,528.22$$

Which shows clearly that z=1 is the optimal ordering policy since ₦28,194,754.30 is greater than ₦26,038,528.22 with accumulated profit of ₦28,194,754.72 with an EOQ of 8 units.

CONCLUSION

This paper is an extension of Hassan and Sani (2020). In the paper, we develop a model for stock replenishment under a periodic review inventory system with stochastic demand using a three-period dynamic programming technique. With the aid of dynamic programming, the decision to order or not to order additional units is modeled as a multi-period decision problem. We demonstrate the working of the model with a numerical example. The model involves a three-period planning horizon, and based on the tree diagram of figure 1, the transition matrices for the problem are 3-multiple transition matrices.

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