

# A Survey on the Different Adaptive Algorithms Used In Adaptive Filters

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**Abstract:-Adaptive filters finds application in various fields which includes echo cancellers, noise cancellation, system identification etc. There are many algorithms available and the choice of a particular type of algorithm is dependent on the requirement of the algorithms in the particular environment. Among the various types of algorithms, LMS is very commonly used because of its simplicity. In this paper a comparison of the variants of LMS algorithms with respect to their convergence behavior, tracking capability, robustness, computational complexity, steady state error is made .**

**Keywords:-Adaptive filter ,LMS.**

## 1. Introduction

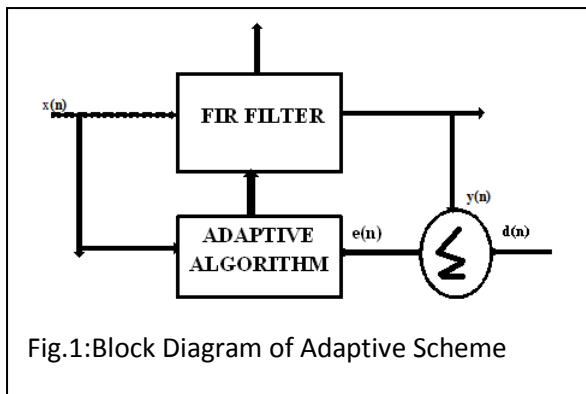
Adaptive filters are one of the important tool in the digital signal processing. They are mainly used whenever the statistical characteristics of the signal is said to be non-stationary in nature. In these type of filters ,the coefficients tend to get updated as per the conditions prevailing so as to minimize the error. Hence the key component of the adaptive filter is the adaptive algorithm. These algorithms are the set of rules which defines how the updation is made. The important requirements of these adaptive algorithm are that they should adapt to the changing statistics and track the solution as the time changes. Based on the adaptive algorithms, adaptive filters are classified broadly into two broad classifications. One is the sample by sample approach and the other is the block approach[1].Sample by sample algorithms are further classified as time domain and frequency domain algorithms[1],[2],[3].The sample by

sample time domain approach is further classified as least mean square and recursive least square algorithms. Similarly the sample by sample frequency domain approach are classified as based on sliding discrete Fourier transform, frequency sampling method, sub band techniques.

Similarly the block adaptive algorithms are classified as time domain and frequency domain adaptive algorithms. They update the filter coefficients only in blocks therefore tracking is very poor.

The choice of a particular type of algorithm depends mainly on the application. The various application of adaptive systems includes the adaptive equalizers [1] used to remove the inter-symbol interference in the communication systems, the acoustic echo cancellation systems.[2]and noise cancellation systems[3] where the adaptive filters generate the echo or noise which is subtracted from the corrupted signal to get back the original signal. Identification of unknown system transfer function is another important application of adaptive filters [2].

In this paper we discuss only the sample by sample time domain LMS algorithms and its variants with respect to various performance criteria. The LMS algorithm is one of the most popular algorithm used in adaptive filters.The advantages of LMS algorithm lies in its simplicity and no requirement of ensemble average to be known in advance. The basic structure [2] of a LMS linear adaptive scheme is shown in figure1.



It consists of a filter whose input is  $x(n)=[x(n),x(n-1),x(n-2),\dots,x(n-k)]$  where  $k$  is the number of delay elements and  $w(n)=[w(n),w(n-1),w(n-2),\dots,w(n-k)]$  is the tap weight vector. These values are estimated from the adaptive algorithm whose expected value may come close to the Weiner Hofp solution as  $n$  tends to infinite.

The desired signal  $d(n)$  and the estimated value of the desired signal  $y(n)$  from the filter are used to generate the error signal  $e(n)$ . This error signal along with the input signal determines the update to the weight vector for the filter.

## 2. Mathematical description of the LMS and its variants

All LMS algorithms have a weight update equation given as

$$w(n+1)=w(n)+\mu e(n)x^*(n)$$

where  $x(n)$ =input vector,  $w(n+1)$ =value of the new weight at time  $n+1$ ,  $w(n)$ = value of the weight at time  $n$ ,  $\mu$ =step size chosen to be a value between

$$0 \leq \mu \leq 2/\lambda_{\max}$$

$$e(n)=d(n)-y(n)$$

where  $y(n)=\sum w_n(k)x^*(n-k)$

### 2.1 Least Mean Square algorithm:

The LMS[2] is the most popular algorithm due to its simplicity. We can notice that the update function is obtained by the multiplication of the step size with the current value of the error signal and input signal and does not depend on any other previous value. The main drawback

of the LMS algorithm is that the convergence is slow due to the step size restriction which depends on the eigen value of the auto correlation matrix of the input signals. Several methods have been used to speed up the convergence of the LMS algorithm at the cost of increased complexity.

### 2.2 Normalized least mean square algorithm:

The normalized least mean square algorithm is the normalized form of the LMS algorithm. In this algorithm the difficulty encountered by the LMS in the selection of step size is eliminated by normalizing the step size. The main advantage of NLMS is that the gradient noise amplification is very much reduced due to the norm function present in the denominator of the weight update equation. The convergence is also faster than the LMS. The main disadvantage with this algorithm is that the computational complexity is more than LMS. The modified equation governing the NLMS is given by

$$w(n+1)=w(n)+\mu e(n)x(n) / (||x(n)||+p)$$

Where  $p$ =small positive value which is used when norm of  $x(n)$  becomes very small.

### 2.3 Proportionate normalized least mean square algorithm

The equations governing the PNLMS algorithm is given by [4]

$$w(n+1)=w(n)+\mu e(n)x(k-n) / (N \sigma_x^2(k)).$$

The need for faster convergence led to the development of PNLMS. The main concept behind this algorithm is that the gain is adapted in proportional to the

magnitude of the estimated impulse response. The adaptive gain at each tap position varies from one position to the other. The total adaptive gain is carefully monitored and controlled so as to have misadjustment constant throughout. The main limitation with this algorithm is that the small coefficients receive less gain therefore the time needed to reach the steady state error is increased. Also the initial convergence is faster and since the impulse response is dispersive, the overall convergence is slower than NLMS.

#### 2.4 Affine Projection Algorithm

Affine Projection Algorithm [2] is a variant of the NLMS algorithm. It converges faster at a cost of computational complexity [5]. Its performance also degrades in the presence of impulse interference [6]. In this algorithm when the convergence is increased with the projection order there is simultaneous increase in the computational complexity. The equations are

Min

$w(n+1) = \|w(n+1) - w(n)\|^2$  subject to the constraint

$$y(n) - X^T(n)w(n+1) = 0$$

#### 2.5 Affine projection sign algorithm

The APSA [6] combines the benefits of APA and SA by updating its weights vector according to  $L_1$  norm optimization criteria while using multiple projections. Due to the combined feature it has got improved speed of convergence, steady state error than APA and SA. The computational complexity [7] is also less than APA, NLMS algorithms.

The equation are  $w(n+1) = w(n) + \mu x(k)$

$$\text{Sqrt}(X_s^T(k)X_s(k) + c)$$

Where  $X_s(k) = X(k) \text{sgn}(e(k))$  &  $c$  is the regularization parameter.

#### 2.6 Variable step size Normalized LMS algorithms

Variable step size NLMS adaptive filters were developed to meet the objective of fast convergence with a low excess mean square error.

In the paper [8] the power of the instantaneous error is used to derive the variable step size LMS filter. Here the step size is larger when the estimated error is larger and vice versa. In [9] the norm of the filter coefficients error vector is the criteria for selecting the variable step size. In [10] the mean square error and the estimated system noise power is used to control the step size. The algorithm is a variable step size non parametric VSS-NLMS algorithm. In [8] a comparison of the VSS-NLMS is made with other VSS-NLMS and is found that [8] performs well with faster convergence, low steady state error and good tracking capability. In [11] a variable step size APA which is robust is presented. This algorithm is based on the minimization of the square norm of the a posteriori error subject to a time dependent constraint on the norm of the filter update. The performance analysis finds that this algorithm RVSS-APA has a steady state error lower without reducing the speed of convergence or tracking.

### 3. Conclusion

This paper has discussed the LMS adaptive algorithms used in adaptive filters. LMS algorithm is used when cost is the major criteria. However convergence and tracking of the APA is more than LMS algorithm. The performance of hybrid algorithms are better than their parents. Hence in future it is required to make an adaptive algorithm which has less computational complexity, more stability, less steady state error and with good converge and tracking capability.

Comparison table depicting the features of the adaptive algorithm.

Sl No	Adaptive Algorithm	Convergence	Tracking	Computational Complexity	Steady State Error	Stability	Limitation
1	LMS	Poor due to the restriction of the step size on the eigen value of the autocorr function of the input signal .	Poor	$O(M)$ where $M$ =length of the filter	Slightly more than the minimum mean square error due to gradient noise	good	The convergence and tracking are poor and mean square error is more.
2	NLMS	Faster than LMS	Better than LMS	$2L+2$ -additions $2L+3$ - multiplications 1 -division	Lesser than LMS	More stable than LMS	Complexity is more.
3	PLMS	Faster convergence than LMS,NLMS since the step size is proportional to the impulse response of the signal	Better than NLMS	50% more complex than NLMS	Almost constant throughout due to distribution of the total adaptive gain over the taps.	good	More complex
4	APA	Improved convergence	Better than LMS and NLMS	As the projection order is increased to improve the convergence the computational	Due to regularization the near end noise	good	Most costlier due to complexity in computation.

				complexity also increases	amplification is avoided		
5	RVSS-APA	Faster than APA since it is base on L1 and L2 norm optimization	Better than APA	More complex	Slightly better than APA	Very robust and highly stable than APA	Computational complexity is more.
6	APSA	Faster convergence than APA due to concept of L1 norm optimization	Better than APA	$2L+3M+4$ Multiplications $Ml+1+2M^2+3M$ $+1$ additions 1 square root and 1 division	Better than APA and SA	Better than APA and SA with higher projection order.	No of additions in the efficient implementation is still high of the order of $O(ML)$
7	VSS-NLMS	Faster convergence	Better than NLMS	More	Better than NLMS	good	Complexity in computation is more.

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