A survey on image denoising using different denoising methods

Abstract— Visualization is the creation of visual representation of an image or data. Several visual design techniques are used for the visualization of natural image statistics. The visual design techniques such as PCA, KPCA, FPSS have seen to be ineffective in denoising and this also increases the complexity of natural image statistics. By introducing foGSM, the effectiveness in denoising can be improved. The foGSM is a visualization technique, where the image is first segmented into several segments and the mixtures of GSM are computed for each segment. The GSM mixtures is computed by using local wiener estimation. These computed segments are combined to provide a denoised image. The damaged features of an image can be restored by using foGSM in image inpainting.

Keywords— gaussian noise, subbands, foGSM, local wiener estimator

I. INTRODUCTION

Image processing is a method to convert an image into digital form and perform some operations on it, in order to image. Image Processing system includes treating images as two dimensional signals while applying already set signal processing methods to them. Image processing basically includes the following three steps: Importing the image with an optical scanner or by digital photography. Analyzing and manipulating the image which includes data compression and image enhancement and spotting patterns that are not to human eyes like satellite photographs. Output is the last stage in which result can be altered image or report that is based on image analysis. The purpose of image processing is divided into 5 groups. They are: Visualization, Image sharpening and restoration, Image retrieval, Measurement of pattern, Image recognition.

Noise is any undesired signal that contaminates the image, which is the result of errors in the image acquisition process that result in pixel values not reflecting the true nature of the scene during acquisition, transmission, storage and retrieval process, any image signal gets contaminated with noise. an image which is being sent electronically from one place to another place is contaminated by a variety of noise sources. noise can be caused in images by random fluctuations in the image signal. The main objective of the image processing is to extract the clear information from the images corrected by noise. such techniques for noise removal is called filtering or denoising. An image gets corrupted with different types of noise. Noise may be classified as substitutive noise(impulsive noise: salt and pepper noise, random value impulse noise, etc) and additive noise (eg: additive white gaussian noise). In many occasions, noise in digital image is found be additive in nature with uniform power in the whole bandwidth with gaussian probability distribution. Such a noise is referred to as additive white gaussian noise.

Some of the most common file formats are: GIF - An 8-bit (256 colours), non-destructively compressed bitmap format. Mostly used for web. Has several sub-standards one of which is the animated GIF. JPEG - A very efficient (i.e. much information per byte) destructively compressed 24 bit (16 million colours) bitmap format. Widely used, especially for web and Internet (bandwidth-limited). TIFF - The standard 24 bit, publication bitmap format. Compresses non-destructively with, for instance, Lempel-Ziv-Welch (LZW) compression. PS - Postscript, a standard vector format. Has numerous sub-standards and can be difficult to transport across platforms and operating systems. PSD - A dedicated Photoshop format that keeps all the information in an image including all the layers.

I.A. GAUSSIAN NOISE

Gaussian noise is statistical noise that has a probability density function (pdf) of the normal distribution (also known as gaussian distribution). It is a major part of the "read noise" of an image sensor, that is, of the constant noise level in dark areas of the image. Gaussian noise is also called as amplifier noise.

I.B. SALT AND PEPPER NOISE

Salt-and-pepper noise in image will have dark pixels in bright regions and bright pixels in dark regions. This type of noise can be caused by dead pixels, analog -to- digital converter errors, bit errors in transmission, etc. It represents itself as randomly occurring white and black pixels.

I.C. POISSON NOISE

Poisson noise are shot noise is a type of electronic noise that occurs when the finite number of particles that carry energy, such as electrons in an electronic circuit or photons in an optical device[11].
II. IMAGE FILTERING

Image filtering improves the quality of the image by the way of enhancement of edges of the images. So many techniques such as mean filter, median filter and some other denoising techniques have been developed to suppress the gaussian noise. Noise cannot be removed without the loss of some information in the form of image detail. Nevertheless, noise - reduction algorithms have been developed to reduce noise without degrading image information too much. Here foGSM method is used for denoising the gaussian noise.

III. VISUAL DESIGN TECHNIQUES.

III.A. FOURIER POWER SPECTRAL SIGNATURES (FPSS)

FPSS[10] are used in image statistics research. Given an image x, a two-dimensional Fourier transformation is first applied to the image, resulting in a frequency domain representation $T_f(x)$. In the Fourier domain, the magnitudes of Fourier coefficients are calculated as a power spectral signature of the image x. The power spectrum, $M_{fpss}(T_f(x))$, is a statistical measure of the power of the signal (energy per unit space) falling in different frequency bins. It is assumed that FPSS in different resolutions may contain different useful information. Then subdivide each image in a hierarchical manner and obtain FPSS contours for each subdivided image. This is referred to as a scale space Fourier transform. gradient orientation, or a two-dimensional histogram where bins are sorted by both magnitude and orientation. Hence, $[b_1,b_2,........b_m]=M_{b}(T_f(x))$

III.B. PRINCIPAL COMPONENT ANALYSIS (PCA)

PCA[1,2] is typically applied to a collection of data, for example, a class of images $X= \{x_1,x_2,....x_n\}$. It linearly transforms X to a new coordinate system so that the greatest variance of the data is encoded by the variance on the first few axes. The transformation requires the computation of a mean image $\bar{X}$, which can be considered a class level statistics. The difference between each image $x_i$ and $\bar{X}$ is then encoded by coefficients $[p_1,p_2,....p_m]=M_{PCA}(x,X)$ corresponding to different eigenvectors. Eigenvectors are typically sorted in descending order according to the corresponding eigenvalues. Thus, the first few eigenvectors of PCA are expected to capture the significant variance of the processed data and non significant signals (possibly noise) lie on those eigenvectors with smaller eigenvalues. As a result, PCA is able to remove the noise from the data, while making use of fewer quantities to describe the images.

III.C. KERNEL PRINCIPAL COMPONENT ANALYSIS (KPCA)

KPCA is an extension of PCA[9] obtained by introducing different kernels, such as linear kernels, radial basis function kernels, or polynomial kernels. The use of kernels allows one to solve only the eigenvectors and eigen values of a kernel, instead of a very high - dimensional feature space where other projection methods are used to map the data. In this work, a linear kernel is adopted as it is the only kernel that does not require any parameters to be tuned, so is a fully unsupervised statistical method.

IV. LIMITATIONS

High frequency components of FPSS indicate sharp changes and texture details in image, while low frequency components represent the main image structure, so FPSS can reflect different types of image patterns. PCA is only powerful for the images with highest variance. Thus PCA is not useful for the images with the lowest variance. In KPCA, the generic kernels do not perform well, therefore more data oriented kernels are to be defined. These visual design techniques are not effective in denoising the images. Here PCA, KPCA do not estimate a clean version of a noise corrupted image.

V. FIELD OF GAUSSIAN SCALE MIXTURE (FoGSM)

The foGSM (field of Gaussian Scale Mixtures)[4] is a visualization technique, where the image is first segmented into several segments and the mixtures of GSM are computed for each segment. These computed segments are combined to provide a denoised image. The damaged features of an image can be restored by using foGSM in image inpainting. Consider an image decomposed into oriented subbands at multiple scales. Thus denote as $(x^{oGSM}(n,m))$ the coefficient corresponding to a linear basis function at scale s, orientation o, and centered at spatial location $(2^n, 2^m)$. Thus denote as $x^{oGSM}(n,m)$ a neighborhood of coefficients clustered around this reference coefficient.

The neighborhood may include coefficients from other subbands (i.e., corresponding to basic functions at nearby scales and orientations), as well as from the same subbands. For notational simplicity, Drop the superscripts s, o and indices (n,m) in the following development. Assume the coefficients within each local neighborhood are characterized.

Assume that, the coefficients within each local neighborhood are characterized by a Gaussian scale mixture (GSM) model. Formally, a random vector x is a Gaussian scale mixture if and only if it can be expressed as the product of a zero-mean Gaussian vector $u$ and an independent positive scalar random variable $\sqrt{\chi}^2$:

$$x \triangleq \sqrt{\chi^2} u,$$  

where $\Delta$ indicates equality in distribution. The variable z is known as the multiplier. The vector x is thus an infinite
mixture of Gaussian vectors, whose density is determined by the covariance matrix $C_u$ of vector $u$ and the mixing density, $p_z(z)$:

$$p_x(x) = \int dz \ p(x|z) \ p_z(z)$$

$$= \int dz \ \frac{\exp\left(-\frac{x^T (C_u + 1)^{-1} z}{2}\right)}{(2\pi)^{N/2}(C_u + 1)^{1/2}} \ p_z(z) \quad (2)$$

where $N$ is the dimensionality of $x$ and $u$ (in this case, the size of the neighborhood). Without loss of generality, assume $\mathbb{E}\{z\}=1$, which implies $C_z = C_u$.

The conditions under which a random vector may be represented using a GSM have been studied. The GSM family includes a variety of well-known families of random variables such as the $\alpha$ stable family (including the Cauchy distribution), the generalized Gaussian (or stretched exponential) family and the symmetrized Gamma family. GSM densities are symmetric and zero-mean, and they have leptokurtotic marginal densities (i.e., heavier tails than a Gaussian). A key property of the GSM model is that the density of $x$ is Gaussian when conditioned on $z$. Also, the normalized vector $x/\sqrt{z}$ is Gaussian. Some authors have suggested division by a local estimate of standard deviation as a means of “Gaussianizing” a highly kurtotic signal.

GSM model can account for both the shape of wavelet coefficient marginal and the strong correlation between the amplitudes of neighbor coefficients. In order to construct a global model for images from this local description, one must specify both the neighborhood structure of the coefficients, and the distribution of the multipliers. The definition of (and calculations using) the global model is considerably simplified by partitioning the coefficients into non-overlapping neighborhoods. Then specify either a marginal model for the multipliers (treating them as independent variables), or specify a joint density over the full set of multipliers. Unfortunately, the use of disjoint neighborhoods leads to noticeable denoising artifacts at the discontinuities introduced by the neighborhood boundaries.

An alternative approach is to use a GSM as a local description of the behavior of the cluster of coefficients centered at each coefficient in the pyramid. Since the neighborhoods overlap, each coefficient will be a member of many neighborhoods. The local model implicitly defines a global (Markov) model, described by the conditional density of a coefficient in the cluster given its surrounding neighborhood, assuming conditional independence on the rest of the coefficients. But the structure of the resulting model is such that performing statistical inference (i.e., computing Bayes estimates) in an exact way is intractable. In this paper, the estimation problem for the reference coefficient at the center of each neighbourhood independently is solved.

V.A. PRIOR DENSITY FOR MULTIPLIER

To complete the model, specify the probability density, $p_z(z)$, of the multiplier. Several authors have suggested the generalized Gaussian (stretched exponential) family of densities as an appropriate description of wavelet coefficient marginal densities: $p_z(z) \propto \exp(-\frac{z^p}{2})$ where the scaling variable $s$ controls the width of the distribution, and the exponent $p$ controls the shape (in particular, the heaviness of the tails), and is typically estimated to lie in the range $[0.5; 0.8]$ for image subbands. Although they can be expressed as GSMs, the density of the associated multiplier has no closed form expression, and thus this solution is difficult to implement.

The density of the log coefficient magnitude, $\log |x|$, may be written as a convolution of the densities of $\log |u|$ and $\log \sqrt{z}$. Since the density of $u$ is known, this means that estimation of the density of $\log \sqrt{z}$ may be framed as a deconvolution problem. The resulting estimated density may be approximated by a Gaussian, and thus proposed a lognormal prior for the $z$. This solution has two important drawbacks. First, it assumes all coefficients in a neighborhood have the same marginal statistics, and thus they must typically all belong to the same subband. Second, it is derived in terms of the noise-free coefficients, and it is difficult to extend it for use in the noisy case.

A more direct maximum likelihood approach for estimating a nonparametric $p_z(z)$ from an observed set of neighborhood vectors:

$$\hat{p}_z(z) = \arg \max_{p_z(z)} \sum_{m=1}^{M} \log \left( \int dx \ p(x_m|z) p_z(z) \right) \quad (3)$$

where the sum is over the neighborhoods estimate, $\hat{p}_z(z)$, must be constrained to positive values, and must have unit area. An efficient algorithm is developed for computing this solution numerically. One advantage of the ML solution is that it is easily extended for use with the noisy observations, by replacing $x_m$ the noisy observation. A fourth choice is a so-called non informative prior, which has the advantage that it does not require the fitting of any parameters to the noisy observation. The most widely used solution is examined, known as Jeffrey’s prior. In the context of estimating the multiplier $z$ from coefficients $x$, this takes the form:

$$p_z(z) \propto \sqrt{I(z)} \quad I(z) = \mathbb{E}\left(-\frac{\partial^2 \log p(x|z)}{\partial z^2}\right)$$

where $I(z)$ is the Fisher information matrix. Computing this for the GSM model is straightforward:

$$-\frac{\partial^2 \log p(x|z)}{\partial z^2} = \frac{\partial^2}{\partial z^2} \left[ N \log(z) + \log|C_u| + \frac{x^T C_u^{-1} x}{z} \right]$$

$$= \frac{N}{z^2} + \frac{x^T C_u^{-1} x}{z^3}$$
Taking the square root of the expectation, and using the fact that \( \{x^2\}^{-1} = \frac{1}{z} \). Jeffrey's prior is obtained:

\[
p_z(z) = \frac{1}{z}, \tag{4}
\]

which corresponds to a constant prior on \( \log(z) \). Note that this prior does not constitute a legitimate probability density, as its integral does not converge. Nevertheless, it is common to ignore this fact and use such a solution as a pseudo-prior, whenever the integrals involved in the estimation converge. In this case, to ensure the existence of the posterior \( p(z|y) \), set the pseudo-prior to zero in the interval \([0; z_{\text{min}}]\), where \( z_{\text{min}} \) is a very small positive constant. The ML-estimated nonparametric prior produces the best results for denoising the pyramid coefficients. However, that a least squares optimal estimate for the pyramid coefficients does not necessarily lead to a least-squares optimal estimate for the image pixels, since the pyramid representation is over complete. The non informative prior typically leads to better denoising performance in the image domain (roughly +0.15 dB, on average).

V.B. IMAGE DENOISING

The procedure for image denoising[3]: (1) decompose the image into pyramid subbands at different scales and orientations; (2) denoise each subband; and (3) invert the pyramid transform[12], obtaining the denoised image. Assume the image is corrupted by independent additive Gaussian noise of known covariance. A vector \( y \) corresponding to a neighborhood of \( N \) observed coefficients of the pyramid representation[7] can be expressed as:

\[
y = x + w = \sqrt{z} u + w \tag{5}
\]

The assumed GSM structure of the coefficients, coupled with the assumption of independent additive Gaussian noise, means that the three random variables on the right side of (5) are independent.

Both \( u \) and \( w \) are zero-mean Gaussian vectors, with associated covariance matrices \( C_u \) and \( C_w \). The density of the observed neighborhood vector conditioned on \( z \) is a zero-mean Gaussian, with covariance \( C_{y|z} = zC_u + C_w \):

\[
p(y|z) = \frac{\exp(-y^T(zC_u+C_w)^{-1}y/2)}{\sqrt{(2\pi)^N|zC_u+C_w|}} \tag{6}
\]

Assume the noise covariance is known apriori in the image domain. Since \( w \) is derived from the image through the (linear) pyramid transformation, it is straightforward to compute the noise covariance matrix \( C_w \). Given \( C_w \), the signal covariance \( C_u \) can be computed from the observation covariance matrix \( C_y \). Taking the expectation of \( C_{y|z} \) over \( z \) yields:

\[
C_y = \mathbb{E}\{z\}C_u + C_w
\]

\( \mathbb{E}\{z\} \) is free to choose, so set it to one, resulting in:

\[
C_u = C_y - C_w \tag{7}
\]

Ensure that \( C_u \) is positive semi definite by performing an eigenvector decomposition and setting any negative eigenvalues to zero.

V.C. BAYES LEAST SQUARES ESTIMATOR

For each neighborhood, estimate \( x_c \), the reference coefficient at the center of the neighborhood, from \( y \), the set of observed (noisy) coefficients. The Bayes least squares (BLS) estimate is just the conditional mean[8]:

\[
\mathbb{E}\{x|y,z\} = zC_u(zC_u + C_w)^{-1} y, \tag{9}
\]

Simplify the dependence of this expression on \( z \) by diagonalizing the matrix \( zC_u + C_w \). Specifically, let \( S \) be the symmetric square root of the positive definite matrix \( C_w \) (i.e., \( C_w = SS^T \)), and let \( \{Q,A\} \) be the eigenvector/eigenvalue expansion of the matrix \( S^{-1}C_u S^{-T} \). Then:

\[
zC_u + C_w = zC_u(zC_u + SS^T) \tag{10}
\]

This diagonalization does not depend on \( z \), and thus only needs to be computed once for each subband. Now simplify (9):

\[
\mathbb{E}\{x_c|y,z\} = zC_uS^{-T}Q \tag{11}
\]

where \( M = SQ \) and \( v = M^{-1}y \). Finally, restrict the estimate to the reference coefficient[11], as needed for the solution of (8):

\[
\mathbb{E}\{x_c|y,z\} = \sum_{n=1}^{zmc+1}\frac{zmc+n^2}{2zmc+n+1} \tag{12}
\]
where $m_{ij}$ represents an element (i-th row, j-th column) of the matrix $M$, $\lambda_n$ are the diagonal elements of $A$, $v_n$ the elements of $v$, and $c$ is the index of the reference coefficient within the neighborhood vector.

V.E. POSTERIOR DISTRIBUTION OF THE MULTIPLIER:

The other component of the solution is the distribution of the multiplier, conditioned on the observed neighborhood values. Bayes rule to compute this is:

$$p(z|y) = \frac{p(y|z)p_z(z)}{\int dp(z)p_z(z)}$$

(13)

Choose a non informative Jeffrey's prior, corrected at the origin, for the function $p_z(z)$. The conditional density $p(y|z)$ is given in (6), and its computation may be simplified using the relationship in (10) and the definition of $v$:

$$p(y|z) = \frac{\exp\left(-\frac{1}{2}N \sum_{k=1}^{N} \frac{x_k^2}{\sigma_k^2}\right)}{\sqrt{2\pi}^N |\Lambda|^{1/2}}$$

(14)

Summarizing the denoising algorithm:

1. Decompose the image into subbands
2. For each subband (except the lowpass residual):
   (a) Compute neighborhood noise covariance, $C_w$
   (b) Estimate noisy neighborhood covariance, $C_y$
   (c) Estimate $C_u$ from $C_w$ and $C_y$ using (7)
   (d) For each neighborhood:
      i. For each value $z$ in the integration range:
         A. Compute $E[|y,z]$ using (12)
         B. Compute $p(y|z)$ using (14)
         C. Compute $p(z|y)$ using (13)&(4)
      ii. Compute $E[p_c|y]$ numerically using (8)
3. Reconstruct the denoised image from the processed subbands and the lowpass residual.

VI. IMPLEMENTATION

Decompose the image into subbands using a specialized variant of the steerable pyramid [14]. The representation consists of oriented bandpass bands at 8 orientations and 5 scales, 8 oriented highpass residual subbands, and one lowpass (non-oriented) residual band, for a total of 49 subbands. The neighborhood structure (i.e., choice of spatial positions, scales and orientations) is hand-optimized. A $3 \times 3$ region surrounding the reference coefficient, together with the coefficient at the same location and orientation at the next coarser scale (the parent), maximizes the denoising performance, on average. Denote this generalized neighborhood as $3 \times 3 + p$. In previous work on compression, inclusion of the parent coefficient was also found to provide a significant improvement in performance. When denoising the subbands at the coarsest scale, which have no parent subband, simply use the $3 \times 3$ spatial neighborhood. Note that the actual spatial extent of the neighborhood depends on the scale of the subband — the basis functions grow in size as $2^s - ds$. Since the parent subband is sampled at half the density of the reference subband, it must be up sampled and interpolated in order to obtain values for neighborhoods at every choice of reference coefficient.

In this implementation, the integral of (8) is computed numerically. This requires us to choose the (finite) range and sample spacing over which the calculations are performed, which must be a compromise between accuracy and computational cost. The typical shape of $p(y|z)$ (considered as a function of $z$ for each observed $y$) extends very far and smoothly as $z$ grows towards infinity, but can exhibit rapid variations as it approaches zero. As such, $z$ is sampled with logarithmically uniform spacing. This adequately captures the shape of the function, in terms of the performance of the algorithm, with only $z = 11$ samples. These samples are spread uniformly over a range extending from $\mu_z - 13\sigma_z$, $\mu_z + 9\sigma_z$, where $\mu_z$, $\sigma_z$ is estimates of the mean and standard deviation of log($z$). These are obtained from the variance and kurtosis of each subband using the method of moments described in [7]. The computational cost of the pyramid transform (both forward and inverse) scales as $I_xI_y \log_2(I_xI_y)$, where $(I_x,I_y)$ are the dimensions of the image. The computational cost of the estimation procedure scales as

$$I_x + \frac{(N_x + B_x)}{2} \left(I_y + \frac{(N_y + B_y)}{2}\right)^{NKs_x}$$

where $N_{x,y}$ are the dimensions of the spatial subband neighborhood (3 in this case). $B_{x,y}$ the dimensions of the bandpass convolution kernels (roughly 9 in this implementation), $N$ the full size of the neighborhood (10 in this case), K the number of orientations, and $S_2$ the number of samples used for the distributions over $z$. The terms added to the image dimensions correspond to the padded boundary region that must be estimated in order to properly reconstruct the image. As a guide, running times in current un optimized Matlab implementation, on a Linux workstation with 1.7 GHz Intel Pentium-III CPU, are roughly 40 seconds for 256x256 images. Finally, the primary memory cost is due to storage of the pyramid coefficients (roughly $7KN_xN_y/3$ floating point numbers).

VII. CONCLUSION

Thus the foGSM method is tested on a set of 8-bit grayscale test images, each contaminated with computer-generated additive Gaussian white noise. For all images, there is very little improvement at the lowest noise level. The foGSM algorithm computes the Bayes least squares estimate (i.e., conditional mean) of individual coefficients based on a local GSM model. The FoGSM algorithm achieves consistent improvements in PSNR over the local GSM based algorithm. Thus foGSM method performs good at some circumstances and worse at other some circumstances. That is performs best
on images with large regions of the same texture, or long contours of similar orientation, and performs less well on images with diverse content. This worse performance on images will be soon reduced in the future ongoing project.

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