A Survey of Non-Local Means based Filters for Image Denoising

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***Abstract*—Image denoising involves the manipulation of the image data to produce a visually high quality image. The Non-Local means filter is originally designed for Gaussian noise removal and the filter is modified to adapt for speckle noise reduction. Speckle noise is a primary source of medical ultrasound imaging noise and it should be filtered out. This paper reviews the existing Non-Local Means based filters for image denoising.**

***Keywords*—denoising; gaussian noise; speckle noise; Non-Local Means;**

I. Introduction

Noise represents unwanted information which deteriorates image quality. Non-Local based filters are mainly used to remove Gaussian and speckle noise. Gaussian noise is statistical noise that has a probability density function of the normal distribution. It is most commonly used as additive white noise to yield additive white Gaussian noise. Gaussian noise is evenly distributed over the signal [1]. Each pixel in the noisy image is the sum of the true pixel value and a random Gaussian distributed noise value.

Speckle noise is a granular noise that inherently exists in and degrades the quality of images. Speckle noise is modeled as spatial correlated multiplicative noise [2]. Noise is introduced at all stages of Image acquisition. Speckle reduction is a critical preprocessing step for extraction of features, analysis and recognition from medical ultrasound image measurements. Commonly used linear low-pass filters, such as the mean filters are not suitable for reducing the speckle noise of ultrasound images since they eliminate the high frequencies and thus tend to smooth out the image edges.

The rest of the paper is organized as follows: Section II presents the Model of Gaussian and Speckle noise. Section III presents a survey of various Non-Local Means based filters for image denoising. Section IV illustrates various parameters used for analyzing the performance of denoising filters. Finally our conclusions are presented in Section V.

II. Noise Modeling

Noise may be modeled either by a histogram or a probability density function which is superimposed on the probability density function of the original image. A noisy image is modeled as,

$$C(x, y) = A(x, y) + B(x, y) \quad (1)$$

where A(x,y) is the original image pixel value, B(x,y) is the noise in the image and C(x,y) is the resulting noise image.

A. Gaussian Noise

The Gaussian noise has a Gaussian distribution [1] which has a bell shaped probability distribution function given by,

$$F(g) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(g-m)^2 / 2\sigma^2} \quad (2)$$

where g represents the gray level, m is the mean or average of the function and σ is the standard deviation of the noise.

B. Speckle noise

Speckle noise [3] is a multiplicative noise. This type of noise occurs in almost all coherent imaging systems such as laser, acoustics and SAR(Synthetic Aperture Radar) imagery. Speckle degrades the quality of ultrasound images and reduces the ability of a human observer to discriminate the fine details of diagnostic examination. Speckle noise follows a gamma distribution and is given as

$$F(g) = \frac{g^{\alpha-1}}{(\alpha-1)!a^{\alpha}} e^{g/a} \quad (3)$$

where the variance is $a^2\alpha$ and g is the gray level.

III. Non-Local Means Based Filters for Image Denoising

Spatial domain filtering is classified into linear and nonlinear filters. Non-Local means filter is one of the spatial domain filter. A single pixel is recovered by averaging all observed pixels in Non-local means filtering [4]. Many changes are performed in the original Non-local means filter to improve the performance.

A. Original Non-Local means algorithm

The non-local means algorithm for noise removal was proposed by A. Buades et al. [4][5]. The estimated value NL[v](i), for a pixel i, given a discrete noisy image $v = \{v(i) \mid i \in I\}$, is computed as a weighted average of all the pixels in the image[4],

$$NL[v](i) = \sum_{j \in I} w(i, j) v(j) \quad (4)$$

where the weights $\{w(i,j)\}_j$ depend on the similarity between the pixels i and j, and the conditions $0 \leq w(i,j) \leq 1$ and $\sum_j w(i,j) = 1$ are satisfied.

Two pixels i and j are similar if the intensity gray level vectors $v(N_i)$ and $v(N_j)$ are similar, where N_k denotes a square neighborhood of fixed size and centered at a pixel k. This similarity is measured as a decreasing function of the weighted Euclidean distance [4] $\|v(N_i) - v(N_j)\|_{2,a}^2$, where a>0 is the standard deviation of Gaussian kernel. The Euclidean distance when applied to noisy neighborhoods raises the following equality [4],

$$E\|v(N_i) - v(N_j)\|_{2,a}^2 = \|u(N_i) - u(N_j)\|_{2,a}^2 + 2\sigma^2 \quad (5)$$

The weights are defined as [4],

$$w(i,j) = \frac{1}{z(i)} e^{-\frac{\|v(N_i)-v(N_j)\|_{2,a}^2}{h^2}}$$

where Z(i) is the normalizing constant

$$Z(i) = \sum_j e^{-\frac{\|v(N_i)-v(N_j)\|_{2,a}^2}{h^2}}$$

and the parameter h acts as a degree of filtering. It controls the decay of the exponential function and therefore the decay of weights as a function of the Euclidean distances. The advantage of this method is that it preserves image details when denoising.

The experiments were simulated by adding Gaussian white noise of standard deviation σ to the true image [4]. The objective was to compare the visual quality of the restored images, the non presence of artifacts and the correct reconstruction of edges, textures and details.

B. *Non-local means filter with maximum likelihood estimator*

Nonlocal-Means (NL-means) filter removes noise and enhances edge information [5]. Y.Guo et al proposed Modified Nonlocal –based (MNL) filter [6] to adapt for the speckle reduction with the Rayleigh distribution noise.

The MNL method consists of two steps. The maximum likelihood (ML) estimation to calculate the initial noise-free intensity is done first. Then the NL-means algorithm is used to restore details. The ML estimator does not retain fine structure details and usually makes edge blurred. MNL speckle filter includes ML estimator and NL-means filter.

1) ML estimator: For each pixel i in the noise image g,

a) Take the window Δi, which is defined as the neighbourhood of pixel i and M_i which is defined as a square neighborhood of pixel i;

b) Compare the average intensity in Δi to discard unwanted ones;

c) Compute the initial noise-free value fML(i) using

$$f(i) = \hat{\sigma}_g\,(\sigma_\eta)^{-1} = \left[\left(\frac{1}{2\pi(\sigma_\eta)^2}\right).\sum_{k=1}^{n} g^2\,(i_k)\right]^{1/2} \quad (2)$$

where g(i) is a noisy pixel, σ_η is the shape parameter.

2) NL-means filter: For each pixel i in the ML filtered image f ML,

a) Take the search window Ωi and the neighborhood window N_i ;

b) For each pixel j in the search window, compute d(i,j), Z(i) Compare the and w(i,j).

$$d(i,j) = G_\rho \|g(N_i) - g(N_j)\|^2 \quad (3)$$

where G_ρ is a normalized Gaussian weighted function with zero mean and ρ standard deviation.

Then w(i,j) is calculated as,

$$w(i,j) = \left(\frac{1}{Z(i)}\right).\exp\left(\frac{-d(i,j)}{h^2}\right) \quad (4)$$

$$Z(i) = \sum_{j\in 1} \exp\left(\frac{-d(i,j)}{h^2}\right) \quad (5)$$

Here Z(i) is the normalized constant. The parameter h acts as a degree of filtering.

c) Given a discrete noisy image g = {g(i) | i ϵ I}, the filtered value NL(g(i)) is calculated as a weighted average of all pixels in the image.

$$NL(g(i)) = \sum_{j\in 1} w(i,j) g(i) \quad (6)$$

To evaluate the performance of the MNL, Y.Guo et al [6] optimized three parameters of the MNL and tested it on synthetic images and clinical ultrasonic images. The three optimized parameters are h (decay of exponential function), radius of similar neighborhood and radius of search window. The MNL performance was compared with six other filters namely NL –means filter, ML estimator, Lee filter, Median filter, SRAD and Med-wavelet filter. The MNL can preserve more true edges, discarding the false ones. It suppresses the speckle in ultrasonic images. Since the MNL filter makes use of the image redundancy, it is time-consuming in 2-dimensional case.

C. *Bayesian Non-Local means based filter*

Coupe et al [7] proposed an adapted method based on Bayesian formulation of non-local means filter for speckle noise reduction. To reduce the computational complexity of the algorithm, a blockwise approach is introduced in which a weighted average of patches is performed instead of weighted average of pixel intensities. This approach includes the following:

1) Paritioning the image Ω into overlapping blocks B_{ik} of size $P=(2\alpha+1)^d$ (d is the dimensionality of image) such as $\Omega=U_k B_{ik}$.

2) Restoration of a block B_{ik} based on a non-local means scheme defined as

$$NL(u)(B_{ik}) = \sum_{B_j \epsilon V_{ik}} w(x_{ik}, x_j) u(B_j) \quad (7)$$

$$w(x_{ik}, x_j) = \frac{1}{Z_{ik}} e^{-\frac{\|u(B_{ik})-u(B_j)\|_2^2}{h^2}} \tag{8}$$

where $u(B_i) = \left(u^{(1)}(B_i), \ldots, u^{(P)}(B_i)\right)^T$ is an image patch containing the intensities of the block B_i, Z_{ik} is a normalization constant.

3) Restoring the pixels values based on the restored intensities of the blocks they belong to. The final restored intensity of pixel x_i is defined as

$$NL(u)(x_i) = \frac{1}{|A_i|} \sum_{i \epsilon A_i} A_i\,(l) \tag{9}$$

Based on Bayesian interpretation of the non-local means filter [8], the blockwise NL means can be written as

$$NL(u)(B_{ik}) = \frac{\frac{1}{|V_{ik}|}\sum_{j=1}^{|V_{ik}|} p(u(B_{ik})|u(B_j))p(u(B_j))u(B_j)}{\frac{1}{|V_{ik}|}\sum_{j=1}^{|V_{ik}|} p(u(B_{ik})|u(B_j))p(u(B_j))} \tag{10}$$

where $p(u(B_{ik})|u(B_j))$ and $p(u(B_j))$ respectively denote the distribution of $u(B_{ik})|u(B_j)$ and the prior distribution of patches.

Evaluations were performed on synthetic data with different noise levels and different speckle simulations [7]. Experiments [7] shows that the filter outperforms the classical implementation of the NL means filter as well as SRAD (Speckle Reducing Anisotropic Diffusion)and the SBF (Squeeze Box Filter).

D. *Median Non-Local means based filter*

Chung et al [9] proposed a method that incorporates a median filtering operation indirectly in the non-local means method. This provides more robust estimation of the weights used to average the pixels in the image. In this approach the weights are estimated from a media filtered version of the image rather than the noisy image. The weights are then used to average the pixels of the original noisy image. An auxiliary vector m, is defined as [9],

$$m=\text{Median}\,(x,s) \tag{11}$$

where x is the observed noisy image, s is the window size for the median operator. The non-local mean is computed as a weighted average of all the pixels in image x in the search window defined as [9],

$$NL(x_i) = \sum_{j \epsilon N(i)} w_{ij}\,(m_i)x_j \tag{12}$$

with

$$w_{ij}(m_i) = \frac{1}{Z(i)} \exp\left\{\frac{\|(m_i - m_j)\|_{2,a}^2}{h^2}\right\} \tag{13}$$

where Z(i) is the normalizing factor, m_i and m_j are two vectors on the auxiliary image 'm' . The median non-local means filter when combined with the anatomical knowledge [9] resulted in effective suppression of noise.

E. *Non-local means filter with optimized weight kernel and novel neighborhood pre-classification strategy*

Rui et al [10] introduced an optimized weight kernel of non-local means filter and an improved neighborhood pre-classification strategy. In original non-local means method, the similarity between two pixels i and j (also called weight) depends on their neighborhoods $N^d\{i\}$ and $N^d\{j\}$ multiplied by a Gaussian kernel. In order to make the neighborhoods with similar structure receive a larger weight, the weght kernel is modified as [10],

$$w(i,j) = \exp\left(-\left[\frac{\|y(N^d\{i\}) - y(N^d\{j\})\|^2}{h^2}\right]^2\right) \tag{14}$$

In the non-local means algorithm, the weight computation of dissimilar patches reduces the precision and increases the computational costs. Rui et al [10] designed a new type of filter to pre-classify the image patches according to similarity. Each patch is divided into four sub-patches evenly and the non-local patches are pre classified according to the ratio of the sum of diagonal sub-patches, followed that the similar patches are used to calculate the weight as,

$$W_{pre}\,(i,j) = \begin{cases} w(i,j), & \eta_1 < \frac{\sum_{j \epsilon P_a, P_d} y(j)}{\sum_{j \epsilon P_{a'}, P_{d'}} y(j)} < \eta_2 \\ \text{and} & \eta_3 < \frac{\sum_{j \epsilon P_b, P_c} y(j)}{\sum_{j \epsilon P_{b'}, P_{c'}} y(j)} < \eta_4 \\ & 0,\ \text{otherwise} \end{cases} \tag{15}$$

It is noted from the denoised results [10], that this method effectively suppresses the noise and preserves more details.

IV. Parameters used for Performance Analysis of Non-Local Means Based Filters

To determine the performance of the denoising filters in terms of efficiency of removal of noise and enhancement of useful image information, the following parameters are analyzed. Table I provides the metrics used for performance analysis.

V. Conclusion

The Non-Local means algorithm not only compares the grey level in a single point but the geometrical configuration in a whole neighborhood. The non-local means filter with maximum likelihood estimator suppresses speckle noise in ultrasound images. It yields better noise attenuation and edge enhancement. It works better for images with fine structures.

Bayesian non-local means based filter introduces the Pearson distance as a relevant measure for patch comparison. This filter outperforms the classical implementation of non-local means filter as well as the SRAD and SBF filters. The median non-local means filter suppress noise effectively without sacrificing low contrast details. The incorporation of anatomical knowledge using segmentation free approach aided in preserving organ boundaries. The measure of similarity of neighborhood with an optimized weight kernel and pre-classification of neighborhoods in the filtering process results in precise and effective non-local means based filter.

TABLE I. PARAMETERS FOR ANALYSIS OF PERFORMANCE OF NON-LOCAL MEANS BASED FILTERS

Denoising filter	Performance metrics used	Range of value for better performance	Range of values of denoising filter	Type of noise filtered
Non-Local means filter [4]	Mean Square Error (MSE)	Lower value indicate small differences between original and denoised image.	68 (Lena image, σ=20) 292 (Baboon image, σ=20)	Gaussian noise
Non-local means filter(NLM) with maximum likelihood estimator [6]	Signal-to-Noise Ratio (SNR)	Higher values show better image quality	Under different noise conditions the values for MNL filter ranges from 18-20	Speckle noise
	Mean Structure Similarity (MSSIM)	Closer to unity for optimal measure of similarity	0.884 to 0.961	
	Figure of Merit (FOM)	Closer to unity for optimal measure of similarity	0.753 to 0.915	
Bayesian Non-local means based filter [7]	Signal-to-Noise Ratio (SNR)	Higher values show better image quality	64.13 (σ=0.2) 53.12 (σ=0.4) 42.13 (σ=0.8)	Speckle noise
Median Non-local means base filter [9]	Figure of Merit (FOM)	Closer to unity for optimal measure of similarity	Effective noise suppression and highest lesion contrast.	Gaussian noise
NLM filter with optimized weight kernel and preclassification strategy [10]	Peak Signal to Noise Ration (PSNR)	Typical value is between 30 and 50 dB. Higher PSNR values show better image quality.	70.14 (σ=0.01) 73.59 (σ=0.005)	Gaussian noise
	Structure Similarity Index Map (SSIM)	Closer to unity for optimal measure of similarity	0.42 (σ=0.01) 0.47 (σ=0.005)	

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