A Study on Different Types of Normalization Methods in Adaptive Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

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Abstract—Operational Research is the way to attain an optimal decision making using MCDM (Multi-criteria Decision Making). Multi-criteria decision making is a method of selecting an alternative from a set of available alternatives or choices according to their criterion. A number of different methodologies are available in MCDM, in which TOPSIS is one of the best methods to rank the alternative. In this paper different normalization techniques are applied for TOPSIS and it also shows the best normalization technique which suits the particular domain in an adaptive manner. This adaptive normalization is implemented using Ontology.

Keywords—Multi-Criteria Decision Making-MCDM; TOPSIS; Adaptive TOPSIS; Normalization; Adaptive normalization; Ontology

I. INTRODUCTION

Multi-Criteria Decision Making (MCDM) was developed in the middle of 1960s and it has prescribed as an important part of decision sciences [1] [2] [3]. It is used to define the ways of ranking and selecting the most suitable alternative among the set of alternatives or choices which is exemplified by multiple and differing criteria.

Moreover, In MCDM problems criteria are not always independent and a possible relationship between a pair of criteria is called prioritization [4] [5] [6] [7] [8]. An example assurance the criteria of safety and cost in the case of buying a bicycle for child [8], buying a car [9], etc., There are many MCDM techniques are available. Most widely used techniques are Technique for the Order Preference by Similarity to Ideal Solution (TOPSIS), Analytic Hierarchy Process (AHP), Elimination and Expressing Reality (ELECTRE), Preference Ranking Organization Method for enrichment evaluation (PROMTHEE) and VIKOR method.

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Most commonly used method is TOPSIS (Technique for Order Performance by Similarity to Ideal Solution). It is familiar while dealing with MCDM problems in reality. TOPSIS describes that the chosen alternative should have the shortest distance from the Ideal solution. It contains many numbers of steps. Normalization is an important process because data should be transferred into comparable values using normalization [16]. In our work has been proposed to determine the behavior of TOPSIS method under different normalization procedures.

Normalization changes the different measurable values into similar units. Criteria are a parameter to normalization which is performed in adaptive manner is called adaptive normalization.

The rest of the paper is set out as follows: In section 2 the literature review are discussed, in Section 3 briefly describes about ontology, in Section 4 describes about TOPSIS, Section 5 explains about various normalization procedures, section 6 describes about experimental design, section 7 describes about result and discussed with normalization and ontology and finally ended up with the conclusion, findings of the study and the future work.

II. PRIOR RESEARCH

TOPSIS is relatively very simple and efficient with systematic procedure. TOPSIS gives the best artificial lift method selection for different situations of oil fields. Normalization is the process of converting different scales and units among different criteria into regular measurable units to allow comparisons across the criteria. There are many types of normalization that can be used in the TOPSIS method such as vector normalization, linear sum based normalization, linear max normalization, linear maxmin normalization.

The literature review look at the scholarly literature concerning to decision analysis. In order to recognize those articles that offered most valuable information.

1. The classical TOPSIS uses vector normalization [10] [11].

$$\mathbf{r}_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^{n} f_{ij}^2}}$$

2. Lai and Hwang (1994) introduced linear normalization into the TOPSIS

$$\mathbf{r}_{ij} = \frac{\mathbf{f}_{ij}}{\mathbf{f}_i^* - \mathbf{f}_i^-}$$

Where,

$$f_i^* = max_j f_{ij}$$

 $f_i^- = min_j f_{ij}$
 $i = 1, 2...m; j=1, 2...n$

Taking into deliberation the notation that normalization procedure may affect the final MCDM solution. This paper is discussed about various normalization and which one is beneficial for particular domain (i.e.) Adaptive Normalization.

A. Review on MCDM

In many cases multi-criteria decision making is a branch of operations research which covenant with the procedure of constructing decisions in the presence of various objectives.

In those methods which can handle both type of quantitative and qualitative criteria, distribute the general characteristics of oppose among criteria incommensurable units and hard in design or selection of alternative [12].

There are many kinds of methods available in MCDM. The mostly used methods are TOPSIS, AHP, ELECTRE and PROMETHEE. MCDM is used in many areas which take decision using the techniques. Among those TOPSIS would be used in many domains such as banking domain, material selection, manufacturing section, bankruptcy, bicycle selection, car selection, course selection teaching performance, student performance and also in academic performance of the teacher and student. MCDM methods are shown by the below diagram:



Fig. 1. MCDM Decision Making Hierarchy

- *B. Major steps involved in MCDM:*
- a) Establishing system evaluation criteria that relate system capabilities to goals
- b) Developing alternative systems for attaining the goals (generating alternatives)
- c) Evaluating alternatives in terms of criteria (the value of the criterion functions)
- d) Applying a normative multi-criteria analysis method.
- e) Keeping one alternative as "Optimal" (preferred)
- f) If the final solution is not accepted, gather new information and go into the next iteration of multicriteria optimization.



Fig. 2. Different Phases of MCDM Process

MCDM process deals with different phases of MCDM like criteria weight, normalization, aggregating and finally select the best alternative as shown in the above Fig.2.

C. Aggregate methods

This is a method which is used to compare MCDM methods. It has 4 steps [13].

• Rank Average Method

Rank average method which ranks alternative on the basis of average of calculated ranks from different MCDM methods.

Borda method

Each and every MCDM method ranks all of the alternatives. If there are k alternatives, each alternative receives **P** points for first choice, **P-1** points for second choice, and so on. The winner which has the most points with the alternative [14].

• Copeland method

Copeland method starts with the end of Borda method. It computes number of losses for all of the alternatives. Subtracting number of loses from number of wins; it concludes the prominence of any alternatives.

Aggregate Stage

In an aggregate stage with considering ranking strategies (rank average, Borda and Copeland method) and through creating one poset (Partially Ordered Test) it will arrive to "consensus"

III. BRIEF DESCRIPTION–ONTOLOGY

In recent years, the development of ontology explicit formal specification of the terms in the domain [15] and Ontology is a specification of conceptualization. Ontology has become common on the World Wide Web.

Ontology is a formal description of concepts in a domain of discourse (classes), properties of each concept describing various features and attributes of the concept (slots), and restrictions on slots (facets). Ontology together with a set of individual instances of classes constitutes a knowledge base.

A. Fundamental Rules of Ontology

First, we would like to emphasize some fundamental rules in ontology design to which we will refer many times. These rules may seem rather dogmatic. They can help, however, to make design decisions in many cases.

- There is no one correct way to model a domain—there are always possible alternatives. The best solution always depends on the application that you have in mind and the extensions that we anticipate.
- Ontology development is necessarily an iterative process.
- Concepts in the ontology should be close to objects (physical or logical) and relationships in your domain of interest. These are most likely to be nouns (objects) or

verbs (relationships) in sentences that describe your domain.

Practically, developing ontology includes:

- Defining classes in the ontology
- Arranging the classes in a taxonomic (sub class– super class) hierarchy
- Defining slots and describing allowed values for these slots
- Filling in the values for slots for instances.

B. Iterative Process

This process of iterative design will likely continue through the entire lifecycle of the ontology.

Step 1: Determine the domain and scope of the ontology

Step 2: Consider reusing existing ontology

Step 3: Enumerate important terms in the ontology

Step 4: Define the classes and the class hierarchy

Step 5: Define the properties of classes—slots

Step 6: Define the facets of the slots

IV. A DETAILED STUDY ON TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was initially proposed by Hwang and Yoon (1981) [10]. According to TOPSIS the best alternative is the one that is nearest to the ideal solution and farthest from the nadir (negative ideal) solution [10] and also the best alternative has the shortest Euclidean distance.

Table I: Ideal Solution (benefit wise, cost wise)

Ideal Solution	Benefit Criteria	Cost Criteria
Positive Ideal Solution	Maximize	Minimize
Negative Ideal Solution	Minimize	Maximize

Note: Please refer [17]

TOPSIS has been shown to be one of the best MCDM methods in addressing the rank traversal issue, which is the change in the ranking of alternatives when a non-optimal alternative is introduced Despite its popularity and simplicity in concept, the conventional TOPSIS is often criticized because of its inability to discuss with uncertainty imprecision inherent in the real world problems.

An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing. TOPSIS allow trade-off between criteria, where a poor result can be negated by a good result in another criterion. This provides more realistic form of modeling than non-compensatory methods which include or exclude alternative solutions based on hard cut-offs. Normalization is usually required as the parameters or the criteria are often incongruous dimensions in multi criteria problems. It is a method of compensatory aggregation.

S. No	Characteristics	TOPSIS	
1	Category	Cardinal information, information on attribute, MADM	
2	Core Process	The distance from PIS and NIS (Cardinal absolute measurement)	
3	Attribute	Given	
4	Weight Elicitation	Given	
5	Consistency check	None	
6	No. of attributes accommodated	Many more	
7	No. of alternatives accommodated	Many more	
8	Others	Compensatory operation	

Table II: Characteristics of TOPSIS

The major steps are involved in TOPSIS method is explained below:

Step 1: The relevant objective or goal, decision criteria and alternatives of the problem are identified in this step.

Step 2: This step produces a decision matrix of criteria and alternatives based on the information regarding the problem. If the number of alternative is M and the number of criteria is N, then the decision matrix having an order of M x N is represented as below:

$$\mathbf{D}_{\mathrm{MxN}} = \begin{bmatrix} a_{11} & \dots & a_{1N} \\ \dots & \dots & \dots \\ a_{M1} & \dots & a_{MN} \end{bmatrix}$$

Where,

an element a_{ij} of the decision matrix D_{MxN} represents the actual value of the ith alternative in term of jth criterion.

Step 3: In this step the decision matrix is converted into normalized decision matrix, so that the scores obtained in different scales becomes comparable. An element r_{ij} of the normalized decision matrix R can be calculated as follows:

$$\mathbf{r}_{ij} = \frac{a_{ij}}{\sqrt{\sum_{j=1}^{m} (a_{ij}^2)}} \tag{1}$$

Step 4: The weighted normalized matrix is obtained by multiplying each column of the normalized decision matrix R with the associated criteria weight corresponding to that column. Hence an element V_{ij} of weighted normalized matrix V is represented as follows:

$$\mathbf{V}_{ij} = \mathbf{W}_{ij} * \mathbf{r}_{ij} \tag{2}$$

Step 5: This step produces the positive ideal solution (A^{*}) and negative ideal solution (A^{-}) in the following manner.

$$A^{*} = \{(\max v_{ij} / j \in J), (\min v_{ij} / j \in J^{-}) \text{ for } I = 1, 2, 3, m\} = \{v_{1}^{*}, v_{2}^{*}, \dots, v_{n}^{*}\}$$
(3)

$$A^{-} = \{(\min v_{ij} / j \in J), (\max v_{ij} / j \in J^{-}) \text{ for } I = 1, 2, 3, \dots, m\} = \{v_{1}^{-}, v_{2}^{-}, \dots, v_{n}^{-}\}$$
(4)

Where,

 $J = \{j=1,2,...,n / j \text{ associated with benefit or positive criteria}\}$

 $J^- = \{j=1,2,...,n / j \text{ associated with cost or negative criteria}\}$

For the benefit criteria the decision maker wants to have the maximum value among the alternatives. Therefore, A^* indicates the positive ideal solution and A^- indicates the negative ideal solution.

Step 6: The N dimensional Euclidean distance method is applied, as shown below, to measure the separation distances of each alternative from the positive and negative ideal solution.

$$\mathbf{S}_{i}^{*} = \left\{ \sqrt{\sum_{j=1}^{n} (V_{ij} - V_{j}^{*})^{2}} \right\}, \, i = 1, 2, \dots, m$$
(5)

$$\mathbf{S}_{i}^{-} = \left\{ \sqrt{\sum_{j=1}^{n} (V_{ij} - V_{j}^{-})^{2}} \right\}, i = 1, 2, \dots, m$$
(6)

Where,

 S_i^* and S_i^- are the separation distances of alternative i from the positive ideal solution and negative ideal solution respectively.

Step 7: In this step relative closeness (C_i^*) value of each alternative with respect to the ideal solution is determined using the equation below. The Value of the C_i^* lies within the range from 0 to 1.

$$C_{i}^{*} = \frac{s_{i}^{*}}{s_{i}^{*} + s_{i}^{*}}$$
(7)

Step 8: All the alternatives are now arranged in descending order according to the value of C_i^* . The alternative at the top of the list is the most preferred one.

TABLE III: Some Applications of TOPSIS

S. No	Application areas	Number of attributes	Number of alternatives	Proposed by
1	Company financial ratios comparison	Four attributes	7 alternatives	Deng
2	Expatriate host country selection	Six major attributes (25 sub-attributes)	10 alternatives	Chen and Tzeng
3	Facility location selection	Five attributes	4 alternatives	Chu
4	Gear material selection	Five attributes	9 alternatives	Milani
5	High-speed transport system selection	Fifteen attributes	3 alternatives	Janic

6	Multiple	Two attributes	18 alternatives	Yang and
	response	(or responses)		Chou
	selection			
7	Manufacturing	Five major	5 alternatives	Yoon and
	plant location	attributes (16		Wang
	analysis	sub-attributes)		_
8	Water	Six attributes	12 alternatives	Srdjevic
	management	(with 3 demand		
	_	points)		
9	Solid waste	Twelve	11 alternatives	Cheng
	management	attributes		_
10	Robot	Four attributes	27 alternatives	Parkan
	selection			and Wu
11	Rapid	Six attributes	6 alternatives	Byun and
	prototyping-			Lee
	process			
	selection			

V. DISCUSSION ON DIFFERENT NORMALIZATION PROCEDURES

Information stored in a decision matrix is usually incommensurable that is performance ratings in relation to different criteria are usually expressed using different units of measure. Therefore, data should be transferred into comparable values using normalization procedure [16].

To compare the alternative on each attribute, the normalized process is usually made column-wise, and the normalized value will be a positive value between 0 and 1. In this way computational problems, resulting from different measurements in the decision matrix are eliminated [26].

There are different types of normalization available in MCDM. They are

1. Vector Normalization The vector normalization has the following general form:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}$$
 for $i = 1, 2, ..., m$;

Where,

aij - Original ratings of decision matrix

j=1,2,...,n

 r_{ij} - Normalized value of the decision matrix

2. Linear Max-Min Normalization

The Linear Max-Min Normalization technique has the following general form:

The normalized value r_{ij} for benefit criteria is acquired by

$$r_{ij} = rac{a_{ij} - a_j^{min}}{a_j^{max} - a_j^{min}}$$
 for $i = 1, 2, ..., m$
 $j = 1, 2, ..., n$

The normalized value r_{ij} for cost criteria is acquired by

$$r_{ij} = \frac{a_j^{max} - a_{ij}}{a_j^{max} - a_j^{min}} \text{ for } i = 1, 2, ..., m$$
$$j = 1, 2, ..., n$$

Where,

 a_j^{max} . Maximum ratings of the alternatives for each criterion C_j (j =1, 2... n)

 a_j^{min} - Minimum ratings of the alternatives for each criterion C_j (j =1, 2,...,n)

3. Linear Sum Based Normalization This method divides the ratings of each

This method divides the ratings of each alternative by the sum of ratings of each criterion as follows:

> $r_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_j}$ for i = 1, 2,...,m j = 1, 2,...,n

Where a_{ij} - Performance ratings of each alternative for criteria C_j (j=1,2,...,n)

4. Linear Max Normalization

The linear max normalization has the following general form:

The normalized value $r_{ij}\ for\ benefit\ criteria\ is\ calculated\ by$

$$\mathbf{r}_{ij} = \frac{a_{ij}}{a_j^{max}}$$
 for i = 1, 2,...,m
j = 1, 2,...,n

The normalized value $r_{ij} \mbox{ for cost criteria} \mbox{ is computed by }$

$$r_{ij} = 1 - \frac{a_{ij}}{a_j^{max}} \qquad \text{for } i = 1, 2, ..., m$$
$$j = 1, 2, ..., n$$

Where,



5. Gaussian Normalization

To normalize the ratings of each alternative i based on the criteria j is calculated by

$$\mathbf{r}_{ij} = \frac{a_{ij} - a_i}{\sqrt{\sum_{j=1}^{n} (a_{ij} - a_i)^2}}$$

for
$$i = 1, 2, ..., m; j = 1, 2, ... n$$

Where,

 a_{ij} - represent the original rating of each alternative based on the criteria. a_i - stands for the averaging rating of alternative i.

6. Linear Normalization (1)

$$x_{ij} = \frac{x_{ij}}{x_j^*}$$
, $i = 1, 2, ..., m; j = 1, 2, ..., n;$

 $x_i^* = \max_i \{x_{ij}\}$ for benefit attributes

$$r_{ij} = \frac{x_j^{-}}{x_{ij}}, i = 1, 2, ..., m; j = 1, 2, ..., n;$$

$$x_j^{\infty} = \min_i \{x_{ij}\}$$

or $r_{ij} = 1 - \frac{x_{ij}}{x_j^{*}}, i = 1, 2, ..., m; j = 1, 2, ..., n;$

$$x_j^{*} = \max_i \{x_{ij}\} \text{ for cost attributes}$$

7. *Linear Normalization (2)*

$$r_{ij} = \frac{x_{ij-x_j^-}}{x_j^* - x_j^-}$$
 for benefit attributes
$$r_{ij} = \frac{x_j^* - x_j}{x_j^* - x_j^*}$$
 for cost attributes

8. *Linear Normalization(3)*

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$$
, $i = 1, 2, ..., m; j = 1, 2, ..., n$.

9. Non-monotonic normalization

$$e^{-\frac{z^2}{2}}, z = \frac{x_{ij}-x_j^0}{\sigma_j};$$

 $x_j^{\mathbb{Q}}$ is the most favorable value and σ_j is the standard deviation of alternative ratings with respective to the jth attribute. It is less used in the literature.

10. Decoupling normalization

This procedure converts the ratings of an item by a user into a probability for the item to be favored by the user. The probabilistic measurement is determined based on the following two assumptions:

- When a large portion of items are rated by a user as no more than category r, items in this rating category r are likely to be favored by the user.
- When more items are rated as category r, it becomes less likely for the user to favor items in the category r.

Based on the assumptions, a special formula named halfway accumulative distribution was proposed [5] to convert the ratings of an item into probability measure. The general form this expression is:

 $p_y(R \text{ is favored}) = p_y(Ratings \le r) - p_y(Ratings = r)/2$

Where,

Py (Ratings \leq r) stands for the percentage of items that are rated no more than category r $p_y(Ratings=r)$ stands for the percentage of items that are razed as r.

VI. EXPERIMENTAL DESIGN

Normalization is the process of normalizing the ratings of different alternatives into same range. Initially all criteria

are dimensionless. So we have to eliminate the units of each criterion. So the criteria are modified into comparable values. We have decided to use MATLAB for computational task.

Various normalization procedures are available in MCDM. MCDM methods use normalization for eliminating different measures of criteria. Any MCDM methods use any one normalization procedure. So there is no proper way of usage normalization. It gives space and time flexibility. We have proposed adaptive TOPSIS that may give changes in the middle steps of the TOPSIS such as Normalization matrix, weighted matrix and distance matrix. So we are going to use ontology for choosing particular Normalization or weighted matrix or distance matrix. In this paper also compute all the normalization by using general formulae of each normalization matrix and uses one of the best normalization depends on the highest value(r).

We have **Proposed Adaptive TOPSIS** work that is described by a below flow chart as follows:



Fig. 3. Different Stages of Adaptive TOPSIS

In the above Fig. 3, first foremost step is to find out the decision matrix and then selects the suitable normalization (adaptive normalization) is done by ontology technique. Then only calculate the normalized matrix for dynamically selected normalization. Then determine the weighted matrix and distance matrix. Finally calculate the Relative Closeness Co-efficient (RCC) and rank the alternatives. The best alternative is one which has the highest value of RCC.

Banking domain, best engineering college selection and bankruptcy in cuddalore and Pondicherry uses different normalization techniques. All normalization techniques are applied for the above mentioned domain in adaptive manner using ontology.

VII. RESULT AND DISCUSSION

This section provides details about result or report. TOPSIS has number of steps to select the best alternative which is a part from MCDM. Adaptive TOPSIS is slightly different from TOPSIS. Adaptive TOPSIS method which made changes on normalization matrix that can be changed according to the application is called adaptive normalization

In the middle steps, Normalization is the first process which is used for normalizing values or change into comparable units from different measures. Adaptive normalization has been applied here according to the user requirements by using ontology. So it gives the best result for the user.

Each and every person uses different normalization techniques in TOPSIS method. So there is no proper way for using best normalization for particular application. Normalization can also be evaluated using time complexity, space complexity, relative closeness coefficient, rank traversal, rank occurrence and sensitivity analysis,. For example, if we take four normalization techniques in TOPIS Ontology gives the rules for selecting the normalization for that particular application. Suitable normalization is accomplished by satisfying the rules or conditions in ontology. Then further steps of the TOPSIS like normalized decision matrix, weighted matrix, distance matrix and relative closeness will be done. Finally, rank the alternatives and select the best one.

VIII. CONCLUSION

This study proposes the best normalization technique for any applications (domain) using ontology. In this paper we have proposed adaptive normalization techniques for any domain. It is been implemented in various domains like banking, bankruptcy and also to find the best engineering college in cuddalore and Pondicherry. All normalization techniques are applied for these domains using the technique adaptive normalization using ontology and select the best normalization according to the domain. It provides best decision making using Ontology technique. In future adaptive methods can be implemented for weight and distance.

REFERENCES

- Koele, P. (1995). "Multiple attribute decision making: An Introduction". In K.P.Yoon, & C.L. Hwang(Eds.), Sage university paper series on quantitative applications in the social sciences decision making" IEEE Transactions on Systems, Man and Cybernetics, 18, 183-190.
- [2] Zhang, S.F., Liu, S. Y., & Zhai, R. H. (2011). "An extended GRA method for MCDM with interval-valued triangular fuzzy assessments and unknown weights". Computers and Industrial Engineering, 61, 1336-1341.
- [3] Chen, S. M., & Wang, C. H. (2009). "A generalized model for prioritized multicriteria decision making systems". Expert Systems with Applications, 36, 4773-4783.
- [4] wang, C. H., & Chen, S. M. (2007). "A generalized model for multicriteria decision Making". In proceedings of international conference on machine learning and cybernetics (pp. 1815-1820).
 [5] Yan, H.B., Huynh, V.N., Nakamori, Y., & Murai, T. (2011). "On
- [5] Yan, H.B., Huynh, V.N., Nakamori, Y., & Murai, T. (2011). "On prioritized weighted aggregation in multi-criteria decision making". Expert Systems with Applications, 38, 812-823.
 [6] Yager, R. R. (2004). "Modeling prioritized multicriteria decision
- [6] Yager, R. R. (2004). "Modeling prioritized multicriteria decision making". IEEE Transactions on Systems, Man, and Cybernetics, 34, 2396-2404.
- [7] Yager, R. R. (2008). "Prioritized aggregation operators. International Journal of Approximate reasoning", 48, 263-274.
- [8] Herrera-Viedma, E., Pasi, G., Lopez-Herrera, A. G., & Porcel, C. (2006). "Evaluating the information quality of websites: A methodology based on fuzzy computing with words". Journal of American Society for Information Science and Technology, 57, 538-549.
- [9] Hwang, C.L. and Yoon, K., (1981). "Multiple Attribute Decision Making: Methods and Applications", Springer-Verlag, Berlin/Heidelberg/New York.
- [10] Triantaphyllou, E., (2000). "Multi-criteria Decision Making Methods: A Comparative 0Study", Kluwer Academic Publishers, Dordrecht.
- [11] Pohekar, SD & Ramachandran, M.(2004) Application of multicriteria decision making to sustainable energy planning A review. Renew Sustain Energy, 8, 365, -8.
- [12] Wang, Y.M. Yanga, J.B., & Xu, D.L. (2005). A Preference aggregation method though the estimation of utility intervals. Computers & Operations Research, 32, 2027-2049.
- [13] Anisseh, M., Rosnah, Y., & Shakarami, A. (2009). Aggregating group MCDM problems using a fuzzy Delphi model for personnel performance appraisal. Scientific research and Essay, 4(5), 381-391.
- [14] Gruber, T. R. (1993). Originally in N. Guarino & R. Poli, (Eds.), International Workshop on Formal Ontology, Padova, Italy. Revised August 1993. Published in International Journal of Human-Computer Studies, special issue on Formal Ontology in Conceptual Analysis and Knowledge Representation (guest editors: N. Guarino and R. Poli) (to appear).
- [15] Zavadskas, E.K, Kaklauskas, A., Turskis, Z., Tamošaitien'e, J. (2009b). Multi-attribute decision-making model by applying grey numbers. *Informatica*, 20(2), 305–320.
- [16] Wang, T.C., Chang, T.H. (2007). Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment. *Expert Systems with Applications*, 33(4), 870–880.