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A Study On c*g-Homeomorphisms In Topological Spaces

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Abstract

In this paper, we have introduced the concept of c*g-continuous, c*g -closed, c*g -open, c*g -irresolute, c*g -homeomorphisms and (c*g)* -homeomorphisms in Topological space.

Key words: c*g-continuous, c*g -closed, c*g -open, c*g -irresolute, c*g -homeomorphisms and (c*g)* -homeomorphisms

1. INTRODUCTION

Levine [6], [28] introduced the concept of generalized closed sets and strong continuity in topological space. Dunham and Levine [2] further studied some properties of generalized closed sets. Sundaram [13] introduced the concept of generalized continuous function and proved that the class of generalized continuous function includes the class of continuous function and studied several properties related to it. Pushpalatha [26] introduced the concept of strongly generalized continuous function and studied several properties related to it.
Pallaniappan[23]introduced the concept of regular generalized continuous function and proved that the class of regular generalized continuous function includes the class of continuous function and studied several properties related to it.

Malghan[20]introduced and investigated some properties of generalized closed maps in topological spaces. The concept of generalized open map was introduced by Sundaram[13]. Biswas [01] defined semi open mappings as a generalization of open mappings and studied several of introduced the concepts of strongly generalized closed maps in topological spaces.

Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science. In 1972, Crossely and Hildebrand [30] introduced the notion of irresoluteness. Many different form of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. It's important in significant in various areas of mathematics and related science.

Several mathematicians have generalized homeomorphisms in topological spaces. Biswas[1], Crossely and Hildebrand[30], Gentry and Hoyle[32], Umehara and Maki[33] have introduced and investigated semi-homeomorphisms, somewhat homeomorphisms and g-Λ-homeomorphisms. Crossely and Hildebrand defined yet another "semi-homeomorphism" which is also a generalization of homeomorphism. Sundaram[13]introduced g-homeomorphism and gc-homeomorphism in topological spaces. Puspalatha [26] introduced strongly generalized homeomorphism and strongly g*-homeomorphism in topological spaces.

In this section, we introduced the concepts of c*g- continuous function, c*g- closed maps, c*g- open maps, c*g-irresolute maps and c*g-homeomorphisms and (c*g)*-homeomorphism in topological spaces and study their properties. It is an extension study of [27] for continuous functions.

2. PRELIMINARIES

DEFINITION: 2.1
A map f: x→y from a topological space x into a topological space y is called
(a) Continuous if $f^{-1}(V)$ is closed in x for each subset V in y.
(b) Strongly continuous if $f^{-1}(V)$ is both open and closed in y for each subset V in y(Due to Sundaram).
(c) Strongly generalized if the inverse image of every closed set in y is strongly g- closed in x.
(d) Regular generalized continuous if the inverse image of every closed set in y is rg- closed in x.

DEFINITION: 2.2
A map f: x→y from a topological space x into a topological space y is called

(a) Strongly generalized closed if for each closed set F in x, f (F) is strongly g-closed set in y.
(b) Regular generalized closed if for each closed set F in x, f(F) is Regular generalized closed set in y.

DEFINITION: 2.3
A map f: x→y from a topological space x into a topological space y is called
(a) Irresolute if $f^{-1}(V)$ is semi-open set in x for every semi-open set V in y[].
(c) Strongly generalized irresolute if $f^{-1}(V)$ is strongly g-closed in x for every strongly g-closed set V in y.
(d) Regular generalized irresolute if $f^{-1}(V)$ is rg-closed in x for every rg-closed set V in y.

3.1 C*g- CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

DEFINITION: 3.1.1 A map f: x→y from a topological space x into a topological space y is called **c***g-continuous if the inverse image of every closed set in y is c*g- closed in x.

THEOREM: 3.1.2 If a map f: x→y is continuous then it is c*g- continuous but not conversely.
Proof: - Let f: x→y be continuous. Let F be any closed set in y. Then the inverse image $f^{-1}(F)$ is closed in x. Since every closed set is c*g-closed, $f^{-1}(F)$ is c*g- closed in x. Therefore f is c*g- continuous.
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE:3.1.3 Let x= y = {a,b,c};with the topology; $\tau = \{\varphi$,x,{a, b}} and $\sigma = \{\varphi$,y,{a}}. Let f: x→y be the identity map. Then f is not continuous, since for the closed set {b,c} in y, f^{-1} ({b,c})={b,c} is not closed in x. But f is c*g- continuous.

THEOREM: 3.1.4 If a map f: x→y is strongly g-continuous then it is c*g-continuous but not conversely.
Proof: - Let f: x→y be a strongly g-continuous map. Let F be any closed set in y. Then the inverse image $f^{-1}(F)$ is strongly g- closed in x. Since every strongly g- closed set is c*g-closed set, $f^{-1}(F)$ is c*g- closed in x. Therefore f is c*g- continuous.
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.1.5 Let x = y = {a,b,c};with the topology; $\tau = \{\varphi$,X,{a}} and $\sigma = \{\varphi$,y,{b}}. Let f: $(x,\tau) \to (y,\sigma)$ be the identity map. Then f is c*g- continuous, but not strongly g- continuous. For {a,c} is closed in y, but f^{-1}({a,c})={a,c} is not strongly g- closed in x. Therefore f is not strongly g - continuous.

THEOREM: 3.1.6. Let f: x→y be a map. Then following statements are equivalent.
a) f is c*g- continuous
b) The inverse image of each open set in y is c*g-open in x.
Proof:-Assume that f: x→y is c*g-continuous. Let G be open in y. Then G^c is closed in y. Since f is c*g-continuous, $f^{-1}(G^c)$ is c*g- continuous in x. But $f^{-1}(G^c)$ = x- $f^{-1}(G)$. Thus $f^{-1}(G)$ is c*g-open in x.
Conversely assume that the inverse image of each open set in y is c*g-open in x. Let F be any closed set in y. Then F^c is open in y. By assumption $f^{-1}(F^c)$ is c*g-open in x. Hence a and b are equivalent.

THEOREM: 3.1.7 If a map f: x→y is c*g- continuous, then it is regular generalized continuous.
Proof: - Let f: x→y be c*g-continuous. Let F be any closed set in y. Then the inverse image $f^{-1}(F)$ is c*g- closed in x. Since every c*g- closed set is regular generalized closed set, $f^{-1}(F)$ is regular

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generalized closed in x. Therefore f is regular generalized continuous.
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.1.8 Let x = y = {a,b,c};with the topology; $\tau = \{\varphi$,X,{b,c}} and $\sigma = \{\varphi$,y,{a,b}}. Let f: x→y be the identity map. Then f is not c*g- continuous. Since for the closed set {c} in y, f^{-1}({c}) ={c} is not closed in x. But f is regular generalized continuous.

We illustrate the relations between various generalizations of continuous functions in the following diagram

Continuity → Strongly g- continuity → c*g- continuity → regular generalized continuity

In the above diagram none of the implications can be reversed.

3.2.c*g-CLOSED MAPS AND c*g-OPEN MAPS IN TOPOLOGICAL SPACES

DEFINITION: 3.2.1 A map f: x→y from a topological space x into a topological space y is called **c***g-closed map if for each closed set F in x, f(F) is a c*g- closed set in y.

THEOREM: 3.2.2 If a map f: x→y is closed then it is c*g- closed but not conversely.

Proof:-. Since every closed set is c*g-closed, the result follows

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.2.3 consider the topological space x = y = {a,b,c};with the topology; $\tau = \{\varphi$,X,{a}} and $\sigma = \{\varphi$,y,{a,b}}. Here c (x, τ) = {φ,X,{b,c }}; c (y, σ) = {φ,Y,{c}} and c*gc(y, σ) = {φ,y,{c},{a,b}{b,c},{a,c}}. Let f be the identity map from x onto y. Then f is c*g closed, but not a closed map. Since for the closed set {b,c} in (x, τ), f{b,c})={b,c} is not closed in y.

DEFINITION: 3.2.4 A map f: x→y from a topological space x into a topological space y is called **c***g-open map if f (u) is **c***g-open in y for every open set u in x.

THEOREM: 3.2.5 If a map f: x→y is open then it is c*g- open but not conversely.

Proof:-. Let f: x→y be an open map. Let U be any open set in x. Then f (U) is an open set in Y. Then f (U) is c*g-open, since every open set is c*g-open. Therefore f is c*g-open.

Converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.2.6 consider the topological space x = y = {a,b,c};with the topology; $\tau = \{\varphi$,X,{a, }} and $\sigma = \{\varphi$,y,{a,b}}. Here c*go (y,σ) = {φ, y, {a}, {b}, {c}, {a,b}}. Then the identity function f: x→y is c*g- open, but not open, since for the open set {a} in (x,τ), f{a})={a} is c*g- open, but not an open in (y, σ). Therefore f is not an open map in y.

THEOREM: 3.2.7. If f: x→y is regular generalized continuous and c*g-closed and A is a c*g-closed set of x, then f (A) is c*g-closed in y.

Proof:-. Let f (A)$\subseteq$ O. where O is regular generalized open set of y. Since f is regular generalized continuous, f^{-1}(O) is regular generalized open set containing A. Hence cl(A) $\subseteq$ f^{-1}(O) as A is c*g-closed. Since f is c*g-closed, f(cl(A)) is a c*g-closed set contained in the regular generalized open set O, which implies that cl[f(cl(A))] $\subseteq$O and hence cl[f(A)] $\subseteq$ O. So f (A) is c*g-closed set in y.

COROLLORY: 3.2.8. If f: x→y is continuous, closed and A is c*g-closed of x, then f(A) is c*g-closed in y.

Proof:-.Since every continuous map is regular generalized continuous and every closed map is c*g-closed, by the above theorem the result follows.

THEOREM: 3.2.9. If f : x→y is closed and h : y→z is c*g- closed then h.f : x→z is c*g- closed.

Proof: - Let f: x→y is a closed map and h : y→z is a c*g-closed map. Let v be any closed set in x. Since f: x→y is closed, f(v) is closed in y and since h : y→z is c*g-closed, h(f(v)) is a c*g-closed set in z. Therefore h.f : x→z is a c*g- closed map.

THEOREM: 3.2.10. If f : x→y is c*g- closed and A is closed set in x. Then f_A : A →y is c*g- closed.

Proof: - Let v be closed set in A. Then v is closed in x. Therefore it is c*g-closed in x. By Theorem 3.2.9. f (v) is c*g- closed. That is f_A (v) =f(v) is c*g-closed in y. Therefore f_A : A →y is c*g-closed.

3.3. c*g- IRRESOLUTE MAPS IN TOPOLOGICAL SPACES

DEFINITION: 3.3.1 A map f: x→y from a topological space x into a topological space y is called **c***g-irresolute map if the inverse of every c*g- closed(c*g-open) set in y is c*g-closed set(c*g-open) in x.

THEOREM: 3.3.2 If a map f: x→y is irresolute then it is c*g-continuous, but not conversely.

Proof:-. Assume that f is c*g-irresolute. Let F be any closed set in y. Since every closed set is c*g- closed, F is c*g- closed in y. Since f is c*g-irresolute, f^{-1}(F) is c*g- closed in x. Therefore f is c*g- continuous.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.3.3 consider the topological space x = y = {a,b,c};with the topology; τ_1= {φ,X,{c}} and τ_2={φ,y,{a},{a,b}}.

Let f: (x, τ_1) →(y, τ_2) be the identity map. Then f is c*g-continuous, but not c*g- irresolute. For the c*g- closed set {a}, the inverse image f^{-1}({a}) = (a) is not c*g- closed in x.

THEOREM: 3.3.4 .Let x, y and z be any topological spaces. For any c*g-irresolute map f: x→y and any c*g-continuous map g: y→z the composition g.f : x→z is c*g-continuous.

Proof: - Let f be any closed set in z. Since g is c*g-continuous,g^{-1}(F) is c*g-closed in y. Since f is c*g irresolute, f^{-1} [g^{-1} (F)] is c*g- closed in x. But f^{-1} [g^{-1} (F)] = $(g.f)^{-1}$ (F). Therefore g.f is c*g-continuous.

THEOREM: 3.3.5. If f: x→y from a topological space x into a topological space y is bijective, c* set and c*g-continuous then f is c*g- irresolute.

Proof: - Let A be a c*g-closed set in y. let $f^{-1}(A)\subseteq O$, where O is c* set in x. Therefore, $A\subseteq$ f(O) holds. Since f(O) is c* set and A is c*g-closed in y, $cl(A)\subset f(O)$ holds and hence $f^{-1}(\bar{A})\subset O$. Since f is c*g-continuous and $\bar{A}$ is closed in y, $cl[f^{-1}(\bar{A})]\subset O$ and so $cl[f^{-1}(A)]\subset O$. Therefore $f^{-1}(A)$ is c*g-closed in x. Hence f is c*g-irresolute.

The following examples show that no assumption of the above theorem can be removed.

EXAMPLE: 3.3.6. Let (x, τ_1) and (y, τ_2) be the spaces defined in 3.3.3. The identity map f: (x, τ_1) →(y, τ_2) is c*g-continuous, bijective and not c*- set. And f is not c*g -irresolute. Since for the c*g- closed set G= {a} in y, the inverse image $f^{-1}(G)$ =G is not c*g-closed in x.

EXAMPLE: 3.3.7. consider the topological space x = y = {a,b,c};with the topology; τ_1= {φ,X,{c}} and τ_2 = {φ,y,{a},{a,b}}. Let f : (x, τ_1)→(y, τ_2) be the identity map. Then f is c*g-continuous and c*-set, but it is not bijective. And f is not c*g-irresolute. Since, for the c*g- closed set G= {a}, the inverse image $f^{-1}(G)$ =G is not c*g- closed in x.

EXAMPLE: 3.3.8. Consider the map f: x → y is defined in example 3.3.3. The map f is c*-set, bijective but not c*g-continuous and f is not c*g- irresolute.

The following two examples show that the concepts of irresolute maps and c*g- irresolute maps are independent of each other.

EXAMPLE: 3.3.9. consider the topological space x = y = {a,b,c} with the topologies,
τ_1= {φ,X,{a}{b},{a,b}} and
τ_2= {φ,y,{a},{b,c}}. Then the identity map f: (x, τ_1) →(y, τ_2) is irresolute. But it is not c*g- irresolute, since for the c*g-closed set G= {a} in y, $f^{-1}(G)$ =G is not c*g- closed in x.

EXAMPLE: 3.3.10. Consider the topological space x = y = {a,b,c} with the topologies, τ_1 = {φ,X,{a,b}} and τ_2 = {φ,y,{c}}. Then the identity map f: (x, τ_1) →(y, τ_2) is c*g- irresolute. But it is not irresolute, since for the semi-closed set G= {c} in y, $f^{-1}(G)$ =G is not semi-closed in x.

THEOREM: 3.3.11. If y is T_S and f: x→y from a topological space x into a topological space y is c*g- irresolute, then f is regular generalized irresolute.

Proof: - Let y be a T_S space and f: x → y is a c*g –irresolute map. Let V be a c*-set in y and since y is T_S, V is c*g-closed in y and since f is c*g –irresolute, $f^{-1}(V)$ is c*g –closed in x. But every c*g-closed set is regular generalized closed, f^{-1} (V) is regular generalized closed. Therefore f is a regular generalized irresolute map.

EXAMPLE: 3.3.12. Consider the topological space x = y = {a,b,c} with the topologies,
τ_1= {φ,X,{a},{b},{a,b}} and
τ_2= {φ,y,{a,b}}. Then the identity map f: (x, τ_1) →(y, τ_2) is c*g-irresolute. But it is not regular generalized irresolute, since for the regular generalized closed set G= {b} in y, $f^{-1}(G)$ =G is not regular generalized closed in x.

THEOREM: 3.3.13. If y is T_S and f: x→y from a topological space x into a topological space y is regular generalized irresolute, then f is c*g-irresolute.

Proof: - Let x be a T_S space and f: x → y is a regular generalized irresolute map. Let V be a c*g-closed set in y. Since every c*g-closed set is regular generalized closed, V is regular generalized closed in y. Since f is regular generalized irresolute, $f^{-1}(V)$ is regular generalized closed set in x. But x is T_S and therefore f^{-1} (V) is c*g-closed in x. Thus f is c*g- irresolute.

EXAMPLE: 3.3.14. Consider the topological space x = y = {a,b,c} with the topologies,
τ_1={φ,X,{a},{c},{a,c}}and
τ_2= {φ,y,{a},{a,b}}. Then the identity map f: (x, τ_1) →(y, τ_2) is regular generalized irresolute. But it is not c*g-irresolute, since for the c*g-closed set G= {b} in y, $f^{-1}(G)$ =G is not c*g-closed in x.

REMARK: 3.3.15. The concepts of regular generalized irresolute and c*g-irresolute are independent as seen from example 3.3.12. and example 3.3.14.

3.4. c*g- HOMEOMORPHISMS IN TOPOLOGICAL SPACES

DEFINITION: 3.4.1. A bijection map f: $(x,\tau) \rightarrow (y,\sigma)$) from a topological space x into a topological space y is called **c***g-homeomorphism if f is both c*g- continuous and c*g-open.

THEOREM: 3.4.2. Every homeomorphism is a c*g-homeomorphism, but not conversely.

Proof:-. Since every continuous function is c*g- continuous and every open map is c*g-open, the proof follows.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.4.3. Consider the topological space x = y = {a,b,c}; with the topology $\tau = \{\varphi, X, \{a\}\}$ and $\sigma = \{\varphi, y, \{a\}, \{b,c\}\}$. Then the identity map f: $(x,\tau) \rightarrow (y,\sigma)$ is a c*g-homeomorphism but not a homeomorphism.

THEOREM: 3.4.4. Every strongly g-homeomorphism is a c*g-homeomorphism, but not conversely.

Proof:-. Let f: x→y be strongly g-homeomorphism. Then f is strongly g-continuous and strongly g-open. Since every strongly g-continuous function is c*g- continuous and every strongly g-open map is c*g-open, f is c*g-continuous and c*g-open. Hence f is a c*g-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.4.3. Consider the topological space x = y = {a,b,c};with the topology $\tau = \{\varphi, X, \{c\}, \{a,c\}\}$ and $\sigma = \{\varphi, y, \{b\}, \{a,b\}\}$. Define the map f: $(x,\tau) \rightarrow (y,\sigma)$ be the identity. Then the set {a,c}is a c*g-homeomorphism but not a strongly g-homeomorphism.

Next we shall characterize c*g-homeomorphism and c*g-open.

THEOREM: 3.4.6. For any bijection f: x→y the following statements are equivalent.

(a) The inverse map f^{-1}:y→x is c*g-continuous.

(b) f is a c*g-open map.

(c) f is a c*g-closed map.

Proof: - (a)⇒(b). Let G be any open set in x. Since f^{-1} is c*g-continuous, the inverse image of G under f^{-1}, namely f(G) is c*g-open in y and so f is a c*g-open map.

(b) ⇒(c). Let F be any closed set in x. Then F^c is open in x. Since f is c*g-open, $f(F^c)$ is c*g-open in y. But $f(F^c)$ =y-f(F) and so f(F) is c*g-closed in y. Therefore f is a c*g-closed map.

(c) ⇒(a). Let F be any closed set in x. Then $(f^{-1})^{-1}(F)$ =f (F) is c*g-closed in y. Since f is c*g-closed map. Therefore f^{-1} is c*g-continuous.

THEOREM: 3.4.7. Let f: $(x,\tau) \rightarrow (y,\sigma)$ be a bijective and c*g-continuous map, the following statements are equivalent.

(a) f is a c*g-open map.

(b) f is c*g-homeomorphism.

(c) f is a c*g-closed map.

Proof: -.(a)⇒(b). By assumption, f is bijective, c*g-continuous and c*g-open. Then by definition, f is c*g-homeomorphism.

(b) ⇒(c). By assumption, f is c*g-open and bijective. By theorem 3.4.4. f is c*g-closed map

(c) ⇒(a). By assumption, f is c*g-closed and bijective. By theorem 3.4.4. f is c*g-open map.

EXAMPLE: 3.4.8. Let x = y = {a,b,c} with topologies $\tau_1=\{\varphi$,X,{c},{a,c},{b,c}};
$\tau_2=\{\varphi$,y,{b}{b,c}};and
$\tau_3=\{\varphi$,z,{b},{c},{b,c}} respectively. Let f and g are identity maps such that f : x→y and g: y→z. Then f and g are c*g-homeomorphism but their composition g.f : x→z is not a c*g-homeomorphism. For the open set {a,c} in x, g[f({a,c})]={a,c} is not c*g-open in z.

THEOREM: 3.4.9. Every c*g-homeomorphism is a regular generalized homeomorphism, but not conversely.

Proof:-. Let f: x→y be c*g-homeomorphism. Then f is c*g-continuous and c*g-open. Since every c*g-continuous function is regular generalized continuous and every c*g-open map is regular generalized open, f is regular generalized continuous and regular generalized open. Hence f is a regular generalized homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.4.10. Let x = y = {a,b,c} with topologies $\tau_1=\{\varphi$,X,{a},{a,b}{a,c}}and $\sigma=\{\varphi$,y,{a},{b,c}}. Define the map f: (x, τ) → (y, σ) be the identity. Then f is regular generalized homeomorphism, but not a c*g-homeomorphism.

DEFINITION 3.4.11. A bijection f: (x, τ) → (y, σ) is said to be a (c*g)*-homeomorphism if f and its inverse f^{-1} are c*g- irresolute maps.

NOTATION: Let family of all (c*g)*-homeomorphism from (x, τ) onto itself be denoted by (c*g)* h (x, τ) and family of all c*g- homeomorphism from (x, τ) onto itself be denoted by c*g h (x, τ). The family of all homeomorphism from (x, τ) onto itself is denoted by h (x, τ).

THEOREM: 3.4.12. Let x be a topological space. Then

i) The set (c*g)*h(x) is a group under composition of maps.

ii) h(x) is a subgroup of (c*g)*h(x).

iii) (c*g)*h(x) $\subset$ c*gh(x).

Proof:- i). Let f,g $\in$ (c*g)*h(x). Then g.f $\in$ (c*g)*h(x) and so (c*g)*h(x) is closed under the composition maps. The composition of maps is associative. The identity map I :x→x is a (c*g)*-homeomorphism and so I $\in$ (c*g)*h(x). Also f.I = I.f =f for everyf $\in$ (c*g)*h(x). If f $\in$ (c*g)*h(x), the $f^{-1}\in$ (c*g)*h(x) and $f.f^{-1}= f^{-1}.f=I$. Hence (c*g)*h(x) is a group under the composition of maps.

ii) Let f: x→y be a homeomorphism. Then by theorem 3.3.5. both of f and $f^{-1}$are (c*g)* irresolute and so f is a (c*g)* - homeomorphism. Therefore every homeomorphism is a (c*g)* - homeomorphism and so h (x) is a subset of (c*g)* h (x). Also h(x) is a group under the composition of maps. Therefore h(x) is a sub group of the group (c*g)*h(x).

iii) Since every (c*g)*- irresolute map is c*g- continuous, (c*g)*h(x) is a subset of (c*g)h(x).

From the above observation we get the following diagram.

Homeomorphism → strongly g-
Homeomorphism → c*g
Homeomorphism → regular
generalized Homeomorphism.

In the above diagram none of the implications can be reversed.

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