A Study On C*G-Closed Sets In Bitopological Spaces

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Abstract

In this paper, we have introduced the concept of c*g –closed and some of their properties in bitopological space.

Key words: (I,j)-c*g-closed

1.INTRODUCTION

A triple (x, τ_1,τ_2) where X is non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly [11] initiated the study of such spaces. In 1985 Fukutake [5] introduced the concepts of g-closed sets in bitopological spaces and after that several authors turned their attention to the generalization of various concepts of topology by considering bitopological spaces instead of topological spaces. In 2004, P.Sundaram [12] introduced the concept of g*-closed sets in bitopological spaces.

Throughout this chapter (X, τ_1, τ_2) (or X)and (Y, σ_1 , σ_2) (or Y) denote two non empty bitopological spaces. In this section we introduce the concept of (i,j)-c*g-closed sets and we obtain some interesting results in bitopological spaces.

2. PRELIMINARIES

DEFINITION 2.1: A subset A of X is called

i) (i,j)*-generalized closed (briefly (i,j)-g-closed) [5] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open in X.

ii) (i,j)- regular generalized closed (briefly (i,j)-rg-closed) [1] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - regular open in X.

iii) (i,j)-generalized pre regular closed (briefly (i,j)-gpr-closed) [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - regular open in X.

iv)(i,j)-weakly generalized closed (briefly (i,j)-wg-closed) [6] if τ_j-cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - open in X.

v) (i,j)-strongly generalized closed (briefly (i,j)-strongly g-closed) [12] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-g-open in X.

vi) (i,j)-weakly closed (briefly (i,j)-w-closed) [7] if τ_j-cl(A) $\subseteq$ U whenever A ⊆ U and U is τ_i-semi open in X.

vii) (i,j) -generalized α-closed (briefly (i,j)-gα-closed) [3] if τ_j-αcl(A) ⊆ U whenever A ⊆ U and U is τ_i- α-open in X.

viii) (i,j) generalized semi-closed (briefly (i,j)-gs-closed) [3] if τ_j-scl(A) ⊆ U whenever A ⊆ U and U is τ_i- open in X.

3. c*g-CLOSED -CLOSED SETS IN BITOPOLOGICAL SPACES

DEFINITION 3.1: A subset A of a bitopological space (X, τ_i, τ_j) is said to be an (i,j)-c*g-closed set if τ_j-cl(A) ⊆ U whenever A ⊆ U and U is τ_i-c*-set in X.

We denote the family of all (i,j)-c*g-closed sets in (X, τ_i, τ_j) by C*(i,j).

THEOREM 3.2: Every τ_j-closed set in (X, τ_i, τ_j) is (i,j)-c*g-closed set in (X, τ_i, τ_j) but not conversely.

Proof: - Let A be a τ_j-closed set in X. Let U be a τ_i -c* -set such that A ⊆ U. Since A is τ_j-closed, that is τ_j-cl(A) = A. Therefore τ_j-cl (A) ⊆ U. Hence A is (i,j) -c*g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 3.3: Consider the topological space X = {a,b,c} with the topologies τ_1 = {φ,X,{a},{a,b},{a,c}}; τ_2 = {φ,X,{c},{a,b}}. The set {b} is (1,2)-c*g-closed set but not τ_2-closed.

THEOREM 3.4: Union of two (i,j)-c*g-closed sets is (i,j)- c*g-closed.

Proof: - Let A and B be (i,j)- c*g-closed sets in X. Let U be a τ_i-c*-set in X. such that AUB ⊆ U. Then A ⊆ U and B ⊆ U. Since A and B are (i,j)-c*g-closed, τ_j-cl(A) ⊆ U and τ_j-cl(B) ⊆ U. Hence τ_j-cl(AUB) = τ_j-cl(A) U τ_j-cl(B) ⊆ U. Therefore AUB is (i,j)-c*g-closed.

REMARK 3.5: Intersection of two (i,j) -c*g -closed sets in X need not be (i,j) -c*g –closed sets in X is proved in the following example.

EXAMPLE 3.6: Consider the topological space X = {a,b,c} with the topologies τ_1 = { φ ,X,{a}};

τ_2= { φ,X,{a}, {c}, {a,c}}. In this topology consider the set {b,c} and {a,c}. Intersection of these two sets not contained in (1,2)-c*g-closed sets. Therefore intersection of two (1,2) -c*g -closed sets in X is not (1,2) -c*g – closed sets in X.

THEOREM 3.7: A subset A of X is (i,j)-c*g -closed in X if and only if τ_j-cl(A)/A does not contain any non empty τ_i-c*- set in X.

Proof:- Suppose that A is a (i,j)-c*g -closed set in X. We prove the result by contradiction. Let U be τ_i-c*- set such that U$\subset$ τ_j-cl(A)/A and U$\neq\varphi$. Then U$\subset$ τ_j-cl(A) ∩ A^c. Therefore U$\subset\tau_j$-cl(A) and U$\subset A^c$ is τ_i-c*- set and A is (i,j)-c*g-closed, τ_j-cl(A) $\subseteq$ U^c. That is U$\subseteq$[τ_j-cl(A)]^c. Hence U$\subseteq\tau_j$-cl(A)∩[τ_j-l(A)]$^c\neq\varphi$.That is U$\neq\varphi$. Which is contradiction. Hence τ_j-cl(A)/A does not contain any non empty τ_i-c*- set in X.

Conversely assume that τ_j-cl(A)/A contains no non empty τ_i-c*-set. Let A$\subseteq$U, U is τ_i-c*- set. Suppose that τ_j-cl(A) is not contained in U. Then τ_j- cl(A) ∩ U^c is a non empty τ_i-c*- set and contained in τ_j-cl (A)/A, which is contradiction. Therefore τ_j-cl(A) $\subset$U. Hence A is (i,j)-c*g -closed.

REMARK 3.8: The converse of the above two theorems is not true as seen from the following example.

EXAMPLE 3.9: Consider X = {a, b, c} with the topology τ_1= { φ, X, {a}, {b}, {a,b},{b,c}} and τ_2 ={ φ, X,{b},{c},{b,c}} . Let A={a},then τ_2-cl(A)/A={{a,c}/{a}} = {c}does not contain any non empty τ_1-c*- set, but A={c} is not (1,2)-c*g -closed set in X.

THEOREM 3.10: Every (i,j)-strongly g-closed set in X is a (i,j)-c*g -closed set in X but not conversely.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 3.11: Consider the topological space X={a,b,c} with the topologies τ_1= { φ,X,{a,b}}; τ_2= { φ,X,{a},{b},{a,b}}.Then the set A= {a,b} is (1,2)-c*g- closed set but not (1,2)-strongly g closed.

THEOREM 3.12: Every (i,j)- c*g -closed set in X is a (i,j)-gpr- closed set in X but not conversely.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 3.13: Consider the topological space X={a,b,c} with the topologies τ_1= { φ,X,{c},{a,b}}and τ_2= { φ,X,{b},{b,c},{a,b}}.Then the set A= {b,c} is not (1,2)-c*g- closed set but (1,2)-gpr closed.

REMARKS 3.14: From the above theorem and example we get the following diagram.

j-closed

↓

(i,j)-strongly g–closed

↓

(i,j)-c*g -closed

↓

(i,j)- gpr-closed.

Figure-3.1.1

In the above diagram none of the implications can be reversed.

REMARK 3.15:

The concept of (i,j)-c*g -set is independent of the following classes of sets namely (i,j)-α- closed, (i,j)-g-closed, (i,j)-wg- closed, (i,j)- β -closed, (i,j)-gα -closed, (i,j)-w -closed, (i,j)-rw-closed, (i,j)-pre- closed and (i,j)- αg –closed.

EXAMPLE 3.16: Consider the topological space X={a,b,c} with the topologies τ_1= { φ,X, {a}}and τ_2= { φ,X,{a},{c},{a,c}}.Then the set A= {c} is not (1,2)-c*g- closed set but (1,2)-α- closed, (1,2)- β –closed, (1,2)-pre- closed and (1,2)-semi-closed. In the same topologies the set A= {a} is (1,2)-c*g- closed set but not (1,2)-α-closed, not (1,2)- β –closed, not (1,2)-pre- closed and not (1,2)-semi-closed.

EXAMPLE 3.17: Consider the topological space X={a,b,c} with the topologies τ_1= { φ,X, {b}}and τ_2= { φ,X,{b},{c},{b,c}}.Then the set A= {c} is not (1,2)-c*g- closed set but

(1,2)- g- closed and the set {a,b} is (1,2)-c*g- closed set but not (1,2)- g- closed.

EXAMPLE 3.18: Consider the topological space X={a,b,c} with the topologies τ_1= { φ,X, {a}}and

τ_2= { φ,X,{b}}.Then

In this bitopologies the set A= {c} is not (1,2)-c*g- closed set but (1,2)- w- closed and (1,2)- wg- closed. For the same topologies,

The set A={a,b} is (1,2)-c*g- closed set but not (1,2)- w- closed.

The set A={b,c} is (1,2)-c*g- closed set but not (1,2)- wg- closed.

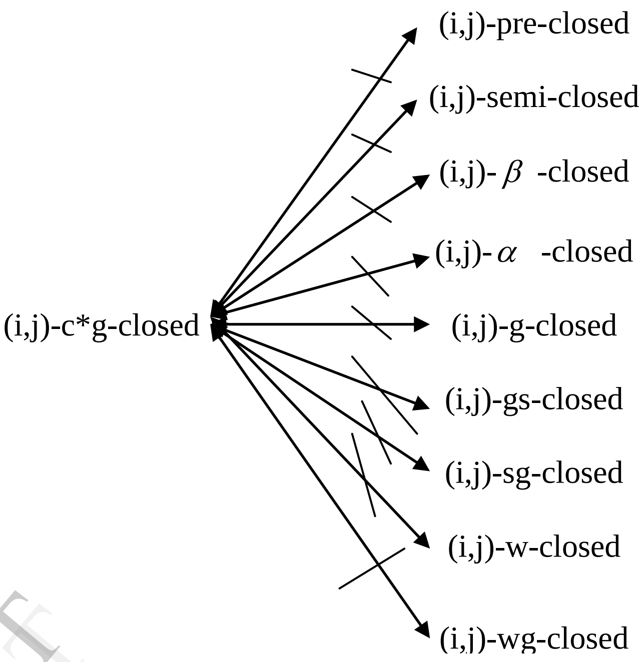
EXAMPLE 3.19: Consider the topological space X={a,b,c} with the topologies τ_1={ φ,X, {b},{c},{b,c}}and

τ_2= { φ,X,{b}}.Then

The set A= {c} is not (1,2)-c*g- closed set but (1,2)- gs- closed and (1,2)- sg- closed.

The set A={a,b} is (1,2)-c*g- closed set but not (1,2)- gs- closed and (1,2)- sg- closed.

REMARK 3.20: From the above discussion and known results we have the following diagram.



REMARK 3.21:

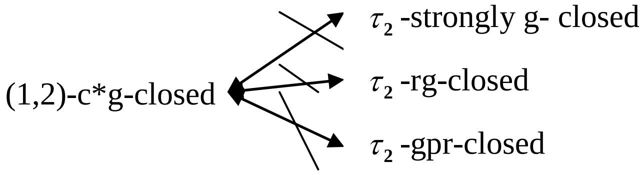
The concept of (i,j)-c*g –closed set is independent of the following classes of sets namely τ_j-strongly g- closed, τ_j-rg-closed and τ_j-gpr –closed.

EXAMPLE 3.22: Consider the topological space X={a,b,c} with the topologies τ_1= { φ,X,{a}}and

τ_2= { φ,X,{a},{c},{a,c}}.Then the set A= {a} is (1,2)-c*g- closed set but not τ_2-strongly g- closed, τ_2-rg-closed and τ_2-gpr –closed.

Consider another topologies τ_1= {φ,X, {b},{a,c}}and τ_2= {φ,X,{a,c}}. Then the set A= {a,b} is τ_2-strongly g-closed, τ_2-rg-closed and τ_2-gpr –closed but not (1,2)-c*g- closed set.

REMARK 3.23: From the above discussion and known results we have the following diagram.



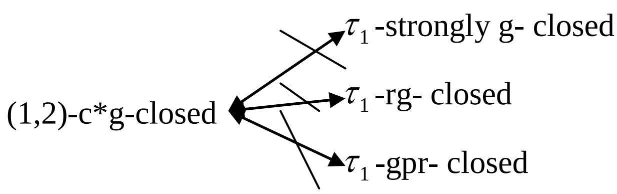
REMARK 3.24:
The concept of (i,j)-c*g -set is independent of the following classes of sets namely τ_i-strongly g- closed, τ_j-rg-closed and τ_j-gpr –closed.

EXAMPLE 3.25: Consider the topological space X={a,b,c} with the topologies τ_1= {φ,X, {c},{a,b}}and τ_2= {φ,X,{b},{a,b},{b.c}}.Then the set A= {a} is (1,2)-c*g- closed set but not τ_1-g*-closed. For the same topology the set A={a,b} is τ_1-g*-closed but not (1,2)-c*g- closed set.

EXAMPLE 3.26: Consider the topological space X={a,b,c} with the topologies τ_1={φ,X, {a}, {b}, {a,b}, {a,c}}and τ_2= {φ,X}.Then the set A= {a,c} is (1,2)-c*g- closed set but not τ_1-rg-closed. For the same topology the set A={a,b} is τ_1-rg-closed but not (1,2)-c*g- closed set.

EXAMPLE 3.27: Consider the topological space X={a,b,c} with the topologies τ_1= {φ,X, {a}, {b}, {a,b}, {a,c}}and τ_2= {φ,X,{a}}.Then the set A= {a,c} is (1,2)-c*g- closed set but not τ_1-gpr-closed. For the same topology the set A={b} is τ_1-gpr-closed but not (1,2)-c*g- closed set.

REMARK 3.28: From the above discussion and known results we have the following diagram.



REMARK 3.29: c*(i,j) is generally not equal to c*(j,i). Consider the following example.

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EXAMPLE 3.30: Consider the topological space X={a,b,c} with the topologies τ_1={ φ,X,{b},{c}, {b,c}}and τ_2= { φ,X,{b}}.Then the set A= {b} is (2,1)-c*g- closed set but A={b} is not c*(1,2).

REMARK 3.31: If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) then c*(2,1) $\subseteq$ c*(1,2). The converse of this remark is not true as seen from the following example.

EXAMPLE 3.32: Let the topological space X={a,b,c} with the topologies τ_1= { φ,X, {a}, {b}, {a,b}, {b,c}}and τ_2= { φ,X,{b},{c},{b,c}}. In this case c*(2,1) $\subseteq$ c*(1,2) but $\tau_1 \not\subset \tau_2$.

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