A Study of Weibull Parameters using Four Numerical Methods to Analyze the Wind Speed at Jogimatti in India

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***Abstract* - Wind speed is the most important parameter in design and study of wind energy conversion system. Wind energy technology is currently making a significant contribution to the electric power generation systems in India. Now India is one of the leading countries in the world for development and utilization of wind energy. According to the growth of wind energy India will achieve high potential in future. The main aim of this study is to asses wind power potential of a location for wind power plant department using Weibull parameters. In this paper we have presented four numerical methods for estimating Weibull parameters using wind speed data collected at Jogimatti located in Chitradurga district, Karnataka, India over a period of 20 years with mast height of 20m at Latitude N at 14˚09"49', Longitude E at 76˚23"56'and its performance is compared to others by using the statistical tests.**

KeyWords: Weibull parameters, Mean wind speed, Numerical methods, Statistical analysis

1. INTRODUCTION

Wind energy is one of the fastest growing renewable sources of energy in both developed and developing countries with total available wind power surrounding the earth, which is several times more than the current global energy consumption. Wind distribution can be used to model the wind speed distribution at a particular location and hence it can help in wind resource assessment of a location. By calculating the shape and scale parameter for Weibull distribution the wind speed frequency curve for a location can be made and the key to perform wind turbine and wind farm energy calculation.

It was believed that the wind characteristics were merely important for builders, bridge designers, architects and ship designers in the past.Wind analysis not only provides extremely valuable information for engineers in the field of structural and environmental designs but also to researchers involved in renewable energy studies. Towards to the end of this century a higher living standard has created a greater demand for energy supply. Limited energy resources coupled with ever increasing demand leads to the necessary and immediate action of seeking a solution in order to overcome the energy shortage. Recently renewable energy such as wind can be thought as one of the alternative energy sources. Wind observations are generally collected in the farm of very large number of points. It is remarkably useful that a wide range of wind character can be summarized by specifying only two parameters are sufficient via shape parameter k and scale parameter c. These two parameters are sufficient to specify the available wind and to enable assessments and evaluations of wind power to be made.

2. RELATED WORK

Weibull scale parameter controls the abscissa, scale of a plot of data distribution. Shape parameter describes the width of data distribution, the larger the shape parameter that narrower the distribution and the higher its peak value [1].For a given data set several numerical methods can be applied to estimate the Weibull parameters. For example, the widely used Moment Method, Empirical Method, Graphical Method, Maximum Likelihood method, Modified Maximum Likelihood Method and Energy Pattern Factor Method [1-5]. Akdag and Dinler [2] reviewed three conventional methods, i.e. the Graphical, Maximum likelihood and Moment Methods and proposed a new method (called Energy Pattern Factor Method) for estimating Weibull parameters. They stated that the new method has better suitability than others based on the comparisons of power density and mean wind speed. Chang [1] used six kind of numerical methods to analyze the wind power density at 46 and 64.7 m heights in Taiwan, two Weibull parameters were estimated and compared. In this study we have used Graphical method, Empirical method, Moment Method and Equivalent energy method to determine the Weibull parameters at Jogimatti.[Table III]

Notations:

k	-	Weibull shape parameter
c	-	Weibull scale parameter
F(v)	-	Cumulative distribution function
f(v)	-	Probability density function
v	-	Wind speed(m/s)
$\bar{v}$	-	Mean wind speed(m/s)
R^2	-	Correlation co-efficient
RMSE	-	Root mean square value
χ^2	-	Chi-square value
Γ	-	Gamma function

3. DATA SOURCE

In this study mean wind speed datas for Jogimatti were collected from the meteorological society from 1989 to 2009 with mean average wind speed rating between 3.98 m/s to 8.33 m/s[Table I]. Our study includes the mean wind speed data for 12-months [Table II]observed at Jogimatti station in India.

Table 1- Sample Wind speed data

Day	*v*	Day	*v*	Day	*v*	Day	*v*
1	3.7260	9	4.6231	17	4.4130	25	2.6493
2	4.4214	10	7.8142	18	4.5121	26	8.6939
3	4.1130	11	5.7969	19	5.3261	27	9.2130
4	5.7160	12	6.4231	20	5.4673	28	7.3411
5	6.4210	13	4.4231	21	6.6001	29	7.0001
6	4.6421	14	2.316	22	6.3266	30	6.9437
7	3.6161	15	2.4244	23	7.4651	31	6.4651
8	2.7349	16	3.9244	24	5.3214		

TABLE II-Monthly wind speed datas

Month	$\bar{v}$	Month	$\bar{v}$
Jan	6.0452	July	10.4671
Feb	5.7826	Aug	11.7204
Mar	6.6847	Sep	10.8460
April	7.5924	Oct	8.2645
May	7.1335	Nov	8.7144
June	8.3169	Dec	6.6211

4. WEIBULL DISTRIBUTION

There are Literature works that deal with the use of several distributions to describe the wind speed. The Two parameter Weibull Distribution is widely accepted and used in the wind energy industry as the better method for describing wind speed variations at a given site. Weibull distribution is a special case of Pearson Type III or Generalised Gamma distribution. In this study, two parameter Weibull Distribution is used for analyzing wind speed pattern variations. The Weibull distribution is characterized by two parameters one is the shape parameter (dimensionless) and the other is the scale parameter c (m/s).[6]

The probability function is given by

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^{k}\right) \qquad (1)$$

And the cumulative distribution function is

$$F(v) = 1 - exp\left(-\left(\frac{v}{c}\right)^{k}\right) \qquad (2)$$

Where v is the wind speed, k is the shape parameter and c is the scale parameter.

5. METHODS FOR ESTIMATING WEIBULLPARAMETERS

5.1. Graphical method

The graphical method is derived through the cumulative distribution function(2) In this method the wind speed data are interpolated by a straight line using the concept of least squares. Equation (2) is interpreted as follows,

$$ln\{-ln[1 - F(v)]\} = k\ln(v) - k\ln(c) \qquad (3)$$

Using equation(3) the two parameters k and c are obtained as follows,

$$k = \frac{n\sum_{i=1}^{n}\ln v\ln[-\ln\{1-F(v)\}] - \sum_{i=1}^{n}\ln v\sum_{i=1}^{n}\ln[-\ln\{1-F(v)\}]}{n\sum_{i=1}^{n}\ln v^3 - \{\sum_{i=1}^{n}\ln v\}^2}$$

$$c = \exp\{\frac{k\sum_{i=1}^{n}\ln v - \sum_{i=1}^{n}\ln[-\ln\{1-F(v)\}]}{nk}\}$$

5.2 Empirical method

In Empirical Method the parameters k and c are defined by

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086} \qquad (4)$$

$$c = \frac{\bar{v}}{\Gamma\left(1+\frac{1}{k}\right)} \qquad (5)$$

Once $\bar{v}$ and σ are calculated for a given data set then k and c can be calculated by solving equations(4) and (5)

5.3 Moment Method

In this method the parameters k and c are determined by the following equations.

$$\bar{v} = c\Gamma\,(1 + 1/k) \qquad (6)$$

$$\sigma = c\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2(1+1/k)\right]^{1/2} \qquad (7)$$

Where $\bar{v}$ and σ are the mean wind speed and the standard deviation of the observed data of the wind speed respectively.

5.4 Equivalent Energy Method

In the equivalent energy method, the parameters k and c are obtained using the equations below

$$\sum_{i=1}^{n}\left[W_{v_i} - e^{-\left\{\frac{(v_i-1)[\Gamma(1+3/k)]^{1/3}}{(v^3{}_m)^{1/3}}\right\}^k} + e^{-\left\{\frac{(v_i)[\Gamma(1+3/k)]^{1/3}}{(v^3{}_m)^{1/3}}\right\}^k}\right]^2 = \sum_{i=1}^{n}(\varepsilon_{v_i})^2$$

$$c = \left[\frac{v^3{}_m}{\Gamma(1+3/k)]^{1/3}}\right]^{1/3}$$

Where W_{v_i} and $v^3{}_m$ are the observed frequency of the wind speed and the mean of cubic wind speed, respectively, and ε_{v_i} is the error of the approximation.

Table III-The Weibull parameters k and c

Weibull parameters	GM	EM	MM	EEM
k	2.6260	2.5987	2.5801	2.5010
c	9.1901	9.2130	9.2142	9.2030

6. STATISTICAL ANALYSIS

A proper statistical analysis of wind data is a very important step in performing a wind resource assessment which provides a wind energy development initiative. Three tests were used to analyze the accuracy of the four methods, namely root mean square value(RMSE) , Correlation Co-efficient (R^2) and Chi-square(χ^2) tests, which are defined by

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{\frac{1}{2}}$$

$$\chi^2 = \frac{\sum_{i=1}^{N}(y_i - x_i)^2}{N - n}$$

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - z_i)^2 - \sum_{i=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(y_i - z_i)^2}$$

Where N is the total number of intervals, y_i is the frequency of observation, x_i is the frequency of Weibull, z_i is the mean wind speed, n is the number of constants used.

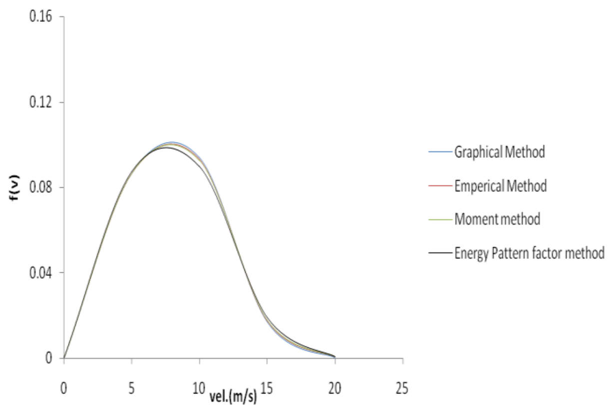
Table IV-Statistical Analysis

TESTS	GM	EM	MM	EEM
RMSE	0.0319	0.0315	0.0314	0.0306
χ^2	0.0011	0.0010	0.0010	0.0009
R^2	0.9585	0.9595	0.9599	0.9619

Table V- Comparison between the above explained methods and measured wind speed

Methods	GM	EM	MM	EEM
$\bar{v}$	8.1657	8.1861	8.1809	8.1655
Error %	-0.2045	0.04519	-0.0183	-0.2069
Measured Mean wind speed: 8.1824				

Fig-1: The Weibull probability density function graph



7. CONCLUSION AND DISCUSSIONS

The present investigation is demonstrated using a sample wind speed data set and the accuracy of each method is compared with measured data obtained from metrological department . Fig-1 shows the probability density function of Weibull distribution using the four methods.As a result Moment Method is the most fitted method to estimate the weibull parameters in our study case (Table V& Fig-1).This fact is clearly validated by means of the statistical Tests (Table-IV). It is also observed from the statistical analysis that the values of RMSE, χ^2 and R^2 have magnitudes very close to each other for all the numerical methods used for the data collected at Jogimatti station. But the suitability of the method varies according to the data size.

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