A Study Of Methods To Solve Complex Spur Gear Problems

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Abstract

Gear is an important part of power transmission machineries. Gear tooth normally fails at its root, due to excessive bending stress concentration at the fillet area of root. Many researchers worked in this area, and suggested number of methods to improve the bending strength of tooth of gear. A detailed study of various methods employed to calculate and optimise the critical stress of the system gear tooth root fillet profile. Several suggestions like circular root fillet instead of trochoidal root fillet, asymmetric involute type as an alternative to non involute teeth etc have been suggested by various authors. After going through the entire literature it is conclude that finite element analysis using ANSYS is more popularly used by researchers in this area. After present study, it is proposed that mat lab simulink may be employed in this field.

1. Introduction

The gear tooth fillet is an area of maximum bending stress concentration. However, its profile and accuracy are marginally defined on the gear drawing by typically very generous root diameter tolerance and, in some cases, by the minimum fillet radius, which is difficult to inspect. In fact, tooth bending strength improvement is usually provided by gear technology, case hardening and shot peening to create compressive residual stress layer, rather than gear geometry. Hence the gear tooth fillet is an area that needs in depth study and analysis.

An attempt has been made in this paper to study the various geometries of root fillet profile like trochoidal fillet profile and circular fillet profile. Several research works have been published in this area and many have concluded that if the gear teeth number is less than seventeen then, circular root fillet is preferable and if the number is more than seventeen then, trochoidal root fillet is suggested. A detailed review of the various methods used in tooth root fillet profile optimisation is carried out on spur gears and is presented in the following paragraphs. The objective of the study is to understand the methods so used and identify the possible improvements that could be applied to improve the existing methods.

2. Methods employed by researchers

Sankar and Nataraj [1] in their work, Profile modification a design approach for increase in the tooth strength in spur gear have suggested a novel method to prevent the tooth failure in the spur gear by introducing a circular root fillet instead of standard trochoidal root fillet in the gear. They carried out the analysis using ANSYS software version 11.0 for the existing standard design gear teeth as well as the proposed design gear teeth to evaluate the performance. In order to facilitate the analysis, they considered the gear tooth as a cantilever beam. They also considered the tooth force being applied normal to the profile at the highest point of single tooth contact.

From their investigations they inferred that the gear tooth deflection in the circular root fillet is less when compared to the trochoidal root fillet. They also identified that there would be considerable reduction in bending stress and contact shear stress for circular root fillet design in comparison to that of trochoidal root fillet design. They concluded circular root fillet design is more apt for less number of teeth whatever may be the pinion speed. The results obtained by them through the ANSYS analysis show that gears made of circular root fillet yield better strength thereby improves the fatigue life of the gear material.

Senthil Kumar, Muni, and Muthuvoorappan [2] carried out work on optimization of asymmetric spur gear drives to improve the bending load capacity. It is...
known that symmetric involute gear designed through conventional approach, the load carrying capacity is restricted at the higher pressure angle due to tipping formation. The use of the asymmetric toothed gear to improve the fillet capacity in bending is investigated in their work. They developed non-standard asymmetric rack cutters with required pressure angles and module to generate the desired pinion and gear of a drive with asymmetric involute surfaces and trochoidal fillet profiles. They optimized the respective profiles thus generated for balanced fillet stresses that were equal and possibly the lowest also.

For this optimization they designed a number of non-standard asymmetric rack cutters to include different combinations in the values of pressure angle, top land thickness ratio, profile shift, speed ratio and the asymmetric factors.

For the purpose of optimization, they considered only two non-standard asymmetric rack cutters under the conditions of any drive with a given center distance and a speed ratio. One for the pinion and the other for the gear used to generate a required number of pinion and gear with different cutter shift values. The influence of these parameters was investigated on the maximum fillet stress to suggest the optimum values of these parameters that improve the fillet capacity in bending.

They carried out the optimization of the asymmetric spur gear drive using an iterative procedure on the calculated maximum fillet stresses through FEM for different rack cutter shifts and finally suggested the optimum values of rack cutter shifts for the given center distance and the speed ratio of an asymmetric gear drive. They compared their results with the results of the AGMA and the ISO codes for symmetric gears to justify the results of the finite element method pertaining to their study. From the obtained results they identified that the peaking that occurs in the gear tooth designed through conventional method at higher pressure angles is overcome by the use of an asymmetric gear drive with suitable top land thickness ratio and coast side pressure angle and this they have achieved by the present non-standard cutter developed. They also identified that the asymmetric gear drive improves the fillet load capacity of the pinion and the gear at higher pressure angles \( \alpha_e \), compared to the conventional symmetric gear.

In their simplified procedure only two non-standard rack cutters that are enough to carry out the optimization were used. It produced an equally acceptable solution with marginal difference in optimum fillet stresses compared to that were developed in the gears synthesized by direct gear design.

They concluded that the procedure developed by them is less laborious as there is only one set of cutters of standard pressure angle and module used for the entire process of optimization of a given drive compared to the direct gear design method which involves more computational work.

Yong Wang [3] in his work optimized tooth profile based on identified gear dynamic model has suggested a method to determine the dynamic model of a practical gear system. This new method for the dynamic gear system was proposed by the identification of the relation between the rotational movement of gears and the acoustical noise signal measured on the gear tester. In his experiment the input signal was taken as the rotational movement is the line of action of gears and is the acoustical noise signal as the output.

For estimation of the parameter of the model he has used least square method. He has taken an example that the gear pair works under a certain design load and rotational speed, the identified model is verified and the optimized gear tooth profile modification is derived based on the identified model.

For a certain design load and rotational speed of the gear model, the optimized gear profile modification from the identified model was derived and tested. From the obtained results he concluded that the identified dynamic model can reflect the relationship between the rotational movement and the acoustical noise. The profile modification determined from his identified model can apparently reduce gear noise in comparison to those without modification.

Costopoulos and Spitas [4] in their research studied the reduction of gear fillet stresses by using one-sided involute asymmetric teeth. This was done for increasing load carrying capacity and to combine the good meshing properties of the driving involute and the increased strength of non-involute curves. Since there are a number of tooth designs alternative to the standard involute proposed for increasing the load carrying capacity of geared power transmissions, by their studies they suggested asymmetric involute type teeth as an alternative of non-involute teeth because the use of non-involute teeth has a number of disadvantages. The novel teeth they have to suggest were intended for constant direction of rotation although they can be used in a limited way for reverse rotation.

They also have investigated the design of hob by which both the flanks were completely generated. By their research they have suggested two main innovative features, use of circular root fillet instead of standard trochoïdal fillet for the reduction of the stress concentration at the driving side and the use of a fully rack or hob-generated, addendum geometry for the
coast side. They used finite element analysis for study of asymmetric half-involute gear teeth and the theory of gearing was used for the geometry of the proposed gears. They obtained reduction of the maximum pinion fillet up to 28% in comparison to standard designs, which means load carrying capacity of the tooth is increased. These results were obtained over a wide range of number of teeth on finite element analysis. Their proposed asymmetric design also increased the strength and pitting resistance.

Fig 1 [4] also shows a comparison of reduction in bending stress of standard teeth with asymmetric teeth. They have found 10-28% reduction in tensile bending stress.

![Non-dimensional bending stress comparison](image)

Fig 1 shows non-dimensional bending stress (tension side) variation for several numbers of teeth. Significant reduction in stress at small number of teeth (pinions) is noted [4].

Sanchez et al. [5] in their research critical stress and load condition for bending calculation of involute spur and helical gear for evaluation of the fatigue tooth root stress, have used a non-uniform model of load distribution along the line of contact of spur and helical gears, obtained from the minimum elastic potential criterion. They have carried out the complete analysis of the tooth bending strength by analysing the critical value of the stress and the critical load conditions.

In their research, they have used a very simple analytical equation for describing the load per unit length at any point of the line of contact and any position of the meshing cycle. The overall studies have been carried out by them on the location and the value of the tooth-root bending stress.

From their studies they identified that the critical fatigue tooth-root stress for spur gears will arise with the tooth loaded with the total load at the outer point of single pair tooth contact. Their results were applicable with restriction for gears with transverse contact ratio between 1 and 2, with non-undercut teeth.

Li [6] in his work effect of addendum on contact strength, bending strength and basic performance parameters of a pair of spur gears obtained basic performance parameters, effect of addendum on tooth contact strength and bending strength of the spur gear. They have also made analysis of tooth load, load-sharing rate, contact stress, root bending stress, transmission error and mesh stiffness of the spur gears.

To carry out their analysis they used face-contact model of teeth, mathematical programming method and three-dimensional finite element method simultaneously to conduct loaded tooth contact analyses, deformation and stress calculations of spur gears with different addendums and contact ratio.

From the obtained results they inferred that there was a slight change in tooth load-sharing rate if addendum was changed and this change did not increase number of contact teeth, and also tooth load-sharing rate can be changed greatly, if the number of contact teeth is increased through increasing addendum. They also explained that, if the addendum was longer and number of contact teeth were not changed, then tooth contact stress and contact width were changed slightly, but if number of contact teeth was increased through increasing the addendum, it reduced greatly. They have also concluded, if addendum becomes longer and number of contact teeth is not changed then Mesh stiffness is reduced, but it can also be increased by increasing the number of contact teeth.

Spitas and Costopoulos [7] carried out work on fast modelling of conjugate gear tooth profiles using discrete presentation by involute segment have suggested a method for the determination of conjugate gear tooth profiles by the discretization of the gear tooth flank in involute segments. According to him the actual tooth flank is considered to be composed of infinitesimal local involutes instead of following a point-to-point analytical approach to the problem of determining the path of contact and the geometry of the generating rack and the mating wheel, so that a closed solution can be achieved.

The method proposed by him is simpler and faster almost six times than standard theory of gearing.

From their investigations they inferred that the method can be applied on both involute and non-involute gears and can be easily programmed on a computer. The advantage of their suggested method is that it is the replica of standard discretization of the tooth flank in linear segments suggested by the theory of gearing to the discretization into infinitesimal local
involutes. Their proposed method works on simple analytical formula instead of calculating complex equations. Their suggested method can also be used with cams and chain drive; it is easily integrate with cad software.

Math, Chand [8] in their work an Approach to the Determination of spur gear tooth root fillet suggest a method for the determination of geometry of spur gear tooth root fillet. For a spur gear without undercutting developed an Equation to determine the point of tangency of involute profile and root fillet on the base circle and the point of intersection of root fillet and involute profile above the base circle for an undercut gear.

They also discussed generation using a hob or rack type cutter with protuberance.

From their research they developed a general procedure to determine the Cartesian coordinates of the trochoidal root fillet. Method proposed by them is also easily programmable on computer and manual computation is also possible because their method has not used any iterative numerical procedure.

Rincon et al. [9] in their study “a model for the study of meshing stiffness in spur gear transmissions” showed an advanced model for the analysis of contact forces and deformations in spur gear transmissions. They have formulated the deformation at each gear contact point formulated as a combination of a global and a local term. They have used Hertzian contact theory to derive an analytical approach for the latter, and by the means of finite element model they obtained the former.

They have imposed complementary and compatibility conditions, leading to a nonlinear system of equations subjected to inequality restrictions so that would be solved once the position of each gear centre is known. Where they have discussed the quasi-static behaviour of a single stage spur gear transmission, also presents a numerical example, which also shows the capabilities of the methodology to obtain the Loaded Transmission Error under several load levels as well as some other related measures such as load ratio or meshing stiffness.

From their investigation they have inferred a procedure to find out transmission, meshing stiffness and load ratio as well as load transmission error for spur gear. The procedure that they showed also allows a better representation of how the load is transferred between teeth pairs. They have specially enlightened coupling between deformations due to contact between adjacent teeth, which has consequences on the load sharing between teeth pairs as well as on the actual contact ratio. And that coupling is considered the deformation is greater than the one estimated by purely kinematic approaches such as those applied in conventional formulations.

Loutrids [10] in his research gear failure prediction using multi scale local statistics suggested multi scale local statistics as a tool for gear failure prediction. He analyzed the Experimental data from gears with localized defects in the form of bending fatigue cracks. His work was shown that second order central moments increased with defect magnitude.

His estimated procedure was carried out in the range of scale where the defect is more prominent. He also suggested empirical law that relates variance at various scales to crack magnitude.

From his results he concluded that the multi scale local statistics only requires data from the damaged pair, as to methods that are based on comparison between signals from healthy and damaged conditions. He also inferred that other than the existence of a local probability distribution around a point. Multi scale statistics make no a-priori assumptions about the nature of vibration time series. He has also concluded that his proposed method is computationally efficient because his method uses a noncritical weighing function and is based on the concept of convolution. He also inferred that his proposed method is alternative to more complicated methods, such as those based on time–frequency analysis.

Worden et.al [11] in their study natural computing for mechanical systems research: A tutorial overview inferred that development of computational algorithms over the past half-century has been motivated by biological system or processes like artificial neural networks. These algorithms are commonly grouped together under the terms soft or natural computing. According to them the common property of all the algorithms is that they allow exploration of, or learning from, data and also the property of these algorithms has proven extremely valuable in the solution of many diverse problems in science and engineering. A tutorial overview of the basic theory of some of the most common methods of natural computing as applied in the context of mechanical systems research, has been highlighted in their work.

They presented the case studies to show the application of some of the main algorithms.

From their conclusion they enlightened how natural computing algorithms have already taken a prominent place in the mechanical systems literature. They provided a number of applications of many algorithms with detailed case studies. They have also inferred as many of the algorithms which have proved valuable have emerged over time from the machine learning committee.
Ristic [12] in his work numerical model for the critical stress determination in spur gears has enlightened gear carrying capacity and stress state depends to a large extent on main gear profile configuration parameters. For this he has investigated a tooth root form and the fillet radius by analyzed gear kinematics, carrying capacity, strength, production and some other characteristics. He has primarily focussed on gear tooth root strength. And also he has given special attention to the analysis of the impact of the gear tooth fillet radius at the critical cross section on stress value and distribution. He analysed that stress intensity factor and the gear working life depend directly on the tooth root stress. According to him tooth root stress concentration depends on many parameters and real gears are statically undetermined systems. He has inferred that main problems of gears design were to obtain an optimal gear form relative to stress concentration. So that he has focussed on to found the optimal fillet tooth root radius to minimize the root stress intensity.

He applied numerical methods for obtaining results. He used finite element method and real working conditions simulation. He has analysed the results in order to form an effective numerical model for tooth root geometrical discontinuity phenomena at static loading.

From his obtained results he concluded as the tooth fillet radius was increased the stresses were reduced. And he also inferred the reduction of stresses affect directly to the service life which increases because it deflects danger of initial cracks appearance and increases the safety factor on that. His research showed that an appropriate fillet radius selection can increase tooth root stresses in its critical section even by 30%.

Pedersen [13] in his work improving bending stress in spur gears using asymmetric gears and shape optimization enlighten that the bending stress plays a significant role in gear design and the magnitude of bending stress is controlled by nominal bending stress and the stress concentration due to the geometrical shape. He has also inferred that the bending stress is indirectly related to shape changes made to the cutting tool. According to him the use of asymmetric gear teeth and by shape optimizing the gear through changes made to the tool geometry can reduce the bending stress significantly.

He has also inferred that to obtain the largest possible stress reduction a custom tool must be designed depending on the number of teeth, but also but the stress reductions found are not very sensitive to small design changes.

From the obtained results he suggested that the improvement in the bending stress for gears can be found by the use of asymmetric gears. He has achieved the bending stress reduction by two contributions, a thicker tooth root and a root shape change where we have the stress concentration. The factor that has the largest influence is the enlargement of the root thickness. He has proposed the use of a standard rack or two standard racks. He also enlighten that reduction in the bending stress is lower than that reported for specific optimizations, but the difference is not significant, especially for gears with a higher number of teeth.

Saravanan et al. [14] have worked on fault diagnosis of spur bevel gear box using artificial neural network. They have shown that the condition of operating machine can be predicted by the vibration signals extracted from rotating parts of machine. This carries a lot many information within them. They have also inferred processing of these raw vibration signatures measured at a convenient location of the machine unravels the condition of the component or assembly under study. Their work dealt with the effectiveness of wavelet-based features for fault diagnosis of a gear box using artificial neural network and proximal support vector machines.

They classified the statistical feature vectors from Morlet wavelet coefficient using J48 algorithm. The predominant features were fed as input for training and testing using artificial neural network and proximal support vector machines. The relative efficiency in classifying the faults in the gear box was compared using the above method.

Wang et al. [15] carried out accurate bending strength analysis of the asymmetric gear using the novel ES-PIM with triangular mesh. They have introduced a method viz. the edge-based smoothed point interpolation method to the bending strength analysis of asymmetric gear with complex outlines. They have used a special designed rack cutter for generation of five sets of asymmetric gears with the pressure angles of 20°/20°, 25°/20°, 30°/20°, 35°/20°, 40°/20° respectively. They have kept four key factors for those five models were accuracy, convergence; convergence rate and computational efficiency of the present ES-PIM are checked in great detail.

They have also used the finite element method for calculating the above factors for comparing the results with ES-PIM. They have showed that the ES-PIM method using the background mesh of triangular cells is used to conduct the asymmetric gear bending stress analysis.

From the obtained result they inferred that ES-PIM could provide more efficient and accurate solutions in stress field than the finite element method, and also
they suggested ES-PIM is more suitable for stress analysis of the complicated asymmetric gears. They have also inferred their proposed method can better solve the stress concentration problem, and also the ES-PIM method convenient for bending strength analysis of complicated asymmetric gear which involves complex outlines, stress concentration at the fillet and concentration load at the HPSTC.

Park and Yoo [16] in their research on deformation overlap in the design of spur and helical gear pair they have investigated the deformation overlap by analysis of the elastic deflection of gear teeth. They have made displacement analysis of the deformation overlap, which was the quantity calculated numerically, and that was the piled region of a contact tooth pair due to the elastic deformation.

For computing the contact force and teeth deflection they have used finite element method. They have calculated the contact forces between teeth from the transmitted torque, and the contact problem was defined as a QP problem. They have used the contact forces as boundary conditions for calculating the deformation overlap. They have used the deformation overlap of a spur gear pair for the basis of the tooth tip relief, for a profile shifted gear pair made analysis of deformation characteristics, and also considered teeth deflection of profile shift.

From the obtained results they have inferred that for the design of a profile shifted gear pair and the tooth tip relief calculation deformation overlap were applicable. They have proposed the standard amount of removable material for the tip relief and also they have obtained deformation characteristics and optimal shift for long and short addendum system by using deformation overlap. They have also inferred that for helical gear pair and three dimensional problems the deformation overlap is applicable, and also it showed good impact on three dimensional problems. They have suggested that the deformation overlap may be used for other types of problem arises in gear pair.

Sfakiotakis, and Anifantis[17] in their research on Finite element modelling of spur gearing fractures introduced a numerical procedure of solution for the study of stress singularities in gear teeth fractures incorporating kinematic rotations and varying contact conditions. They have used the global design variables for parametric description of both the geometry and the operation, and also they have used the finite element method for approximation of elastic deformations which enables the development of a computational design procedure which simulates the operation of spur gear drives.

They have considered the members of the drive as individual sub domains which could elastically deform and rotate; the engagement of operating surfaces at successive points enables the simulation of conjugate action. They have evaluated the stress intensity factor by displacement formulas and in the presence of cracks; the behaviour of stress singularities is approximated by quarter point elements.

From the obtained results they have inferred that the model which they developed enables the design engineers with a powerful computational tool, which eliminates any modelling requirements on load sharing. And also their approach may be further extended to describe three-dimensional problems arising in other types of gearing and defects.

Fig 2 (a) and (b) [17] it is clear that the stress intensity factor is completely dependent on the location of the contacts. Stress intensity factor of the mode I becomes minimum, when the contact lies in the region of pitch point and the stress intensity factor of mode II rises and finally becomes maximum. Stress intensity factors of mode I and mode II increase with increase in ratio α/m. Where m is module and α is edge crack of the depth.

Fig 2 variation of stress intensity factors with respect to the relative contact position for an edge crack on gear tooth face (a) mode I and (b) mode II. [17]

Songa and Imb [18] in their study the applicability of process design system for forward extrusion of spur gears showed through experiment the applicability of the automated process design system developed for cold forward extrusion of solid or hollow spur gears was confirmed. They have used the developed system for designing the extrusion die set.

They considered an elastic analysis of dies for shrink fit and carried out to determine level of hoop and effective stresses depending on its ratios. In their investigation they were extruded hollow and solid gears that depend on types of lubrication and types of materials.

From their results which they obtained from experiments they have inferred that filling status of the hollow gear was improved in comparison with that of
the solid gear irrespective of forming conditions and the predicted forming load obtained from the developed design system matched well with the experimental load requirements. They have also highlighted that solid and hollow spur gear was easily extruded without any forming defects through the die design set-up determined from their developed method.

Atan and Ozdemir [19] in their work Intelligence modelling of the transient asperity temperatures enlighten that the rise in temperature at the contact zone of meshing gears is a serious problem in gear design. According to them the if the temperature of lubricating surface is increases then surfaces may result in the significant decrease on the material strength and lubricant viscosity which reduces the film thickness, causing solid to solid contact.

They have used the equations and evaluations of temperature rise for establish an artificial intelligence model where a multi layer feed forward neural network has been employed.

Their proposed model accepts surface roughness, gear ratio, horsepower and the number of teeth as input variables, and outputs calculated pinion surface asperity temperatures. They have also inferred that the advantage of the model is the simplification, and the lessening of the iterative efforts. From the obtained results they concluded the analytical method requires many iterations between the viscosity and the temperature and through the iterations, many data have to be entered by hand. But for the artificial neural network method none of the difficulty exists.

3. Conclusion

After combing through the literature it is concluded that finite element analysis using Ansys software can be used for optimization of gear tooth root fillet, however with a drawback that it requires an accurate modelling of gear which is time consuming and difficult. Least square method can be used for preparation of a gear dynamic model for optimization of gear profile to reduce noise in gear system is also a laborious and difficult and it involves complex calculations. Multi scale local statics method could be applied as a tool to predict gear failure, which is also a good method but it takes time to prepare statics.

After understanding the various methods used for gear tooth root fillet optimisation it is suggested that software can be prepared by using equation of curve fitting in mat lab simulink for parameters like number of teeth, speed, deflection which predict the relation between above parameters and gives smooth results for bending stress for different types of root fillet like trochoidal or circular of a spur gear. This reduces the iterations during the design process. Matlab simulink actually could be used as a supporting tool to reduce the effort and time in the design and simulation process using finite element method.

References


