

A Study of Darcian Effect on Heat and Mass Transfer Flow over an Inclined Surface in Presence of Chemical Reaction

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Abstract: The steady laminar two dimensional flow past an inclined porous plate in the presence of thermophoresis, chemical reaction and radiation has been studied. With the appropriate transformations the boundary layer equations are transformed into a set of non-linear ordinary differential equations. The similarity solutions of the transformed dimensionless equations are solved numerically using Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth order iteration scheme. In this paper, the profiles of dimensionless velocity, temperature and concentration distributions are shown graphically for various values of chemical reaction parameter and different order of chemical reaction.

Keywords: Darcy flow, Chemical reaction, Thermophoresis, Radiation, Runge-Kutta iteration scheme, laminar flow.

I. INTRODUCTION

The study of the behavior of the fluid through porous media is of principle interest in many scientific and engineering applications. Quite apart from its theoretical interest, for purely scientific reasons, this type of flow is of great importance in petroleum engineering, soil mechanics, ground water hydrology, water purification, industrial filtration and ceramic engineering. On the other hand, the study of flow through porous media is also of great interest in astrophysics and geophysics. The usual starting point in the study of problems concerning flow through porous media is Darcy's law. In the fluid mechanics of porous media, the place of momentum equations of force balances is occupied by the numerous experimental observations summarized mathematically as the Darcy law. These observations were first reported by Darcy [1] who discovered that the area-averaged fluid velocity through a column of porous material is directly proportional to the pressure gradient established along the column. Subsequent experiments proved that the

area-averaged velocity is inversely proportional to the viscosity of the fluid seeping through the porous media. The velocity of Darcy's law has been tested on many occasions and it has been shown that it is valid for a wide domain of flows. For liquids it is valid for arbitrary small pressure differentials, but for liquids of high velocities or for gases at extreme velocities, Darcy's law becomes invalid.

By considering the importance of radiation in which heat is transmitted from one place to another without heating the intervening medium, Perdikis and Raptis [2] studied the heat transfer of a micropolar fluid in the presence of radiation. Later Raptis [3] studied the same fluid flow past a continuously moving plate in the presence of radiation. Recently El-Arbawy [4] extended the model of Raptis by taking the effects of suction and injection. In all these studies only forced convection was taken into account. Rahman and Sattar [5] studied transient convective flow of micropolar fluid past a continuously moving vertical porous plate in the presence of radiation.

Thermophoresis is a phenomenon which makes submicron sized particles to be driven toward cold surface. This phenomenon has gained importance for engineering applications; include particle deposition onto wafers in the microelectronics industry, particle surfaces produced by condensing vapor-gas mixtures, particles impacting the blade surface of gas turbines, and others such as filtration in gas-cleaning and nuclear reactor safety. The thermophoretic velocity is the velocity acquired by the particles and the thermophoretic force is the force experienced by the suspended particles due to the temperature gradient. The magnitudes of the thermophoretic force and velocity are proportional to the temperature gradient and depend on many factors like thermal conductivity of aerosol particles and the carried gas.

Goren [6] was one of the first to study the role of thermophoresis in laminar flow over a horizontal flat plate

where the analysis covered both cold and hot plate conditions. The laminar tube flow solution for thermophoretic deposition of small particles has been studied by Waker et al. [7], Epstein et al. [8] carried out a thermophoretic analysis of small particles in a free convection boundary layer adjacent to a cold vertical surface. The thermophoretic deposition of the laminar slot jet on an inclined plate for hot, cold and adiabatic plate conditions with viscous dissipation effect were presented by Grag and Jayaraj [9].

Another situation which can arise in practice is a chemical reaction produced in the porous medium. The main aspects occurring in the modeling of a chemical reaction in a porous medium are discussed by Nield and Bejan [11]. The rate equation for a reagent of concentration C is:

$$\frac{dc_m}{dt} = -kC_m^n$$

Where $C_m = \frac{c}{m}$ is the concentration in moles per unit volume of the fluid mixture, m denotes the molar mass of the reagent and n is the order of the reaction. Further, the rate coefficient k is in general a function of the temperature, described for instance by the Arrhenius equation. However, in many studies k is considered as a constant.

In Anjalidevi and Kandasamy [12], a work done for viscous fluid, the reaction between species A and B was considered to be the first order, homogeneous and with a constant rate. The study of Chamka [13] of MHD flow over a uniformly stretched vertical permeable surface used similar assumptions for the chemical reaction. Afifi [14] analyzed the MHD free convective flow and mass transfer over a stretching sheet with homogeneous chemical reaction of order n (n was taken 1,2 and 3), and with a constant rate. A diffusion equation with a chemical reaction source term was included by Abo-Eldahad and Salem [12] in the analysis of flow and heat transfer of an electrically conducting and non-Newtonian fluid over a moving cylinder in the presence of a uniform magnetic field and the order of the reaction $n = 1, 2$ and 3 .

In this paper, the present investigation has been done on free convection fluid flow past over an inclined porous plate with thermal radiation, chemical reaction and thermophoresis in presence and absence of magnetic field taking into account the Rosseland diffusion approximation. The investigation is based on the known similarity analysis and the solutions are obtained numerically.

II. MATHEMATICAL FORMULATION

A two-dimensional steady boundary layer heat and mass transfer flow of a viscous incompressible fluid past over an inclined infinite porous plate is considered in the presence of thermophoresis. It is assumed that the fluid properties are isentropic and constant, except for the fluid viscosity, which is assumed to vary as inverse linear function of temperature given by I. Pop [14]:

$$\frac{1}{\mu} = \frac{1}{\mu_0} [1 + \gamma(T - T_\infty)] \text{ or } \frac{1}{\mu} = a(T - T_r) \quad (1)$$

Where $a = \frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$

Here μ_∞ and T_∞ are the fluid free stream dynamic viscosity and the fluid free stream temperature, a and T_r are constants

$$\gamma > 0 \quad a > 0$$

and their values depend on the reference state and the thermal property of the fluid. In general for liquids. The flow is assumed to be in the x -direction which is taken along the plate in the upward direction and the y -axis is taken normal to it. Allowing for the thermophoretic transport of particles the governing boundary-layer equations are:

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_\infty} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + g\beta(T - T_\infty) \cos \alpha + g\beta^*(c - c_\infty) \cos \alpha - \frac{v}{k} u - \frac{bu^2}{k} \quad (3)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho_\infty c_p} \frac{\partial q_r}{\partial y} \quad (4)$$

Concentration equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - \frac{\partial(v_T C)}{\partial y} - k_1(C - C_\infty)^n \quad (5)$$

Where u, v are the velocity components in the x and y directions respectively, μ is the fluid dynamic viscosity, ρ_∞ is the ambient fluid density, σ is the electrical conductivity, k is the permeability of the porous medium, T , T_w and T_∞ are temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream respectively, C , C_w and C_∞ are the corresponding concentrations, D_m is the chemical molecular diffusivity, c_p is the specific heat at constant pressure, n order of chemical reaction.

The boundary conditions for the model are:

$$u = U \quad v = 0 \quad T = T_w \quad C = C_w \text{ at } y = 0 \\ u = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (6)$$

By using Rosseland approximation,

$$q_r = -\frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y} \quad (7)$$

Where σ_1 is Stefan-Boltzmann constant and k_1 is mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that may be T^4 expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series and neglecting higher order terms, thus

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad (8)$$

Using (7) and (8) in equation (4), we have

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{16\sigma T_\infty^3}{3\rho_\infty c_p k_1} \frac{\partial^2 T}{\partial y^2} \quad (9)$$

The thermophoretic velocity, which appears in equations (5), may be expressed in the following form:

$$v_T = \frac{-k\nu \partial T}{T \partial y}$$

Where the value $k\nu$ represents thermophoretic diffusivity, T is some reference temperature, and k is the thermophoretic coefficient, which range in value from 0.2 to 1.2 as observed by Bachelor and Shen [10],

III. SIMILARITY ANALYSIS

To transform Equations (3), (4) and (9) into a set of ordinary differential equations the dimensionless variables are introduced as:

$$\eta = \frac{y}{x} \sqrt{Re}$$

$$u = Uf'(\eta)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\Phi(\eta) = \frac{c - c_\infty}{c_w - c_\infty}$$

where η is the dimensionless distance normal to the sheet, f is the dimensionless stream function, θ is dimensionless fluid temperature and ϕ is dimensionless concentration.

We also consider,

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{-1}{\gamma(T_w - T_\infty)}$$

Introducing by Tsai [10], the thermophoretic parameter τ , is given by

$$\tau = \frac{-k(T_w - T_\infty)}{T}$$

$$\Rightarrow \frac{1}{T} = \frac{-\tau}{k(T_w - T_\infty)}$$

Finally we get the following similarity equations

$$(\theta - \theta_r)f'''' - f''\theta' - (G_r\theta + G_c\phi) \frac{(\theta - \theta_r)^2}{2\theta_r} \cos \alpha - \frac{(\theta - \theta_r)^2}{2\theta_r} f f'' + \frac{2}{D_a} (f' + F_s f'^2) \frac{(\theta - \theta_r)^2}{2\theta_r} = 0 \quad (10)$$

$$(6N + 8)\theta'' + 3NP_r f \theta' + 6NP_r E_c f''^2 = 0 \quad (11)$$

$$2\phi'' - 2S_c \tau (\theta'' \phi + \theta' \phi') - 2S_c \gamma \phi^n + S_c f \phi' = 0 \quad (12)$$

The transformed boundary conditions are:

$$f = 0, f' = 1, \theta = 1, \phi = 1 \text{ at } \eta = 0$$

$$f' = 0, \theta = 0, \phi = 0 \text{ as } \eta \rightarrow \infty \quad (13)$$

The dimensionless parameters appeared into the above equations are defined as follows:

$$G_r = g\beta(T_w - T_\infty) \frac{2x}{U^2} \text{ is the Grashof number}$$

$$G_c = g\beta^*(c_w - c_\infty) \frac{2x}{U^2} \text{ is the modified Grashof}$$

$$D_a = \frac{kU}{\nu_\infty x} \text{ is the Darcy flow}$$

$$F_s = \frac{bU}{\nu_\infty} \text{ is the Forchheimer number}$$

$$Pr = \frac{\nu_\infty}{\alpha} = \frac{\rho_\infty \nu_\infty C_p}{k} \text{ is the Prandtl number}$$

$$Ec = \frac{U^2}{c_p(T_w - T_\infty)} \text{ is the Eckert number}$$

$$N = \frac{\kappa K_1}{4\sigma_1 T_\infty^3} \text{ is the Radiation parameter}$$

$$Sc = \frac{\nu_\infty}{D_m} \text{ is the Schmidt number}$$

$$\gamma = \frac{k_1 x}{U} C_w^{n-1} \text{ is the chemical reaction parameter}$$

$$\tau = \frac{-k(T_w - T_\infty)}{T} \text{ is the thermophoretic parameter}$$

IV. RESULTS AND DISCUSSION

The transformed governing equations (10), (11) and (12) γ together with boundary conditions (13) were integrated numerically by means of the Runge-Kutta method incorporated with the shooting technique for a systematic guessing of $f'(0)$, $-\theta'(0)$ and $\phi'(0)$. Numerical results for

$f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ for different values of chemical reaction parameter and the order of chemical reaction n .

Here we assume the governing parameters are as follows:

$$Pr = 0.2, Sc = 0.5, G_r = 2.5, G_c = 0.2, \theta_r = 4.0,$$

$$D_a = 0.7, N = 7.0, Ec = 9.4, \tau = 1, \alpha = 0^\circ$$

Figure 1-3 shows the effect of chemical reaction parameter on the velocity, temperature and concentration profiles for the first order chemical reaction respectively. In figure 1 and 2 as expected we see that there is no effect of chemical reaction parameter on the velocity and temperature profiles. In figure 3 we see that the concentration decreases with the increase of chemical reaction parameter. Also this is consistent with the fact that the concentration boundary layer thickness decreases with the increase of chemical reaction parameter.

Figure 4-6 shows the effect of chemical reaction parameter on the velocity, temperature and concentration profiles for the second and third order chemical reaction respectively. In figure 4 and 5 as expected we see that for second and third order chemical reaction the velocity and temperature remain same in the absence of chemical reaction parameter, but in

$$\gamma = 0.$$

the presence of chemical reaction velocity decreases with compared to However, the temperature increases for $\eta < 1.1$ and $\eta > 5.3$ and decreases for $\eta \in [1.1, 5.3]$ with the increase of γ . In figure 6 for fixed γ the concentration increases with the increase of order of chemical reaction n.

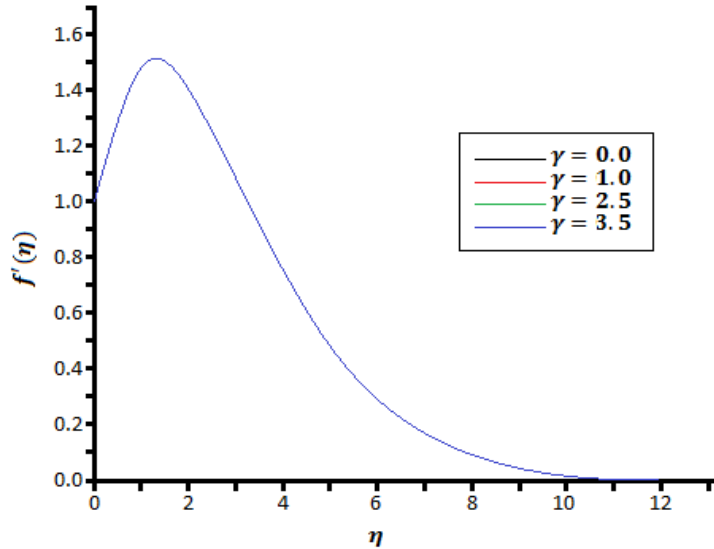


Fig. 1. Velocity profiles for various values of chemical reaction parameter.

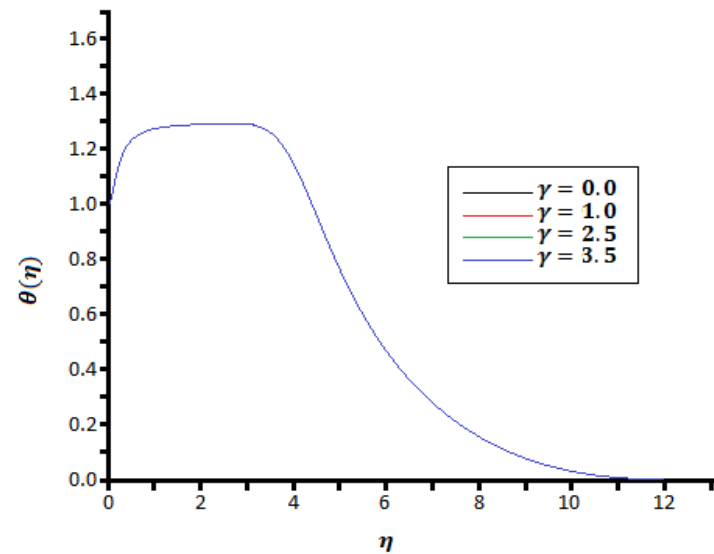


Fig. 2. Temperature profiles for various values of chemical reaction parameter.

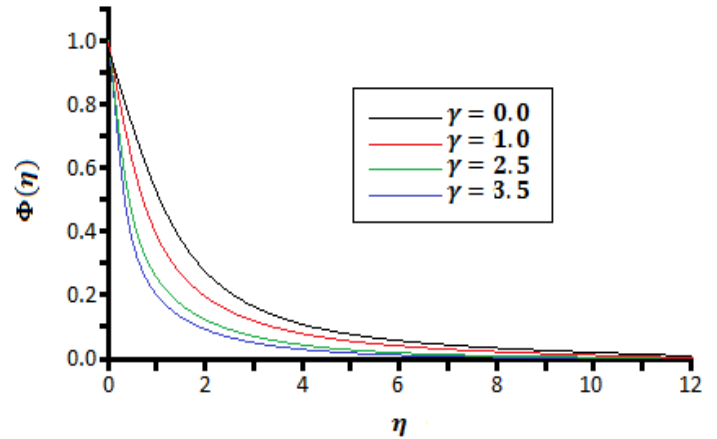


Fig. 3. Concentration profiles for various values of chemical reaction parameter.

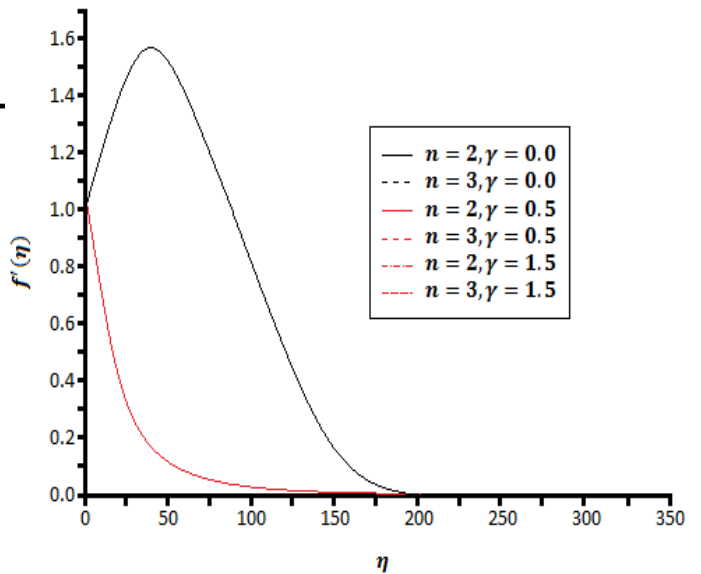


Fig. 4. Velocity profiles for second and third order.

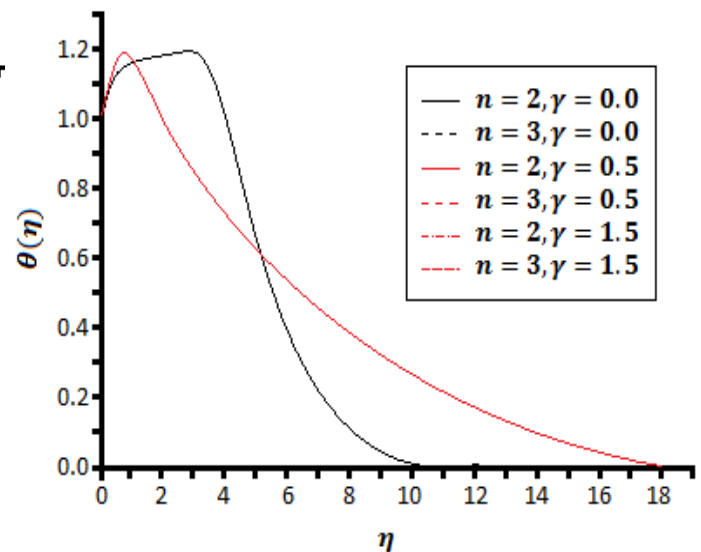


Fig. 5. Temperature profiles for second and third order.

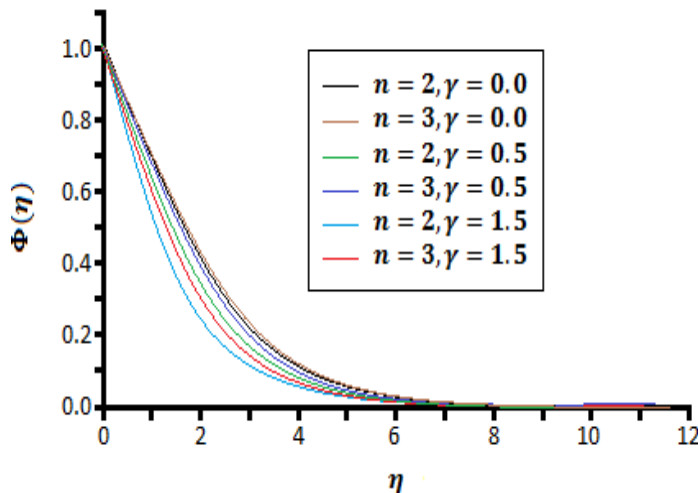


Fig. 6. Concentration profiles for second and third order.

V. CONCLUSION

In this study, we have investigated the solutions to the boundary layer equations for momentum, temperature and concentration of a steady fluid flow past over an inclined porous plate in the presence of chemical reaction, thermophoresis and radiation. Using usual similarity transformations the governing equations have been transformed into non-linear ordinary differential equations and were solved for similar solutions by using Runge-Kutta shooting iteration technique. The effects of chemical reaction parameter and order of chemical reaction n are examined to observe boundary layer behavior. From the present investigation we may conclude that for first order chemical reaction there is no effect of chemical reaction parameter on the velocity and temperature profiles but the concentration decreases with the increase of chemical reaction parameter. For second and third order chemical reaction the velocity and temperature remain same in the absence of chemical reaction parameter but in the presence of chemical reaction velocity decreases with compared to $\gamma = 0$. However, the temperature increases for $\eta < 1.1$, $\eta > 5.3$ and decreases for $\eta \in [1.1, 5.3]$

with the increase of chemical reaction parameter. In figure 6 for fixed γ the concentration increases with the increase of order of chemical reaction n . It is hoped that the present investigation of the study of physics of flow over an inclined surface can be utilized as basis for many scientific and engineering applications.

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