A Study in ISO Geometric Analysis

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***Abstract* - In modern era, designers generate Computer Aided Design files which are scaled into analysis suitable geometries. These geometries are analyzed through finite element codes. Although it is an advanced method form the times when a geometry was worked on a drawing board with pencils and later passed to a stress analyst, the task still is far from simple. For complex geometries it is estimated that over 80% of the overall analysis time is required [1]. Moreover, the engineering designs are becoming more complex. This paper is an attempt to study a new method of analysis namely isogeometric analysis which was first proposed by Thomas J.R. Hughes and aims at integration of CAD and FEA to reduce analysis time. This analysis reconstitutes design and analysis part so as to produce a geometric model readily available for analysis.**

Keywords - Isogeometric Analysis, CAD, FEA, NURBS, B-splines, T-splines.

I. INTRODUCTION

A well established area of mathematics solely devoted to representation and manipulation of curves and surfaces is Computer Aided Geometric Design (CAGD). This area has become a major interest for researchers since emergence of field of isogeometric analysis. The reason for the success of this field is the fact that shape presentation in CAD is not well suited for FEA. FEA is an integral part of many product development processes. Still little attention is paid to requirements of FEA in CAD-systems. FEA requires meshing of a CAD part. However, meshing can be perceived as an approximation of geometrical representations in CAD. This approximation is used in numerical simulation tools which results in crude approximation of design shapes. Isogeometric analysis proposed in 2004 by T.R. Hughes builds this gap between FEA and CAD models. Isogeometric Analysis uses different tools including standard curves such as NURBS which are already a standard being used as CAD tool. T-splines by Sederberg are a forward and backward generalization of NURBS technology. T-splines are very robust and extend NURBS to use local refinement and coarsening. Besides these, there are also other computational geometry technologies that can be a basis of isogeometric analysis. One such method is subdivision surface. This method uses a limiting process to define smooth surface from a mesh. The mesh generally contains triangular or quadrilateral elements. This is demonstrated in [2] (Cirak et.al 2000). The subdivision method is very promising as there is no restriction on topology of the control grid. This can be seen even in revenue terms as Pixar animation uses this method to model its animations. This paper is an attempt to study isogeometric analysis applications through B-splines[3], T-splines[4] to an extent to Kirchoff-Love Shells[5] , subdivision methods[2] till the application to viscous incompressible flow[6].

A more simplified and graphical analysis between manufacturing time and complexity is shown in [4] in form of following Fig.1.

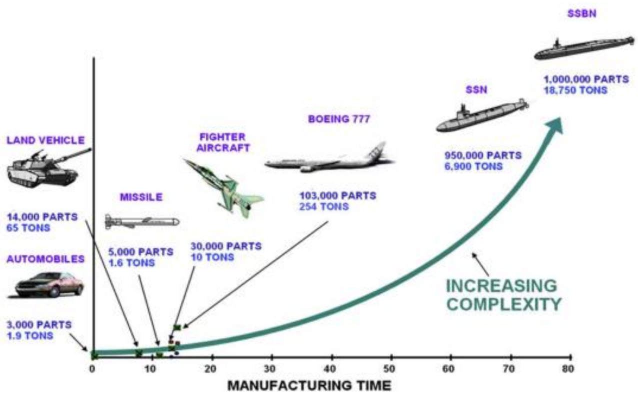

Fig. 1

In the same work by Bazilves et.al [4] a more detailed time analysis is shown as in Fig. 2:

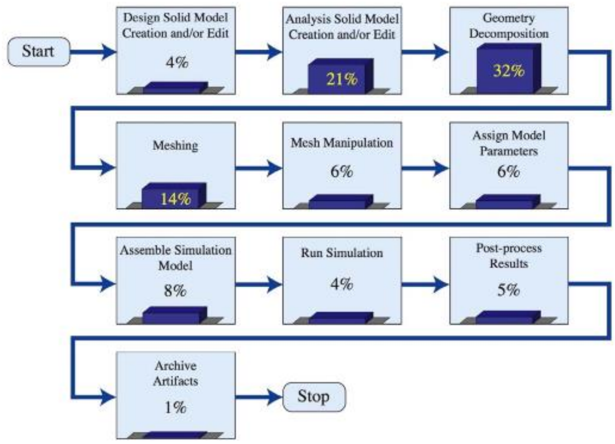

Fig. 2

It can be seen in the Fig. 2 that the majority part of total time for design and analysis part is headed by analysis. However, the design part only consumes approx. four percent of the total time. It can be easily concluded that if the feasible design model is for FEA, better are the chances of reducing analysis time and hence reducing total manufacturing time. Isogeometric Analysis attempts to do the same. In the next section we shall discuss various techniques through which this can be achieved.

II. IGA THROUGH NURBS & T-SPLINES

B-Spline curves provide minimum span or so to say maximum possible control over the curve. Non-Rational B-Spline (NURBS) as a basis function are very effective in CAD geometries due to their various properties such as convex hull and diminishing properties. A B-spline is a non-interpolating piecewise polynomial curve defined by a set of points (Control Points) and a knot vector, which is a set of parametric coordinates which divide B-spline into sections. If the these knot vectors divide B-spline equally then they are called uniform knot vectors.

A lot of researches have been done for using NURBS for isogeometric analysis. This technique has been used effectively in various analysis. T-splines are yet another smooth function equivalent to NURBS basis with a geometrical flexibility which can keep original geometry with parameterization unchanged.

A basic difference between NURBS and T-splines can be understood through Fig.3 [4] and Fig. 4 [4].

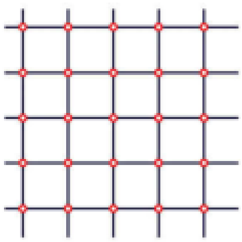

Fig. 3

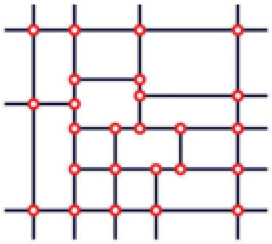

Fig. 4

Fig, 3 is topology of NURBS control grid where it can be seen that NURBS control points lie in a rectangular grid. Fig. 4 is topology of a sample T-spline control grid where it can be seen that T-spline control point can be incomplete. An advantage of T-spline geometry can be easily understaood through blow up of a model a human hand. Fig. 5 [4] and Fig. 6 [4] show the blow up model pertaining to human wrist generated through NURBS control grid and T-spline control grid.

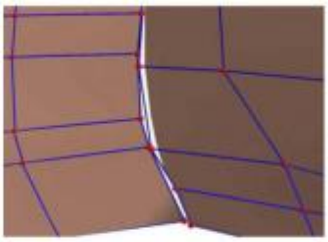

Fig. 5

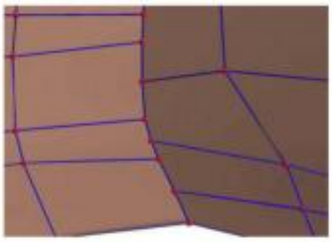

Fig. 6

It can be seen in Fig. 5 that NURBS control grid are unable to complete the geometry. However, T-splines provide an independency where less control points are required to complete the complex geometry. If two NURBS surfaces do not share a common curve C^0 continuity cannot be achieved. To join two surfaces in such case would require inserting knot vectors from one surface to other. This is disadvantageous as knot insertion would affect entire geometry, as it is a global operation. J.A. Cottrell et. Al [1] proposed a method of local refinement was considered which involved multiple patches [4].

However, the method was inconvenient as the refinement still propagates through surfaces. T-splines are a better option which follows smooth functions, possesses geometrical flexibility similar to NURBS while permitting local refinement.

An another application of NURBS for IGA has been shown in [5]. For thin shells i.e. radius to thickness ratio is greater than 20, Kirchoff-Love theory is applicable. C^1 continuity is required between elements for generating Kirchoff Shell elements. NURBS are ideally suited for generating these elements. In [5] Kiendl et.al. have demonstrated the same by generating Kirchoff-Love elements through NURBS and then integrating CAD-CAE elements. For mesh refinement they have used both knot insertion (for *h*-refinement) and order elevation (for *p*-refinement). Their method shows universality of NURBS and its effectiveness for existing theories. This work shows isogeometric shell elements modeled as solids have advantage of fewer degree of freedoms which is compensated by more involved element formulation[5]. Still a development of same models through T-spline seems to be promising for compatible multiple patches in a complex geometry.

A new class of compatible isogeometric discretization for incompressible flow is also possible as discussed in [6]. Geometric structures of Navier-Stokes flow has to be constructed for this purpose. While doing this it should be kept in mind that conservation laws are satisfied. The discretization would satisfy incompressibility criteria in a pointwise manner. Compatible B-spline spaces was detailed and various mathematical properties of resulting spaces are presented in [6]. This also shows that IGA can be applied to fluid flow problems as effectively as to structural analysis.

III. B-SPLINES, PB SPLINES & T-SPLINES

As discussed in various citations above, it is clear that B-splines are an essential tool in isogeometric approach. Almost all the design and analysis software prefer NURBS. But it also has certain disadvantages also pertaining to its continuity. For example, as shown in fig. 5, NURBS control grids are unable to complete the geometry. Similar type of problem arises if two different patches based on NURBS are attempted to join. To overcome this, PB splines are used. PB spline stands for point based splines.

PB splines add flexibility. Unlike in B splines, a local knot vector of arbitrary length is provided in PB splines. For better control in B splines, the knot vector provided is global in nature. The insertion of knot vector provides better local control without affecting the whole shape of geometry. One can argue its effectiveness with subdivision method also. However PB splines are found to be equally equally effective. In this case, a term control cloud is used instead of control mesh[4]. Each point corresponds to one function. Choosing points arbitrarily will result in degenerate geometries but basic concept of splines remains the same. In NURBS, there is no clear ordering of control points and the same goes for PB splines. Properties of NURBS still hold good in PB-splines but in an unstructured environment. PB splines are built upon local knot vectors and will have as many continuous derivatives. The blending functions formed in this case have enough spaces between them, which can again be used for reproducing arbitrary linear polynomials.

A problem which comes along with PB splines can be easily understood once concept of local knot vector is clear. Due to the random selection, there is no clear region which can be identified as an element. As each function has been constructed without considering another, a refinement is not possible. New blending functions can be added as required but there is no guarantee that control points generated would preserve the original geometry. A PB spline formed is shown in Fig. 7[4] with four control points.

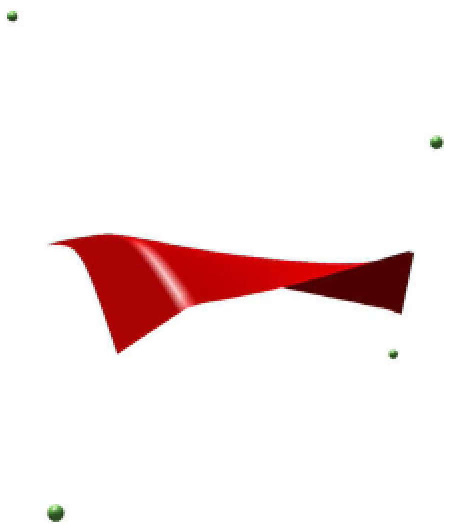

Fig. 7

T splines emerges as an alternative which combine flexibitlity of PB splines with topology and structure of NURBS. PB splines possess the property of local refinement while NURBS provide smoothness. These both properties can be provided by T-splines. Moreover properties that make T-splines useful for geometric modeling also make them useful for finite element analysis.

Knot vectors in NURBS are global in nature, while in PB splines, they are local in nature. In T splines each function has its own local knot vector but these local knot vectors are derived from global structure. This is demonstrated in Fig. 8 [4].

In Fig. 8 each line in the mesh corresponds to knot value. The presented paper does not go through the numerical analysis of these functions rather than focuses on the outcomes of these techniques. Another advantage that T-splines provides is continuity. An abrupt change in continuity is well dealt by T-splines as shown in Fig. 9 [4].

For control in FEA, T-splines provide feasible solution. The extension of T-spline geometry in three dimension is also fairly simple. An index space version of T-mesh is defined as a prism where every face has a positive integer value. A degree of freedom is chosen for T-spline and for each index space direction a knot vector is chosen. As knot vector is local in nature, it provides a local control over elements which will be helpful in creating FEA based elements.

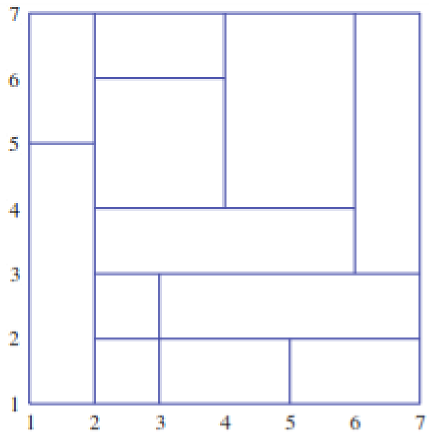

Fig. 8

Fig. 9

IV. FEA

FEA stands for finite element analysis. A partial differential equation solved through finite element method will consist of a variational formulation and trial and weighing function. These space functions depend upon respective basis functions. These basis functions are defined by finite elements. Finite elements can be seen as local representation of spaces. These elements discretize the domain into simple shapes for example, triangles, quadrilateral, tetrahedral, hexahedral etc. These elements are defined in form of interpolator polynomials. Mostly Lagrange and Hermite polynomials are used for FEA.

When it comes to understanding application of FEA onn CAD models, it should be kept in mind that CAD models are based on NURBS and for FEA we need interpolatory functions. In NURBS, the basis function is usually not interpolatory. While meshing for FEA we deal with two ideas – control mesh and physical mesh. Control points define control mesh. This control mesh interpolates control points. The control mesh consists of multilinear control elements. For one dimensional meshes this control element is a straight line defined by two consecutive control points. For 2 dimensional meshes, elements are bilinear quadrilaterals defined by four control points. For a three dimensional mesh, trilinear hexahedra is taken as control element which is defined by eight control points. The control mesh can be distorted from model geometry, but physical geometry may still remain valid for sufficiently smooth NURBS.

Physical mesh is a decomposition of actual geometry. Physical mesh constitutes patch and knot span. Patch can be idealized as images of rectangular meshes in the parent domain mapped into actual geometry. Topology of patches for one dimension are in form of curves, for two dimensional, in form of surfaces and for 3 dimensional, in form of volumes. These can be decomposed into knot spans. Topology of these knot spans are nothing but elements. For one dimensional these are curved segments connecting consecutive knots. For 2d, these are curved quadrilaterals bounded by four curves and for 3d these are curved hexahedra bounded by six curved surfaces.

REFERENCES

[1] J.Austin Cottrell, Thomas R Hughes, Yuri Bazilves, “Isogemetric Analysis Towards Integration of CAD & FEA”John Wiley & Sons Publications

[2] F. Cirak, M. Ortiz, P. Schröder, Subdivision surfaces: a new paradigm for thin shell analysis, Int. J. Numer. Methods Engrg. 47 (2000) 2039–2072.

[3] T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exactgeometry and mesh refinement. Computer Methods in Applied Mechanics and Engineering, Elsevier, 2005, 194 (39-41), pp.4135-4195.

[4] Y. Bazilevs, V.M. Calo, J.A. Cottrell, J.A. Evans, T.J.R. Hughes, et al.. Isogeometric analysis using T-splines. Computer Methods in Applied Mechanics and Engineering, Elsevier, 2010, 199, pp.229-263.

[5] J. Kiendl, K-U Bletzinger, J. Linhard, R. Wiichner, “Isogeometric Shell Analysis with Kirchoff-Love Shell Elements”, Computer Methods in Applied Mechanics and Engineering, September 14, 2009

[6] Y. Bazilevs, V. Calo, J. Cottrell, T. Hughes, A. Reali, G. Scovazzi, \Variational multiscale residual-based turbulence modeling for large eddy simulation of incompressible ows"and Engineering, 197(1-4): 173{201, 2007.