A Stochastic Differential Equation Model

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Abstract : In this paper, we propose a stochastic differential equation model where the underlying stochastic process is a jump- diffusion process.The stochastic differential equation is represented as a Partial Integro Differential Equation(PIDE) using the Fokker Planck equation. The solution of the PIDE is obtained by the method of finite differences. The consistency, the convergence of the solution and the stability of the finite difference scheme are discussed. The model is applied to forecast the daily price changes in a commodity derivative. The observed values are compared graphically with the values expected from the proposed model.

Keywords: jump-diffusion process, partial integro differential equation, stability of the finite difference scheme, consistency and convergence of a numerical solution, commodity derivative.

Introduction

There are two reasons for studying a SDE. One motivation is that, many physical phenomena can be modelled as random processes (e.g. thermal motion). When such a process enters a physical system, we get a SDE model. The second reason is that, in statistical modelling, unknown forces are modelled as random processes. This again leads to a SDE. Unknown forces are often found to occur in modelling financial quantities like asset price, interest rate, options and other derivatives. In this paper, we present a SDE model and apply it for pricing a commodity derivative. In recent years the complexity of the models used has increased and in turn has led to complicated equations. Of particular interest is a type of differential equation containing an integral term. Such equations are called Partial- Integro Differential Equations. I. Florescu and Mariani[3]studied these type of problems and proved the existence of a solution under a general hypothesis about the integral term. R.Cont suggested a finite difference scheme with discretization of the integral term[9] .

In this paper, we discuss a particular type of PIDE . The paper is organised as follows: In sections 1 and 2 we give a review of a SDe with jump diffusions and the finite difference scheme for second order partial derivatives. In section3, we recall the eigenvalues and spectral value of a tri diagonal matrix. Also we recall the definitions of consistency, convergence and stability of the solution obtained by the numerical scheme. Finally in section 4, the proposed model and the assumptions made are described. We prove the consistency, the uniqueness of the solution to the system of equations under consideration and also obtain the conditions of stability of the finite difference scheme.The proposed model is tested with a set of real time data. The observed and the estimated values are compared graphically.

Section 1: A SDE and jump- diffusion process:

Generally a SDE is in the form $dX(t) = \mu(t, X(t))\,dt + \sigma(t, X(t))\,dW(t)$, where W denotes a Wiener process. This equation should be interpreted as an integral equation as

$X(t) - X(0) = \int \mu(t, X(t))\,dt + \int \sigma(t, X(t))\,dW(t)$, X(0) being the initial value of X.Sometimes in a SDE, the stochastic process may consist of three components namely: a drift term, a diffusion term and a jump term. The jump term accounts for 'abnormal changes' in value.The jumps occur at random times and are usually assumed to follow Poisson distribution. The magnitudes of the jumps may be assumed to follow Poisson or other distributions like beta, exponential, log normal or Pareto[1],[9].There is a direct relationship between a stochastic differential equation and a boundary value problem for a parabolic partial differential equation (diffusion equation) given by the Fokker –Planck equation:

If X(t) is a jump diffusion process satisfying the SDE ,

$dX_t = f(X(t),t)dt + g(X_t,t)dW_t + h(X(t),t,Q)dP(X_t ,t)$ then the expectation $v(x,t) = E(u(X(T)|X(t)=x)$ is the solution to the PIDE ,

$$0 = \frac{\partial v}{\partial t} + f\frac{\partial v}{\partial x} + \frac{1}{2} g^2 \frac{\partial^2 v}{\partial x^2} + \lambda \int_Q \left[v\big(x + h(x,t,q)\big) - v(x,t)\right]\varphi(q)\,dq$$

where f ,g,h are continuously differentiable functions, Q is the random variable denoting the jump amplitude mark, P is the jump process, W(t) is the Weiner process , φ(q) is the density of Q and T is the terminal time. Florescu and Mariani studied these types of equations and proved the existence of a solution under a general hypothesis on the integral term. To solve such kind of equations, a finite difference scheme with discretization of the partial derivative terms and integral term is suggested by R.Cont.

Section 2: The finite difference scheme

Let C_0^∞ be the space of continuously differentiable functions with a norm $\|.\|$

Let $v \in C_0^\infty([0,T]xR)$ be continuously differentiable as many times as required for $t \in [0,T]$ and $x \in R$. The first and second order partial derivatives of v with respect to x are discretized as follows:

For $(t_n,x_i) \in [0,T] \times R$,

$$\left(\frac{\partial v}{\partial x}\right)_i = \frac{(v_{i+1} - v_i)}{\Delta x} \;;$$

$$\left(\frac{\partial^2 v}{\partial x^2}\right)_i = \frac{(v_{i+1} - 2v_i + v_{i-1})}{(\Delta x)^2}.$$

Moreover an integral of the form $\int_\gamma^\infty [v(x + h(x,t,q),t) - v(x,t)]\varphi(q)dq$ is approximated to a sum $\sum_{j=K_1}^{k_2} \varphi_1(q)\,(v_{k+j} - v_j)$ using trapezoidal quadrature formula after replacing the limits of integration by a suitable bounded interval say [γ,δ]. By the introduction of a uniform grid on [0,T] x [γ,δ] and discretization of the derivative as well as the integral terms, the PIDE may be reduced to a system of linear equations in the form Ax = B.We apply this discretization process for the PIDE considered in this paper.

Section3: Stability and uniqueness of solution of a tri diagonal system of equations

Let Ax = B be a system of equations where A is a tri diagonal matrix of order n having each of its entry on the main diagonal as 'a', the sub diagonal entries are all 'c' and super diagonal entries as 'b'.The eigen values of this matrix are $\lambda_k = a + 2\sqrt{bc}\cos\frac{k\pi}{n+1}$ for k = 1,2,3…….n provided $bc > 0$.

The spectral radius of A is $\rho(A) = \max_k |\lambda_k|$ and the criterion for the stability of the scheme is $\rho(A) < 1$.

A matrix of order n x n is said to be diagonally dominant if the sum (in modulus) of all the off-diagonal elements in any row is less than the modulus of the diagonal element in that row.

Consistency and Convergence of the solution

A finite difference scheme is said to be consistent with the PIDE it represents, if for any sufficiently smooth solution u of this equation, the truncation error of the scheme , ϵ tends uniformly to zero as $(\Delta t,\Delta x)\to 0$. Moreover, if there exist C_1 and C_2 independent of Δt and Δx such that $|\epsilon| \le C_1 (\Delta t)^h + C_2 (\Delta x)^k$ then the order of convergence is (h,k).

The Lax-Ritchmeyer equivalence theorem:

Given a properly posed linear initial or boundary value problem, if a linear finite difference approximation to it is consistent and stable , then the scheme is convergent.

A problem is properly posed if:

(i) the solution exists, (ii) the solution is unique and (iii) the solution depends continuously on the initial data[2].

Section 4:The Proposed Model

4.1 Assumptions for the model:

Let X(t) be a stochastic process and f, g, h_1, h_2 be continuously differentiable functions of X and t. Let Q be the random variable denoting the jumps.

Define $Q^+ = \max(Q,0)$ and $Q^- = \max(-Q,0)$.Then Q^+ and Q^- are respectively the positive and negative parts of Q. Moreover, $Q = Q^+ - Q^-$. We shall assume Pareto distribution for Q^+ and Q^-.The density functions of Q^+ and Q^- are :

$$\Phi(q^+) = \frac{\lambda\gamma^{\lambda}}{(q^+)^{\lambda+1}}, \quad q^+ \ge \gamma, \lambda > 0. \quad (\gamma, \lambda \text{ are the parameters of the distribution of } Q^+)$$

$$\Phi(q^-) = \frac{\mu\beta^{\mu}}{(q^-)^{\mu+1}}, \quad q^- \ge \beta \quad, \mu > 0. \quad (\alpha \;, \mu \text{ are the parameters of the distribution of } Q^-)$$

The expectations of these two random variables are given by :

$$E(Q^+) = \frac{\gamma\lambda}{1-\lambda} = \hat{\lambda} \text{ and } E(Q^-) = \frac{\mu\beta}{1-\beta} = \hat{\mu}.$$

We shall assume λ and μ are different from 1.The parameters λ, μ, γ and β of the distributions may be estimated from historical data. Let P^+ and P^- be the densities of Q^+ and Q^- respectively.

We represent the process as $X = Z + Q^+ - Q^-$ where Z is normal in [β, γ] , Q^+ and Q^- are Pareto variables in the intervals [γ,∞] and [β,∞] respectively..Now we construct the stochastic differential equation in the form

$$dX(t) = f(X(t),t)dt + g(X(t),t)dW(t) + h_1 dP^+ 1_{[\gamma,\infty]} + h_2 dP^- 1_{[\beta,\infty]} \text{ --------- } (4.1)$$

where 1 is the indicator function.

4.2 The PIDE

Let v(x,t) be continuously differentiable in x and t as many times as required while bounded at ∞. Then the conditional expectation of the process V(X(t),t) namely v(x,t) = E[V(X(t)|X(t_0)=x_0] is the solution to the PIDE

$$0 = \frac{\partial v}{\partial t} + f\frac{\partial v}{\partial x} + g\frac{\partial^2 v}{\partial x^2} + \hat{\lambda}\int_{\gamma}^{\infty}[v(x + h_1(x,t,q^+),t) - v(x,t)]\varphi(q^+)dq^+ - \hat{\mu}\int_{\beta}^{\infty}[v(x + h_2(x,t,q^-),t) - v(x,t)]\varphi(q^-)dq^- \quad \text{---------}(4.2)$$

The domains of Q^+ and Q^- are truncated to the bounded intervals [γ,δ] and [α,β] by choosing δ and α suitably.

In equation (4.2), q^+ and q^- denote the upward and downward jumps respectively in X. φ_1 and φ_2 are the respective density functions.Let P_1, P_2, and N_1, N_2 be the parameters of the distributions of q^+ and q^- respectively. The numbers α, δ are suitably chosen as the bounds of q^+ and q^- such that $\alpha < N_2 (=\beta)$ and $P_2(=\gamma) < \delta$.

Let $D = f\frac{\partial}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}$, $I = \hat{\lambda}\int_{\gamma}^{\delta} - \hat{\mu}\int_{\beta}^{\alpha}$ $= I_1 - I_2$ and $L = D + I$

Now equation 4.1 takes the form $\frac{\partial v}{\partial t} + Lv = 0$. ------------(4.3)

We shall include the initial condition $v_0^0 = x_0$ and the notations

$$\Delta_{h_1}(v) = [v(x + h_1(x,t,q^+),t) - v(x,t)] ,$$

$$\Delta_{h_2}(v) = [v(x + h_2(x,t,q^-),t) - v(x,t)] .$$

4.3 The explicit implicit scheme for equation (4.3):

Let the period of time under consideration be T. Introduce a uniform grid on [0,T] x [α, δ] as $t_n = n\Delta t$, for n = 0,1,2,…..N-1; $x_i = \alpha + k\Delta x$ for k = 1,2……M with $\Delta t = \frac{T}{N}$ and $\Delta x = \frac{\delta-\alpha}{M}$.

Let $\{v_k^n\}$ be the solution of the numerical scheme to be defined. Let K_1^+ and K_2^+ be real numbers such that $[P_2,\delta]$ is contained in the interval $[(K_1^+ - \tfrac{1}{2})\Delta x, (K_2^+ + \tfrac{1}{2})\Delta x]$ and K_1^-, K_2^- be such that $[\alpha, N_2]$ is contained in $[(K_1^- - \tfrac{1}{2})\Delta x, (K_2^- + \tfrac{1}{2})\Delta x]$.

We use an explicit finite difference approximation for the differential operator D and an implicit difference approximation for the integral operator I as in R.Cont[10].

The derivatives are discretized as $\left(\frac{\partial v}{\partial x}\right)_k = \frac{v_{k+1}-v_k}{\Delta x}$ and $\left(\frac{\partial^2 v}{\partial x^2}\right)_k = \frac{v_{k+1}-2v_k+v_{k-1}}{(\Delta x)^2}$.

Also $(J_1(v))_k = \int_{\gamma}^{\delta} \Delta_{h_1}(v)\ \varphi_1(q^+)dq^+ \approx \sum_{j=K_1^+}^{K_2^+} \varphi_1(q_j^+)\ (v_{k+j} - v_j)$

$(J_2(v))_k = \int_{\beta}^{\alpha} \Delta_{h_2}(v)\ \varphi_2(q^-)dq^- \approx \sum_{j=K_1^-}^{K_2^-} \varphi_2(q_j^-)\ (v_{k+j} - v_j)$

Now we replace equation (4.3) by the system of equations:

$\frac{v_{k+1}^n - v_k^n}{\Delta t} + Dv_k^{n+1} + J_1v_k^n - J_2v_k^n = 0$, for k = 1,2…….M ;, $v_k^{n+1} = 0$, k > M ,
n=0,1,2….N-1 and $v_0^0 = x_0$.

4.4 Consistency of the system of equations

Proposition 1:

The system of equations (4.4) is consistent.

.ie. for all $v \in C_2^\infty$ ([0,T] x [α,δ]) and all $(t_n,x_k) \in [0,T] \times [\alpha,\delta]$,

$\left| \left(\frac{v_k^{n+1}-v_k^n}{\Delta t} + Dv_k^{n+1} + J_1v_k^n - J_2v_k^n\right) - \left(\frac{\partial v}{\partial t} + Lv\right)(t_n, x_k) \right| \le \varepsilon_k(\Delta t,\Delta x) \to 0$ as $(\Delta t,\Delta x) \to 0$.

Also there exist C_1 and C_2 such that $|\varepsilon_k| \le C_1\Delta t + C_2\Delta x$.

Proof :

Using Taylor's expansion up to the second order,

$$\left|\frac{v_k^{n+1}-v_k^n}{\Delta t} - \frac{\partial v}{\partial t}\right| \le \left|\frac{\Delta t}{2}\right| \left\|\frac{\partial^2 v}{\partial t^2}\right\|_\infty = \frac{1}{2}\left\|\frac{\partial^2 v}{\partial t^2}\right\|_\infty \Delta t \to 0 \text{ as } (\Delta t,\Delta x) \to 0$$

$$\left| Dv_k^{n+1} - Dv(t_n,x_k)\right| = \left| Dv_k^{n+1} - Dv(t_{n+1},x_k) - \Delta t Dv(\hat{t},x_k)\right| , \text{ where } \hat{t} \in (t_n,t_{n+1})$$

$$\le \left|\Delta t Dv(\hat{t},x_k)\right| + \left| f(t_{n+1},x_k)\left(\frac{v_k^{n+1}-v_k^n}{\Delta x}\right) - f(t_{n+1},x_k)\frac{\partial v}{\partial x}\right| + \frac{\sigma^2}{2(\Delta x)^2}\left| (v_{k+1}^{n+1} - 2v_k^{n+1} + v_{k-1}^{n+1}) - \frac{\partial^2 v}{\partial x^2}(t_{n+1},x_k)\ (\Delta x)^2 \right|$$

$$\leq \left| \Delta t D v(\hat{t}, x_k) \right| + \frac{1}{\Delta x} \left| f(t_n, x_k) \frac{\partial^2 v}{\partial x^2} \right| \frac{(\Delta x)^2}{2} + \left| f(t_n, x_k) \frac{\partial v}{\partial x} \right| \Delta x$$
$$+ \frac{\sigma^2}{2(\Delta x)^2} \left| (v_{k+1}^{n+1} - v_k^{n+1}) + (v_{k-1}^{n+1} - v_k^{n+1}) - \frac{\partial^2 v}{\partial x^2}(t_{n+1}, x_k) \right| (\Delta x)^2$$

$\to 0$ as $(\Delta t, \Delta x) \to 0$.

$$\left| J_1 v_k^n - I_1 v(t_n, x_k) \right| = \left| \hat{\lambda} [\textstyle\sum_{j=K_1^+}^{K_2^+} [v(tn, x_k + q_j^+) - v(tn, xk)] \varphi_1(q_j^+) - \int_\gamma^\delta \Delta_{h_1}(v) \varphi_1(q^+) dq^+] \right|$$

$$= \left| \hat{\lambda} \right| \left| \textstyle\sum_{j=K_1^+}^{K_2^+} \int_{(j-\frac{1}{2})\Delta x}^{(j+\frac{1}{2})\Delta x} [v(tn, x_k + q_j^+) \varphi_1(q_j^+) - v(t_n, x_k + q^+) \varphi_1(q^+)] dq^+ \right|,$$

since $v_k \sum_{j=K_1^+}^{K_2^+} \varphi_1(q_j^+) \approx \int_\gamma^\delta v(tn, xk) \varphi_1(q^+) dq^+$

$$\left| J_1 v_k^n - I_1 v(t_n, x_k) \right| = \left| \hat{\lambda} [\textstyle\sum_{j=K_1^+}^{K_2^+} \int_{(j-\frac{1}{2})\Delta x}^{(j+\frac{1}{2})\Delta x} (q_j^+ - q^+) \frac{\partial u}{\partial x} dq^+] \right|,$$

where $u(t_n, x_i + \theta) = v(t_n, x_i + \theta) \varphi_1(\theta)$.

So $\left| J_1 v_k^n - I_1 v(t_n, x_k) \right| \leq \left| \hat{\lambda} \right| (\delta - \gamma) \left\| \frac{\partial u}{\partial x} \right\|_\infty \int_{(j-\frac{1}{2})\Delta x}^{(j+\frac{1}{2})\Delta x} dq^+$

$$\leq \left| \hat{\lambda} \right| (\delta - \gamma) \left\| \frac{\partial u}{\partial x} \right\|_\infty (K_2^+ - K_1^+) \Delta x$$

$\to 0$ as $(\Delta t, \Delta x) \to 0$.

Similarly ,

$\left| J_2 v_k^n - I_2 v(t_n, x_k) \right| \leq \left| \hat{\mu} \right| (\alpha - \beta) \left\| \frac{\partial w}{\partial x} \right\|_\infty (K_2^- - K_1^-) \Delta x$,where $w(t_n, x_i + \theta) = v(t_n, x_i + \theta) \varphi_2(\theta)$

$\to 0$ as $(\Delta t, \Delta x) \to 0$.

Hence the error term ε_k satisfies

$$|\varepsilon_k| \leq \frac{1}{2} \left\| \frac{\partial^2 v}{\partial t^2} \right\|_\infty \Delta t + [\left| \hat{\lambda} \right| (\delta - \gamma) \left\| \frac{\partial u}{\partial x} \right\|_\infty (K_2^+ - K_1^+) + |\hat{\mu}| (\alpha - \beta) \left\| \frac{\partial w}{\partial x} \right\|_\infty (K_2^- - K_1^-)] \Delta x$$

$= C_1 \Delta t + C_2 \Delta x$, where C_1 and C_2 are independent of Δt and Δx .

4.5 Stability of the finite difference scheme

The system of equations (III) may be written in the form,

$$\frac{v_k^{n+1} - v_k^n}{\Delta t} + f \frac{v_{k+1}^{n+1} - v_k^{n+1}}{\Delta x} + \frac{\sigma^2}{2} \frac{v_{k+1}^{n+1} - 2 v_k^{n+1} + v_{k-1}^{n+1}}{\Delta x} = J_2 v_k^n - J_1 v_k^n, \quad k = 0, 1, \ldots, N,$$

$v_k^0 = x_0$, $v_k^{n+1} = 0$, for $k > N$.

or equivalently, $c \Delta t v_{k-1}^{n+1} + (1 - a \Delta t) v_k^{n+1} + b \Delta t v_{k+1}^{n+1} = v_k^n + (J_2 v_k^n - J_1 v_k^n) \Delta t$, $k = 0, 1, 2, \ldots\ldots N$ wherea $= \frac{f}{\Delta x} + \frac{\sigma^2}{(\Delta x)^2}$, $b = \frac{f}{\Delta x} + \frac{\sigma^2}{2(\Delta x)^2}$ and $c = \frac{\sigma^2}{(\Delta x)^2}$, $v_k^{n+1} = 0$, for $k > N$. It can be seen that $a = b + c$.

Δx may be chosen to satisfy $b > 0$. Obvoiusly $c \geq 0$. Hence we have $a \geq 0$.

Thus the system of equations is of the matrix form AX = B

where $A = \begin{bmatrix} 1-a\Delta t & b\Delta t & 0 & & & & \\ c\Delta t & 1-a\Delta t & b\Delta t & \cdots & & & \\ 0 & c & 1-a\Delta t & & & & \\ & & \vdots & \vdots & & & \\ 0 & 0 & 0 & \vdots & \vdots & c\Delta t & 1-a\Delta t & b\Delta t \\ 0 & 0 & 0 & & 0 & & c\Delta t & 1-a\Delta t \end{bmatrix}$

It is seen that A is a tridiagonal matrix.

Proposition 2:

The solution of the system of equations (4.5) is unique and stable if $\Delta t < \frac{1}{2a}$.

Proof:

Let $\Delta t < \frac{1}{2a}$.

Then $1 - a\,\Delta t > \frac{1}{2}$ whereas $(b+c)\,\Delta t = a\,\Delta t < \frac{1}{2}$. Thus the matrix A is diagonally dominant and the system has a unique solution.

To prove the stability of the finite difference scheme we shall verify that the spectral radius ρ of the matrix A is less than unity.

The eigen values of the tri diagonal system are given by

$$\lambda_k = 1- a\,\Delta t + 2\sqrt{bc}\,\Delta t \cos\frac{k\pi}{N+1} \quad , k = 0,1,2,3\ldots\ldots N.$$

The absolute value of the maximum among the eigen values is $|1- a\,\Delta t + 2\sqrt{bc}\,\Delta t|$

Now $\rho < 1$ requires $|1- a\,\Delta t + 2\sqrt{bc}\,\Delta t| < 1$.

i.e, $$-1 < 1- a\,\Delta t + 2\sqrt{bc}\,\Delta t < 1$$

i.e, $$0 < (a - 2\sqrt{bc})\,\Delta t < 2$$

or $$\Delta t < \frac{2}{a-2\sqrt{bc}}$$

$$< \frac{1}{2a}$$

Hence the condition $\Delta t < \frac{1}{2a}$ is sufficient for stability of the solution.

It should be noted that $b \geq 0$ requires $\frac{f}{\Delta x} + \frac{\sigma^2}{2(\Delta x)^2} \geq 0$. This is true if $f \geq 0$.

If $f < 0$, we shall select $\Delta x < -\frac{\sigma^2}{2f}$.

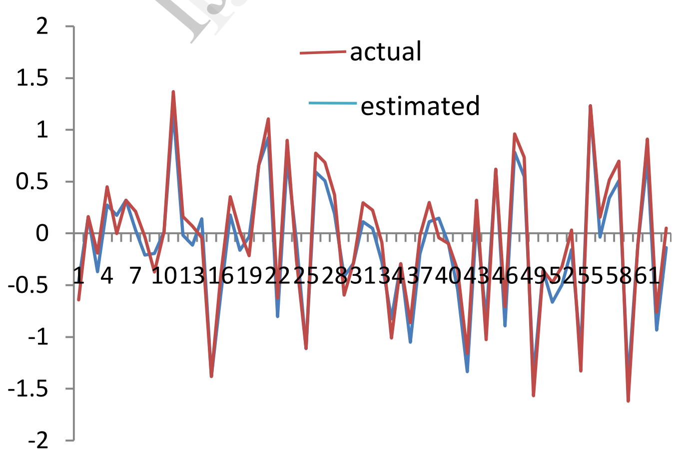
Also it can be easily checked that $\frac{1}{2a} < \frac{1}{a} < \frac{2}{a-2\sqrt{bc}}$. For the sake of simplicity, the stability condition is stated in the form given in Proposition 2.

Section5: Application of the SDE model to estimate the expected variation in the price of a commodity derivative.

A derivative is a financial insruent that gives the owner a certain payment depending on the value of the underlying asset or delivery of the asset according to the agreement between the buyer and seller. A futures contract is an agreement between two parties to buy or sell a certain asset for a certain price at a future date. Trading is done through exchanges and the underlying asset may be stocks, bonds, interest rates, exchange rates or commodities.

We consider the data from MCX (Multi-Commodity Exchange at Mumbai) for all wheat futures contracts from April 2011 having expiry as October 2011. In India, the marketing season for the rabi (or winter) wheat crop begins in the month of April every year. So data is considered at the beginning of the marketing season. We assume f(X(t),t) = m cos($\frac{2\pi Rt}{T}$), where m is the arithmetic mean of the percentage changes in the derivative price obtained from historical data and R is the demand during the period T. We shall assume R is constant throughout the period. We have chosen f as a periodic function because the demand of an agricultural commodity is seasonal, as far as a commodity exchange is concerned. Let the function g = $s^2/2$, s being the standard deviation of the data selected. Based on the test data the parameters of the distributions of q^+ and q^- may be estimated by the use of any one of the statistical techniques available. For the data under consideration the test for goodness of fit for Pareto distribution has been performed. Both Anderson Darling test and Smironov test accept the fit. The parameters of the distributions of q^+ and q^- are denoted by P1, P2 and N1,N2 respectively.

The empirical parameters are found to be P1=0.341, P2=0.016, N1=0.308, N2=0.016. Also m=o.o43819 and s = 0.56341. The limit specified by MCX for the commodity wheat is 3% either way. This means that the price variation in wheat cannot exceed 3% or fall below it. So we shall take the values of α and δ as -3 and 3 respectively. We take Δt =1. The condition Δt < 1/2a now requires Δx > √2s-m. Hence $\Delta x$is taken suitably. Let N=30 and M=5.



Plots showing the observed percentage changes in the futures price and the estimated values. The test data consists of all the wheat futures contracts at MCX for a period of fifty days, beginning 11th April 2011and having expiry month October 2011.

Conclusion

The estimated changes and the actual changes (in percentage relative to the closing price on the first day of the period under consideration) are plotted . It is found that the estimated values agree moderately with the actual values.A MATLAB code has been developed to obtain the solution from the finite difference scheme. Further work will focus on improving the choice of input parameters .

The MATLAB code is given below.

```
function f=price(mean,s,alpha,delta,delx,N1,N2,P1,P2,T,R,xo,N,M)
  K1plus=round((P1/delx)+0.5);
  K2plus=round((delta/delx)-0.5);
  K1min=round((N1/delx)+0.5);
  K2min=round((-alpha/delx)-0.5);
  c=(s^2)/(2*(delx^2));
for n=2:N
    for i=2:M
        b=c+(mean*(cos(2*pi*R*(n-1))/T)/delx);
        a=b+c;
        A(i)=1-a*n/T;
        I=i-1;
        B(I)=b*n/T;
        C(I)=c*n/T;
        IV(i)=xo;
    end
        X=diag(A)+diag(B,1)+diag(C,-1);
        NV=inv(X)*IV';
for i=2:M
    RHS1(i)=NV(i);
        J1=0;
        J2=0;
        for j=K1plus:K2plus
            l=i+j;
            qplus=P2*(l-2);
            if (l <= M)
                 phi1=P1*(P2^P1)/(P2^(P1+1));
                 J1=J1+(phi1*(NV(l)-NV(i)));
            else
                J1=J1+0;
            end
        end
        for k=K1min:K2min
            m=i+k;
            qmin=N2*(m-2);
            if(m <= M)
                phi2=N1*(N2^N1)/(N2^(N1+1));
                J2=J2+(phi2*(NV(m)-NV(i)));
            else
                J2=J2+0;
            end
        end
        RHS2(i)=(n/T)*(J2-J1)*NV(i);
end
price=inv(X)*(RHS2'+RHS1')
end
end
```

The command to execute the code and the input parameters are given below:

```
>> price(0.043819,0.56341,-3,3,1.2 ,0.308,0.016,0.341,0.016,30,5917808.2,0.17628,30,5)
```

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