A Stiffness Index Prediction Approach for 3-RPR Planar Parallel Linkage

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Abstract

This paper presents an analytical approach for calculating the stiffness matrix of parallel manipulators. A way to improve this stiffness along certain paths is important during trajectory tracking. The analytical procedure presented assumes that the stiffness matrix varies along the work-space. The stiffness is directly varies with the Jacobian matrix of the mechanism. Applying the procedure iteratively over the workspace, stiffness maps are also obtained. The methodology is illustrated for a 3-RPR parallel manipulator architecture.

1. Introduction

In recent years, many studies have focused on parallel manipulators. Since their end-effector (moving platform) is sustained by several kinematic chains, they can achieve better structural and dynamic properties with less structural mass. That leads to higher stiffness, where the stiffness can be defined as capacity of a mechanical system to sustain loads without excessive changes in its geometry. The value of the stiffness evolves according to the geometry, the topology of the structure and the position and orientation of the end-effector within its workspace. The stiffness of a parallel robot at a given point of its workspace can be characterized by its stiffness matrix. This matrix combines the forces and moments applied to the end-effector. When inputs of a manipulator are locked, the parallel kinematic machine (PKM) can be considered as a structure and its stiffness is then called static stiffness. Since positioning accuracy is strongly dependent on this stiffness, it is an important characteristic. Such characteristics can be essential in certain applications of PKMs, such as machine tools, micro positioning devices and mechanisms for surgical procedures. Several authors have been studying how to quantify the static stiffness of different parallel robots. In this line, various methodologies were employed to obtain a stiffness matrix which relates an applied external wrench forces to the displacements it produce. Most published studies can be included in the following groups: (a) Jacobian matrix-based methods [1-3], where Jacobian is used to calculate a stiffness matrix and the analysis is carried out for the entire workspace. (b) Matrix product methods (stiffness matrix as the product of several matrices) [4-5] where initially the applied external wrench is related to the local reactions and further local reactions are connected with corresponding deformations and finally other matrices relate these local deformation to the end-effector deformation(c) Structural or Finite Element Methods [6] and (d) Analytical–experimental methods [7], where the experimental results are included in analytical stiffness calculations. Stiffness matrix is often estimated based on the linear approximations. Over the last few years several works reported the importance of stiffness in parallel mechanisms. Pashkevich et al. [8] presented a new stiffness modeling method for over-constrained parallel manipulators with flexible links and compliant actuating joints. Here the approach was implemented for 3-PUU architecture. Li and Gosselin [9] derived the analytical stiffness equation of 3-RPR planar parallel mechanism based on conservative congruence transformation stiffness matrix. Pashkerich et al. [10] presented a methodology to enhance the stiffness analysis of serial and parallel manipulators with passive joints. More recently, Aginaga et al. [11] presented a method of calculating stiffness matrix of a 6-RUS parallel manipulator and employed inverse singularities to enhance the stiffness. In addition, stiffness is taken as an important kinematic metric in several works [12-13] relating to optimum design of parallel linkages.

This paper presents an analytical procedure of computing the stiffness matrix of a planar 3-RPR parallel manipulator, within the workspace. Singularities are identified and an attempt is made to obtain the maximum stiffness poses in the workspace using inverse singular configurations. The paper is organization as follows: section-2 presents the description, kinematics and singularity analysis; section-3 gives the finite element modelling and stiffness evaluation approach. manuscripts must be in English. These guidelines include complete descriptions of the fonts, spacing, and related information for producing your proceedings manuscripts.

2. Kinematic Modelling of Manipulator

The 3-RPR parallel manipulator composes of two triangular (or circular) platforms; one of them is fixed to the ground. One the fixed platform, there are three revolving (R) cylinders in which three moving sliders reciprocate and provide linear actuations for each leg of the linkage. The other end of slider is connected to the corner points of triangular mobile platform with the help of revolute joints (pins). Fig.1 shows a schematic of 3-RPR mechanism.

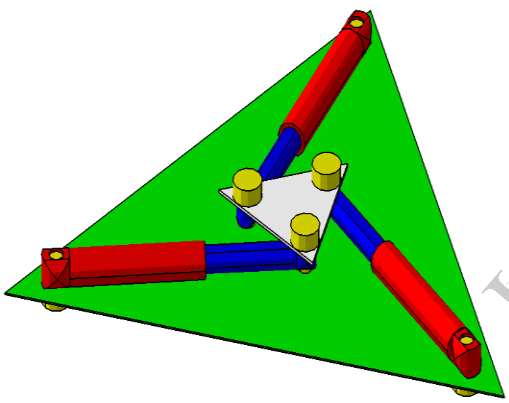

Figure 1. CAD model of 3-RPR linkage

The pose (position of a point C and rotation) of mobile triangular platform is described by two coordinate system as shown in Fig.2. The position of the mobile platform reference point C with respect to fixed frame is represented by {c}=[x y 0]T. The position vector of point B_i (where i=1,2,3) in the fixed and mobile frames are denoted by {bi} and {bi′}. The rotation matrix [R] representing rotation of the platform from X-A_1-Y frame to X′-B_1-Y′ frame is:

$$[R]=\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where ϕ refers to the platform rotation around the axis perpendicular to the plane. The inverse and forward kinematics and Jacobian analysis of a 3-RPR parallel robot were extensively studied in literature [14-15].

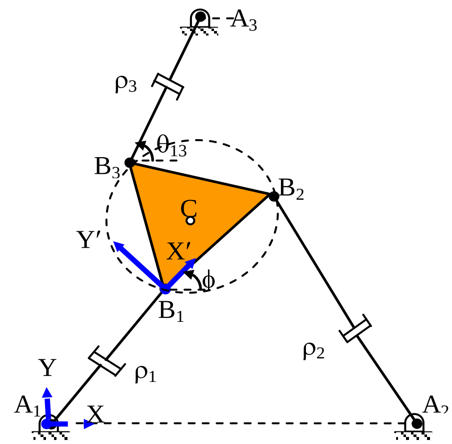

Figure 2. Parameterization of 3-RPR linkage

The position of considered point C of the platform in the fixed and moving frames are respectively $\{c\}=[x\ y]^T$ and $\{c'\}=[x'\ y']^T$ related to each other as:

$$\{c\}=\{c_r\}+[R]\{c'\}$$
$$\text{(or)} \quad \{c_r\}=\{c\}-[R]\{c'\} \quad (2)$$

Hence, the position of B_i (i=1,2,3) in the fixed frame can be expressed as:

$$\{b_i\} = \{c_r\}+[R]\{b_i'\}=\{c\}+[R](\{b_i'\}-\{c'\}) \quad (3)$$

Now the inverse kinematics can be written as distance between points A_i and B_i.
Therefore:

$$\rho_i^2= (\{b_i\}-\{a_i\})^T(\{b_i\}-\{a_i\}) \quad (4)$$

Differentiating this equation with respect to time, one obtains:

$$[J_x]\{v\}=[J_q]\{\dot{\rho}\} \text{ (or) } [J]\{v\}=\{\dot{\rho}\} \quad (5)$$

where $\{\dot{\rho}\}=\begin{bmatrix}\dot{\rho}_1 & \dot{\rho}_2 & \dot{\rho}_3\end{bmatrix}^T$ denotes the velocity of sliders and $\{v\}=\begin{bmatrix}\dot{x} & \dot{y} & \dot{\phi}\end{bmatrix}^T$ is Cartesian velocity vector of the platform point. $[J_x]$ and $[J_\rho]$ are two Jacobian matrices. Workspace is one of the most important factors for designing parallel robots. It is the set of space configuration that the platform point (C) can reach. This space is defined by its limits which are imposed by the joints (active and passive), the length of

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segments and by the internal collisions. The constant orientation workspace of 3-R$\underline{P}$R mechanism is the common area of the vertex spaces generated by each limb. The vertex space of each limb is the ring centered at $\{a_i\}+\{b_i\}$, where $\{a_i\}$ is position vector of base joints A_i, with ρ_{min} and ρ_{max} as the radii for internal circle and external circle respectively. Generally, it is assumed that all the prismatic actuators have identical strokes defined by ρmin and ρmax. Under static condition, the articulated forces $\{\tau\}$ and generalized forces at mobile base (external wrench) $\{F\}$ are related to each other according to the relation:

$$\{\tau\}=[J]^T\{F\} \qquad (6)$$

where $[J]=[J_q]^{-1}[J_x]$. In singular configurations, the performance of the mechanism degenerates and structure may be damaged. Therefore, their determination is primordial. Several methods exist to determine these configurations. From Jacobian matrices, their state can be ascertained. If determinant of matrix $|J_x|=0$, it corresponds to the appearance of uncontrollable mobilities of the mobile platform because it is possible to move it even the actuated (prismatic) joints are locked. Here the manipulator gains one or more degrees of freedom and the stiffness is locally lost. This state is known as direct singular configuration. On the other hand, if $|J_q|=0$, it is not possible to generate some velocities of mobile base in some directions. Such inverse singularities represent the limits of the reachable workspace. In these configurations, manipulator loses one or more degrees of freedom. As it is proved in literature that the equilateral triangular platforms produce the maximal singularity free workspace, such a configuration is only selected in this work. The stiffness value evolves according to the geometry, the topology of the structure and the position and orientation of the mobile platform within the workspace. Stiffness at any given point in workspace can be characterized by its stiffness matrix. This matrix combines the forces and moments applied to the end-effector and can be obtained using kinematic and static equations. If $[k]=\mathrm{diag}(k_i)$, $i=1,2,3$, is diagonal matrix of stiffness with each non-zero diagonal corresponding to stiffness of an actuator, one can express the stiffness matrix of manipulator as:

$$[K]=[J]^T[k][J] \qquad (7)$$

There are many ways for stiffness evaluation, such as the determinant, the condition number, and the eigenvalues of the stiffness matrix. The minimum and maximum values of stiffness can be obtained by calculating the eigenvalues of the matrix [K]. Here, the minimum and maximum eigenvalues are selected. In real application, the minimum eigenvalue is paid more attention, because we always hope that the minimum stiffness over the workspace should be larger than a specified value to ensure the accuracy of the operation everywhere in the workspace. In present work, the stiffness constant k_i for all actuators is set to 10^6N/m.

3. Results and Discussion

Geometric model of the 3-R$\underline{P}$R architecture is developed using a base equilateral triangle of side 160 µm and mobile equilateral triangle of side 30 µm. The maximum and minimum lengths of the sliders are respectively 200 µm and 0. Inverse kinematics problem is solved for all of the possible positions and orientations (poses) of the moving platform of the mechanism to obtain the volume of the workspace. For each pose in the Cartesian space (x,y,ϕ), if the solution of the inverse kinematics verifies the conditions and constraints of all joints, then this point is considered as a valid point and lies within the workspace. Using such an inverse kinematics of the manipulator, the three-dimensional work volume of manipulator is obtained for $\phi\in[-180^o, 180^o]$. Fig.3 shows the work-volume obtained from a program developed in MATLAB for the four inputs entered interactively..

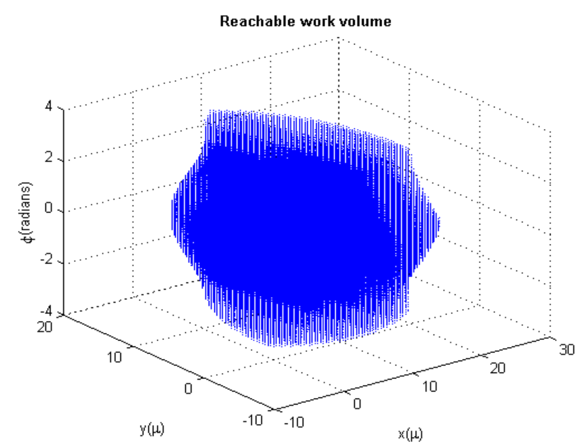


Figure 3. Work-volume of the manipulator

Using a geometric standpoint, kinematic sensitivity is defined as the maximum displacement of the moving platform of the mechanism, under a unit displacement/rotation in the joint space. The corresponding 2-D workspace at different platform orientation angles is also shown in Figures 4 and 5.

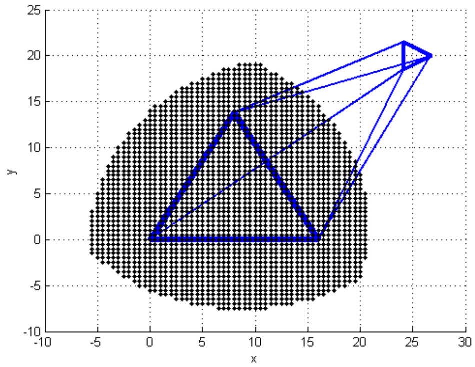


Figure 4. Work-space at ϕ=30^o

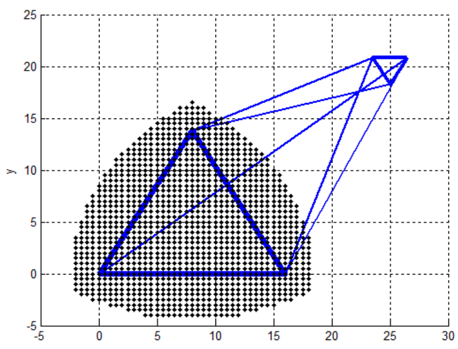
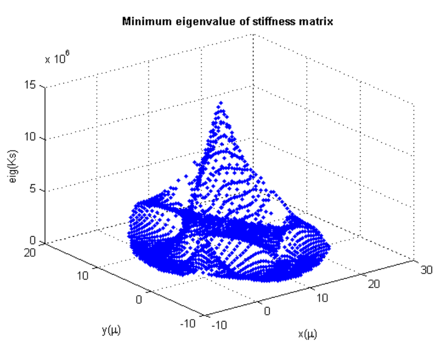
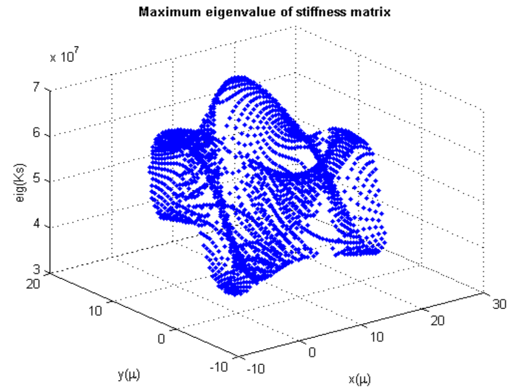


Figure 5. Work-space at ϕ=180^o

Fig.6 shows the maximum and minimum eigenvalues at each pose within the prescribed workspace of manipulators computed at ϕ=30^o and 180^o orientations. It is seen that the minimum stiffness eigenvalue close to a singular configuration is almost equal to zero.



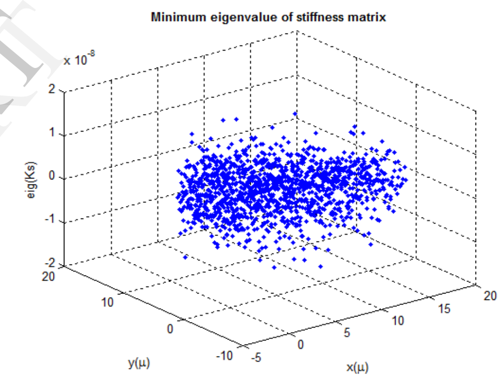
(a) Minimum eigenvalue



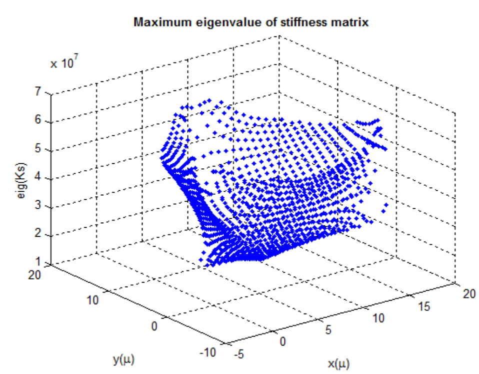
(b) Maximum eigenvalue

Figure 6. Stiffness index at platform orientation ϕ=30^o.

The same data for 180^o pose are shown in Fig.7. It is seen that at 180^o orientation, the platform comes to complete singular configuration as the index is close to zero.



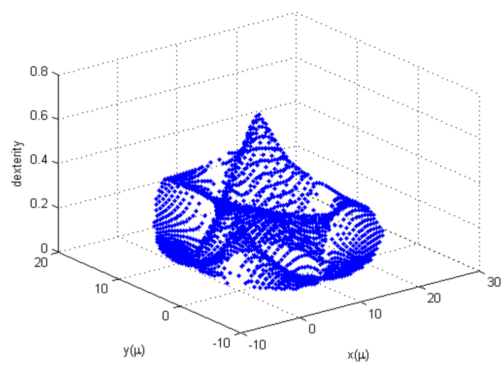
(a) Minimum eigenvalue



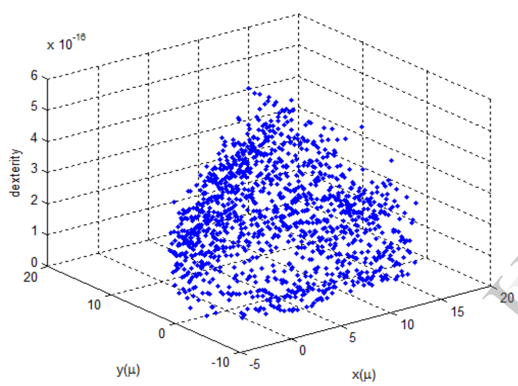
(b) Maximum eigenvalue

Fig. 7. Stiffness index at platform orientation ϕ=180^o.

Fig.8 shows the variation of dexterity index which is ratio of minimum and maximum singular values of Jacobian matrix of manipulator. This is shown for two different platform orientations..



(a) Orientation $\phi=30^o$



(b) At orientation $\phi=180^o$

Fig.8. Dexterity index of non-redundant manipulator

4. Conclusions

In this work, the importance of stiffness index for planar parallel linkage was highlighted. As with other measures, stiffness index is also very important in design of parallel linkage. The methodology was illustrated with a 3-RPR non-redundant parallel mechanism. The minimum eigenvalue selected as a stiffness index in present work runs at-par with the dexterity index indicating the contours of singular points. The work can be extended for a redundant parallel linkage stiffness analysis.

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